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# A special lecture series on **Galaxy Formation**

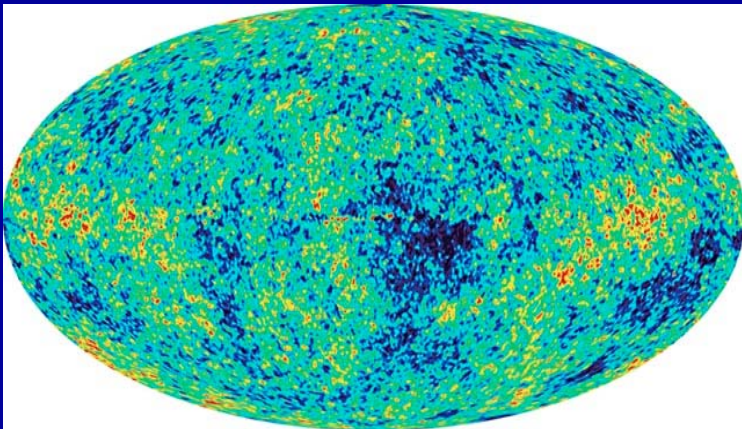
by Avishai Dekel (Chaire Internationale Blaise Pascal)

for graduate students and researchers; IAP/OP Wednesdays 17:00-19:00

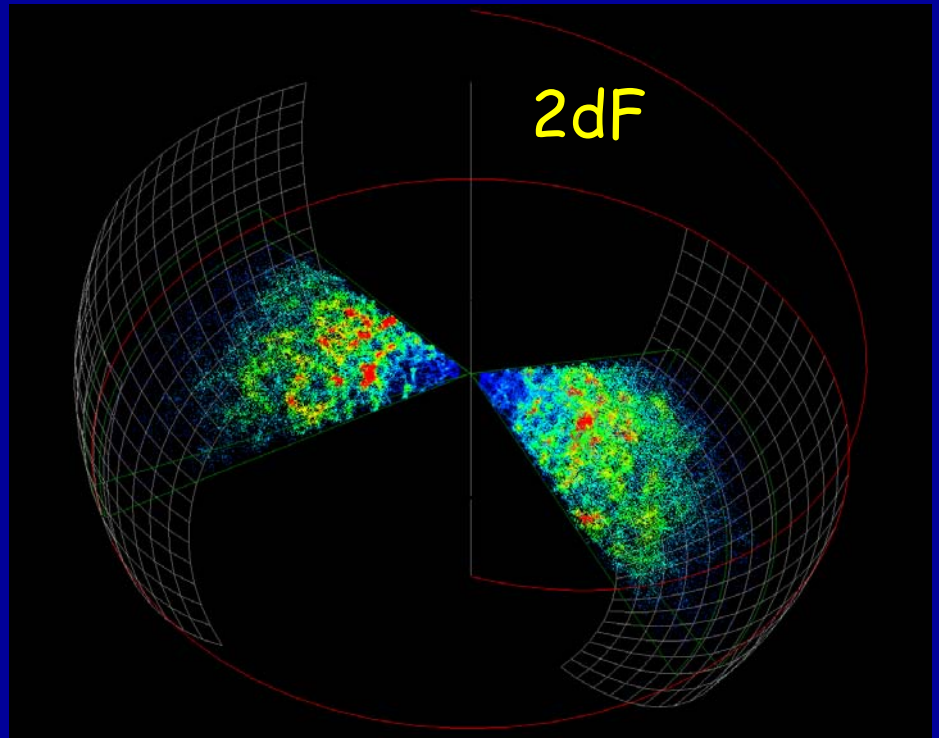
- |             |  |
|-------------|--|
| Octobre 20  | 1. the standard cosmology<br>2. linear growth of fluctuations by gravitational instability                                 |
| Novembre 17 | 3. statistics of density fluctuations: the CDM scenario<br>4. nonlinear growth: spherical model, filamentary structure     |
| Decembre 8  | 5. numerical simulations of structure formation<br>6. hierarchical clustering: Press-Schechter formalism, biasing          |
| Decembre 15 | 7. dark-matter halos: density profile, cusp/core problem<br>8. halo substructure: dynamical friction, tidal effects, HOD   |
| Janvier 5   | 9. angular momentum problem: tidal torques, disk formation<br>10. the origin of galaxy scaling relations and their scatter |
| Janvier 12  | 11. semi-analytic modeling: cooling, star formation, mergers<br>12. feedback processes: supernova, AGN and black holes     |
| Fevrier 9   | 13. cold flows versus shock heating<br>14. origin of bi-modality in galaxies   |
| Fevrier 16  | 15. dwarf galaxies and the "fundamental line"<br>16. dark-dark halos: effect of cosmological photoionization               |

# Lecture 2: Linear Growth of Fluctuations by Gravitational Instability

WMAP



2dF

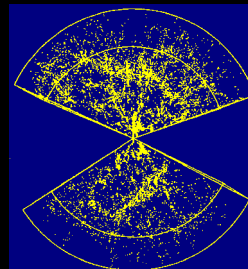


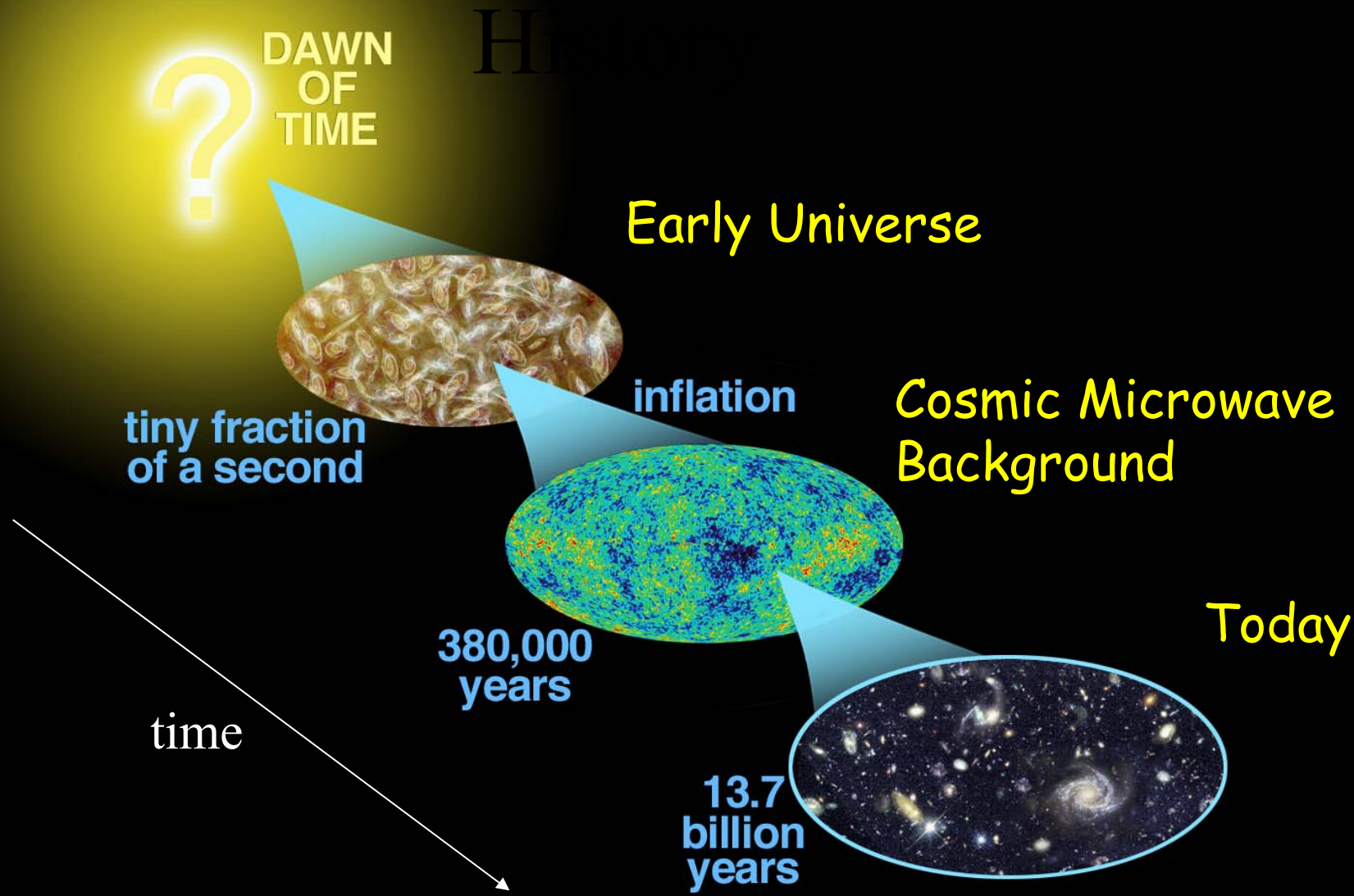
# 2dF Galaxy Redshift Survey

$\frac{1}{4}$  M galaxies 2003

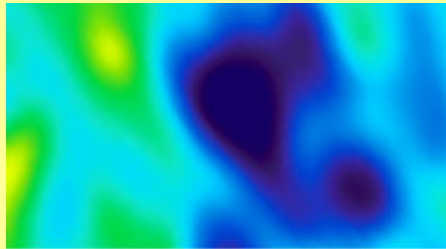
$\frac{1}{4}$  of the horizon

CFA Survey  
1980





# Late Cosmological Epochs



380 kyr  $z \sim 1000$

recombination  
last scattering

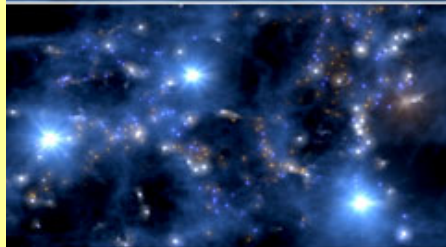


dark ages



180 Myr  $z \sim 20$

first stars  
reionization



galaxy formation

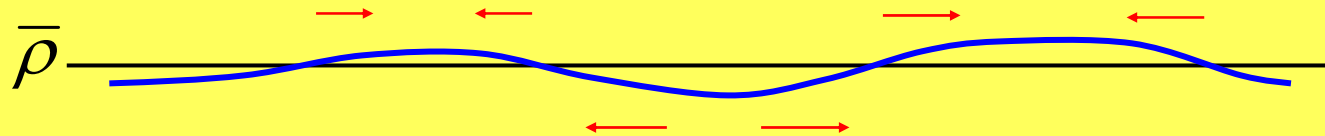


13.7 Gyr  $z = 0$

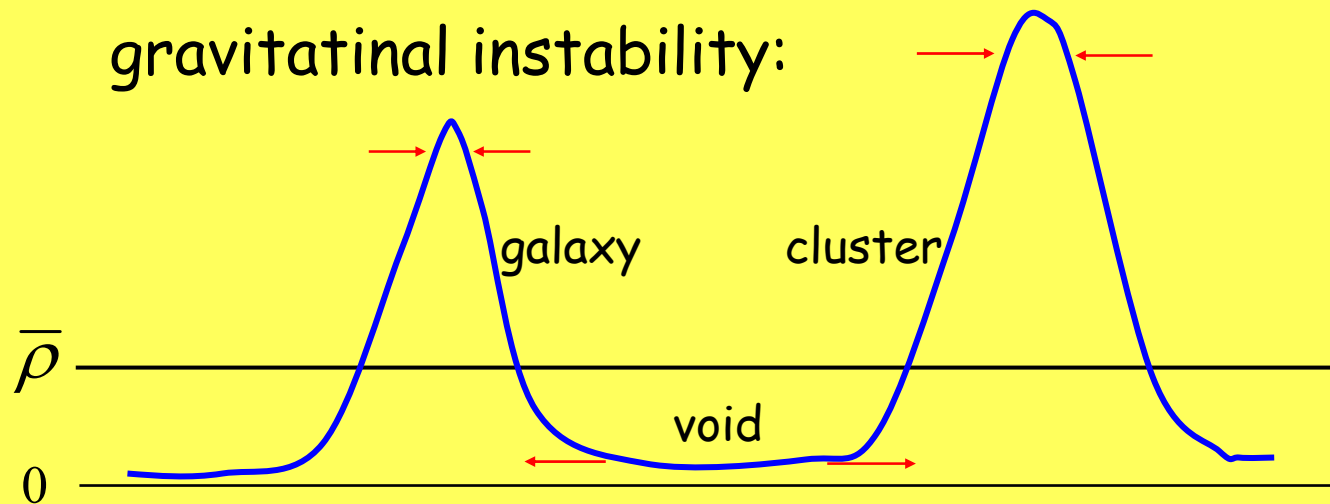
today

# Gravitational instability

small-amplitude fluctuations:



gravitational instability:



# Gravitational Instability: linear, matter-era

Fluid equations :

$$(1) \quad \dot{\rho} + \vec{\nabla} \cdot (\rho \vec{V}) = 0 \quad \text{continuity}$$

$$(2) \quad \dot{\vec{V}} + (\vec{V} \cdot \vec{\nabla}) \vec{V} = -\vec{\nabla} \Phi - \vec{\nabla} P / \rho \quad \text{Euler}$$

$$(3) \quad \nabla^2 \Phi = 4\pi G \rho \quad \text{Poisson}$$

Uniform background :  $\rho(\vec{r}) = \text{const.}$

$$\vec{r} \equiv a\vec{x} \quad \vec{v} = \frac{\dot{a}}{a} \vec{r} \quad \rho^{(1)} = \frac{\rho_0}{a^3} \quad \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3} \rho \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \rho$$

$$H \equiv \dot{a} / a \quad \dot{\rho}^{(1)} = -3\rho H$$

Perturbations :  $\rho(\vec{r}, t) = \rho_u(t)[1 + \delta(\vec{r}, t)] \quad \vec{V} = H(t)\vec{r} + \vec{v} \quad \Phi = \Phi_u + \varphi \quad P = p$

1<sup>st</sup> order + :  $\delta \ll 1$  etc.

$$(1) \quad \dot{\delta} + H\vec{r} \cdot \vec{\nabla} \delta + \vec{\nabla} \cdot \vec{v} + \vec{\nabla} \cdot (\delta \vec{v}) = 0$$

$$(2) \quad \dot{\vec{v}} + H(\vec{r} \cdot \vec{\nabla}) \vec{v} + H\vec{v} + (\vec{v} \cdot \vec{\nabla}) \vec{v} = -\vec{\nabla} \varphi - c_s^2 \vec{\nabla} \delta$$

$$(3) \quad \nabla^2 \varphi = 4\pi G \rho_u \delta$$

$$\vec{\nabla} P = \frac{\partial P}{\partial \rho} \vec{\nabla} \rho \quad c_s^2 \equiv \frac{\partial P}{\partial \rho} (\text{ideal gas}) = \frac{P}{\rho} = \frac{kT}{m_p}$$



Comoving coordinates:  $\vec{x} \equiv \frac{\vec{r}}{a}$      $\frac{\partial}{\partial t}\bigg|_x = \frac{\partial}{\partial t}\bigg|_r + \frac{\dot{a}}{a} \vec{r} \cdot \vec{\nabla}_r \bigg|_t$      $\nabla_x = a \nabla_r$

$\vec{w} \equiv \vec{v} / a$      $\psi \equiv \varphi / a$

(1)  $\dot{\delta} + \vec{\nabla} \cdot \vec{w} + \vec{\nabla} \cdot (\delta \vec{w}) = 0$

(2)  $\dot{\vec{w}} + 2H\vec{w} + (\vec{w} \cdot \vec{\nabla})\vec{w} = -\vec{\nabla} \psi - a^{-1} c_s^2 \vec{\nabla} \delta$

(3)  $\nabla^2 \psi = 4\pi G \rho_u \delta$     [= (3/2)H<sup>2</sup>Ωδ]

$a^{-1} \vec{\nabla} \cdot (\text{eq. 2}) \quad \partial / \partial t (\text{eq. 1}) \quad \rightarrow$

$\ddot{\delta} + 2H\dot{\delta} = 4\pi G \rho_u \delta + a^{-2} c_s^2 \nabla^2 \delta$

gravity

pressure

$\delta(\vec{x}, t) \quad a(t) \quad H(t) = \frac{\dot{a}}{a} \quad \rho_u(t) \propto a^{-3}$

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G\rho_u\delta + a^{-2}c_s^2\nabla^2\delta \quad a(t) \quad H(t) = \frac{\dot{a}}{a} \quad \rho_u(t) \propto a^{-3}$$

Static background :  $\dot{a} = 0 \quad a = \text{const} \equiv 1 \quad \rho_u = \text{const} .$

$$\delta \propto \exp[i(\vec{k} \cdot \vec{x} + \omega t)] \quad \omega^2 = k^2 c_s^2 - 4\pi G\rho_u$$

pressure gravity

Jeans scale :  $k_J = \left( \frac{4\pi G\rho_u}{c_s^2} \right)^{1/2} \quad \lambda_J \equiv \frac{2\pi}{k_J} \quad M_J \equiv \frac{4\pi}{3} \rho_m \left( \frac{\pi c_s^2}{G\rho} \right)^{3/2} \propto \frac{T^{3/2}}{\rho^{1/2}}$

$$\lambda \gg \lambda_J \quad (p=0) \quad \rightarrow \delta = Ae^{\omega t} + Be^{-\omega t}$$

$$\lambda \ll \lambda_J \quad \rightarrow \text{stable oscillations}$$

Expanding background ,  $\lambda \gg \lambda_J$  :

$$k = 0 \quad \rightarrow \quad a \propto t^{2/3} \quad \rightarrow \quad \ddot{\delta} + \frac{4}{3t}\dot{\delta} = \frac{2}{3t^2}\delta \quad \rightarrow \quad \delta = At^{2/3} + Bt^{-1}$$

$$k = -1 \quad \rightarrow \quad a \propto t \quad \rightarrow \quad \ddot{\delta} + \frac{2}{t}\dot{\delta} = \frac{3\Omega_0 t_0}{2t^3}\delta \quad \rightarrow \quad \delta = \text{const} . \text{ freezout}$$

## Properties of the linear growing mode:

linear  $\ddot{\delta} + 2H\dot{\delta} = (3/2)H^2\Omega\delta$   $H(t)$   $\Omega(t)$

growing mode:  $\delta \propto D(t)$

$$f(\Omega) \equiv \frac{\dot{D}}{HD} \approx \Omega^{0.6} \quad \rightarrow \quad \frac{\ddot{D}}{D} = H^2 \left( -2f + \frac{3}{2}\Omega \right)$$

continuity  $\rightarrow \delta = -\frac{1}{Hf} \vec{\nabla} \cdot \vec{v}$

Poisson  $\rightarrow \vec{v} = -\vec{\nabla} \varphi_v$  irrotational  $\varphi = \frac{3H\Omega}{2f} \varphi_v$

## The Jeans scale in an expanding universe:

In  $k$ -space:  $\delta = \sum_{\vec{k}} \delta_{\vec{k}}(t) e^{i\vec{k} \cdot \vec{x}}$   $r = ax$

for each  $\vec{k}$ :  $\ddot{\delta}_k + 2H\dot{\delta}_k = (4\pi G\rho - k^2 c_s^2) \delta_k \rightarrow$  same Jeans scale

# Lecture 3

## Statistics of Fluctuations:

## The Cold Dark Matter Scenario

# The Initial Fluctuations

At Inflation: Gaussian, adiabatic

fluctuation field  $\delta(x) = \frac{\rho(x) - \langle \rho \rangle}{\langle \rho \rangle}$  a realization of an ensemble  
ensemble average  $\sim$  volume average

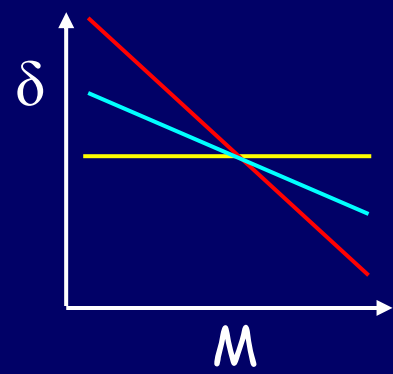
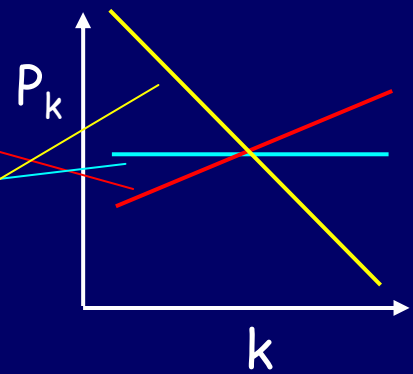
Fourier  $\delta(\vec{x}) = \sum_{\vec{k}} \delta_{\vec{k}} e^{i\vec{k} \cdot \vec{x}}$  Power Spectrum  $P(k) \equiv \langle |\tilde{\delta}(\vec{k})|^2 \rangle \propto k^n$

rms  $\langle \delta^2 \rangle_\lambda \sim \langle \int_{k=0}^K \int_{k'=0}^{K-2\pi/\lambda} \exp[-i(k+k') \cdot x] d^3k' d^3k \delta_k \delta_{k'} \rangle \sim \int_{k=0}^K d^3k \langle \delta_k \delta_{-k} \rangle$   
 $\longleftarrow \delta_{Dirac}(k+k') \longrightarrow$

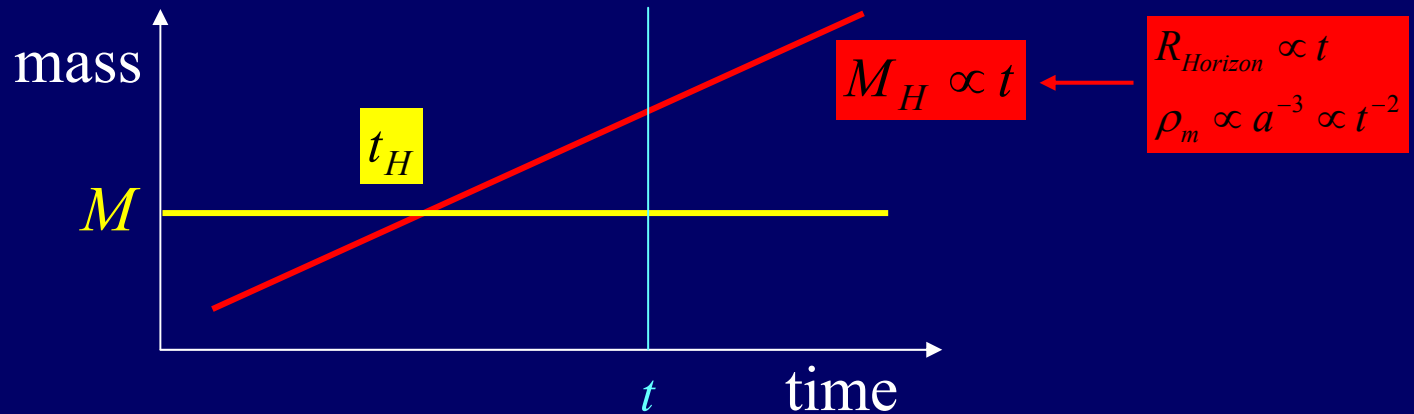
$$\langle \delta_k \delta_{-k} \rangle = \langle |\delta_k|^2 \rangle$$

$$\langle \delta^2 \rangle_\lambda \propto \int_{k=0}^{2\pi/\lambda} P_k d^3k \propto M^{-(n+3)/3}$$

- $n = 1 \quad \delta \propto M^{-2/3}$
- $n = 0 \quad \delta \propto M^{-1/2}$
- $n = -3 \quad \delta \propto const.$

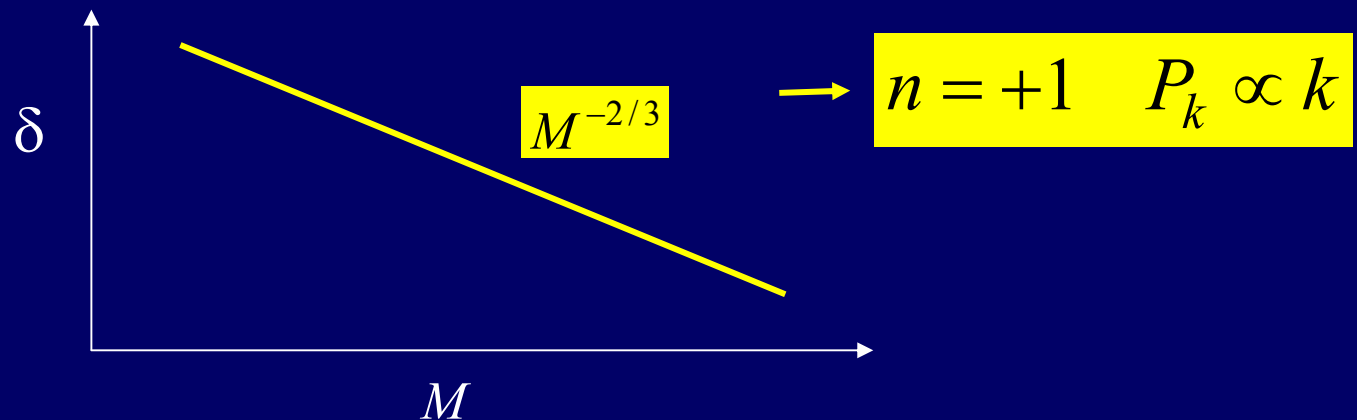


# Scale-Invariant Spectrum (Harrison-Zel'dovich)



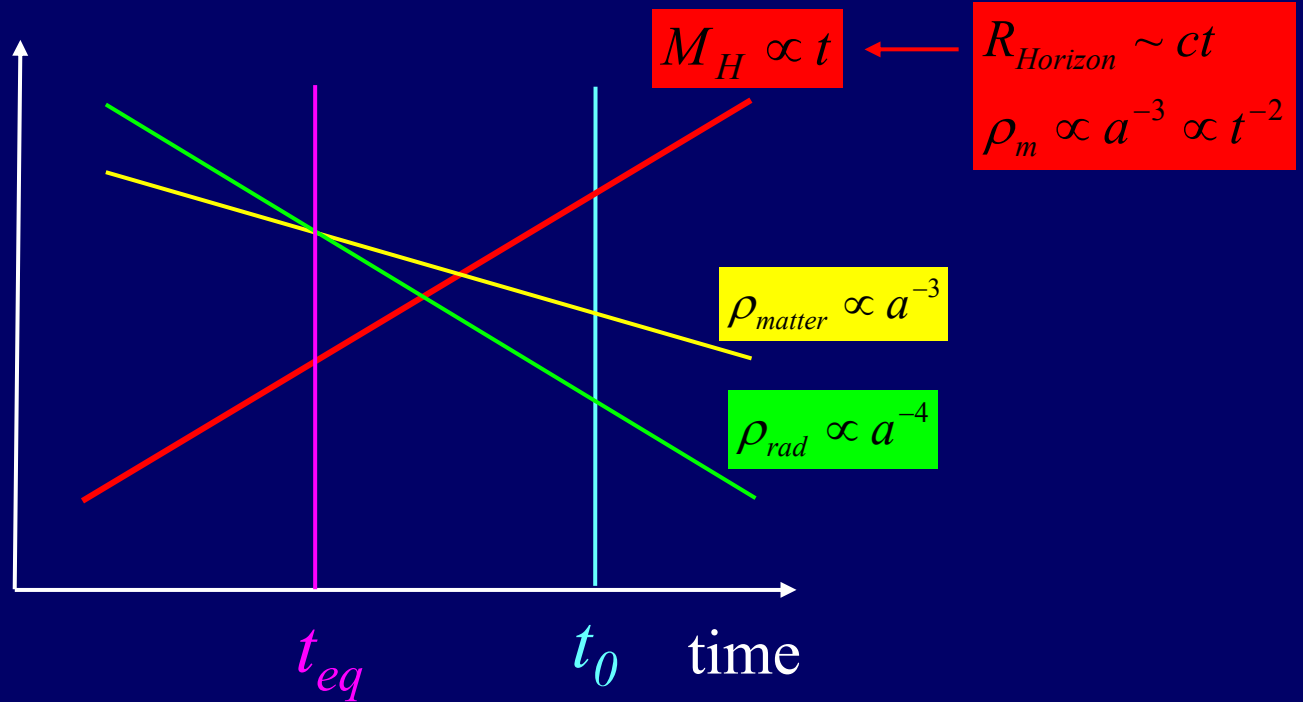
$$\delta(M, t) = \delta_H \left( \frac{t}{t_H(M)} \right)^{2/3} \propto M^{-2/3} t^{2/3}$$

$\delta_H = \text{const.}$



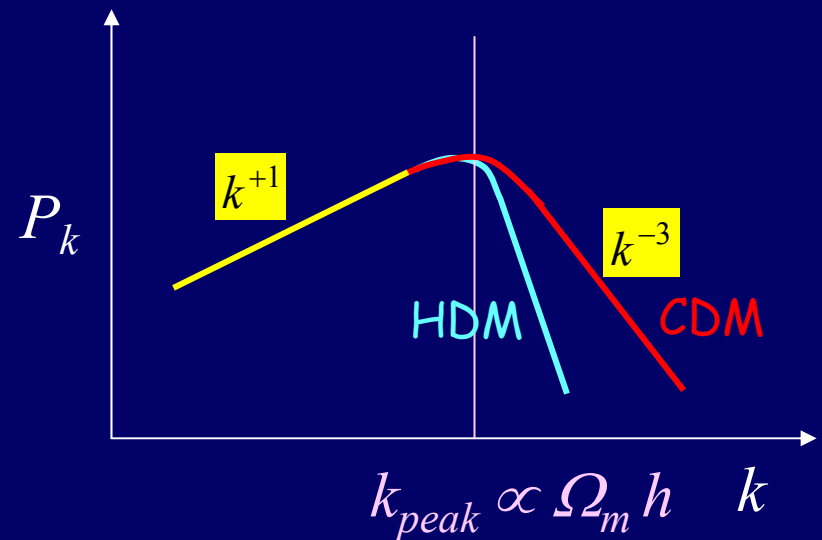
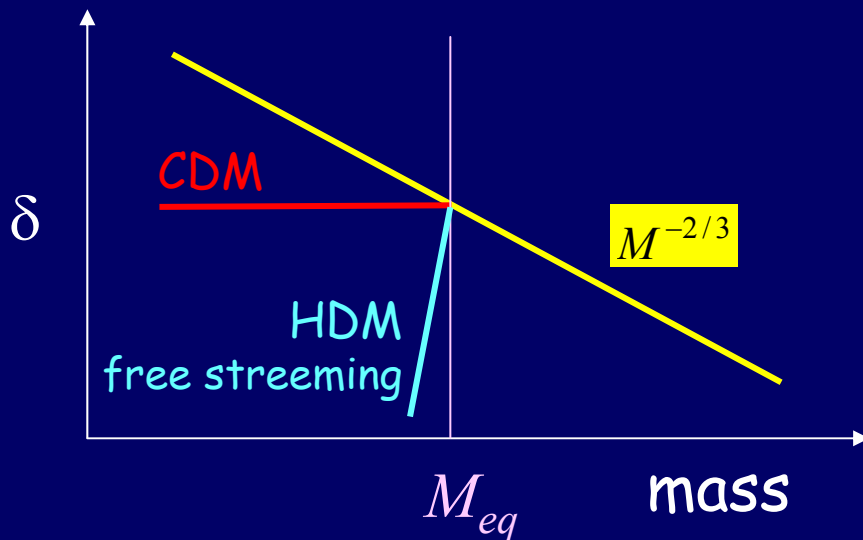
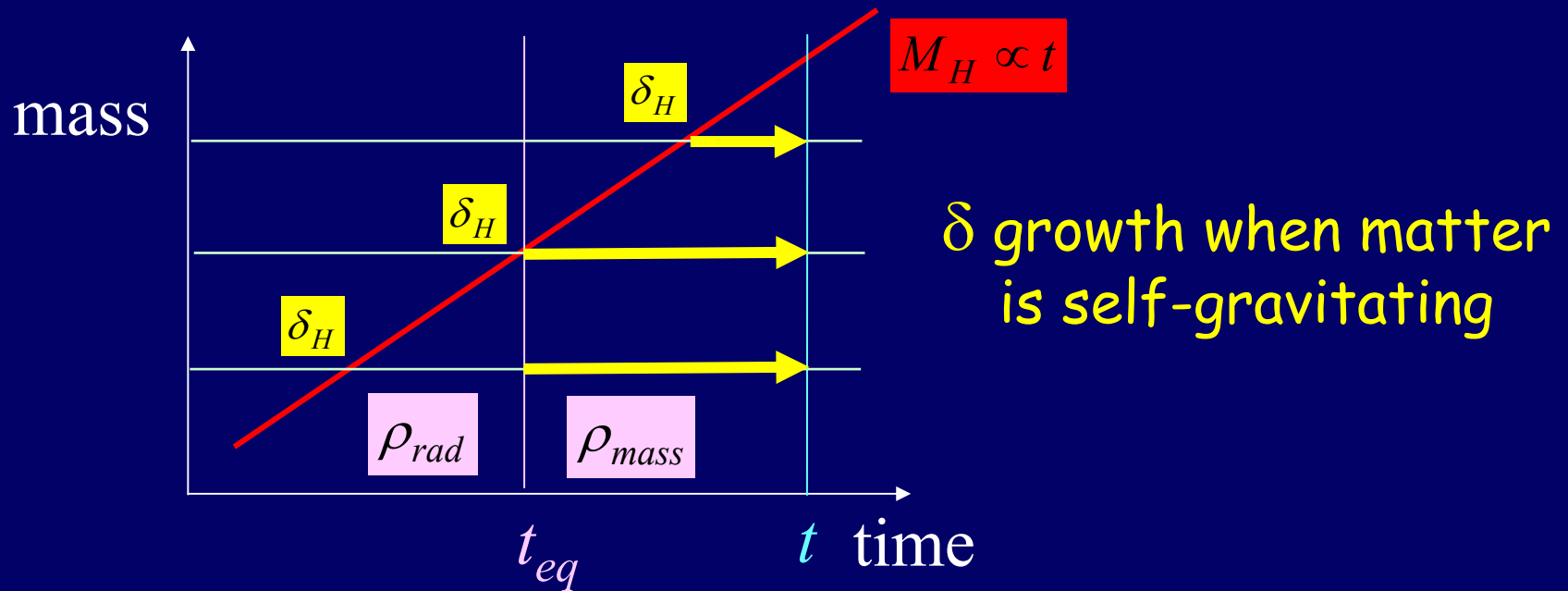
# Cosmological Scales

mass



$$z_{\text{eq}} \sim 10^4$$

# CDM Power Spectrum





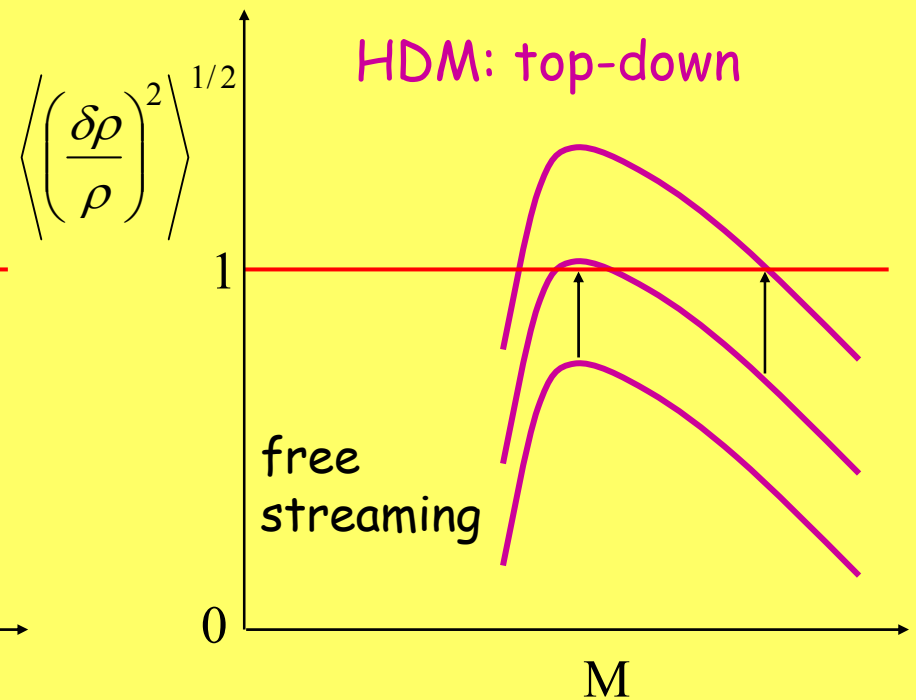
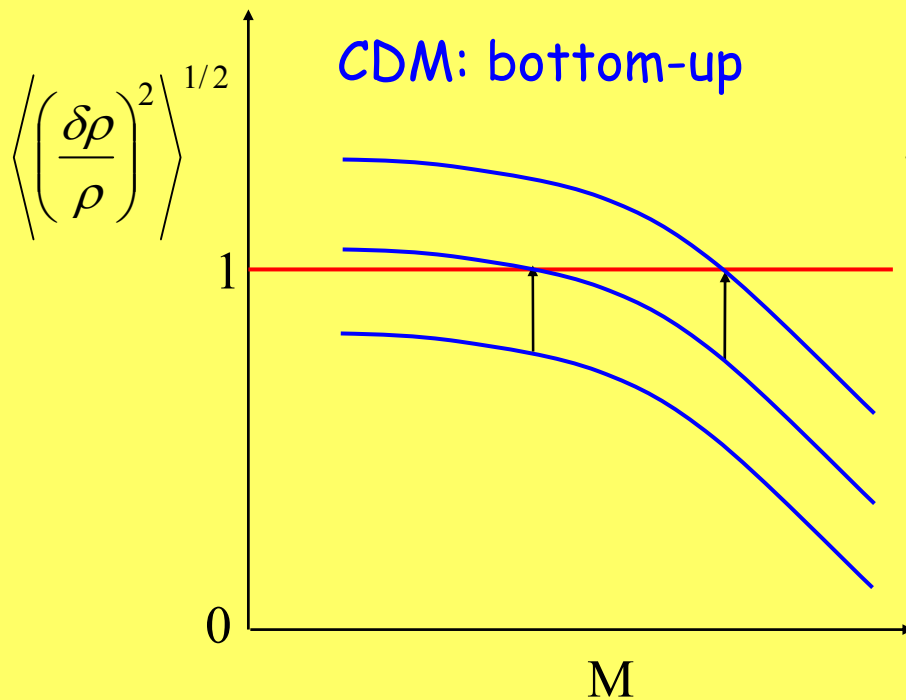
# Formation of Large-Scale Structure

Fluctuation growth in the linear regime:  $\delta \ll 1 \rightarrow \delta \propto a \propto t^{2/3}$

rms fluctuation at mass scale  $M$ :  $\delta \propto M^{-\alpha} \quad 0 < \alpha = (n+3)/6 \leq 2/3$

Typical objects forming at  $t$ :  $1 \sim \delta \propto M^{-\alpha} a \rightarrow M_* \propto a^{1/\alpha}$

example  $n = -2 \rightarrow M_* \propto a^6$



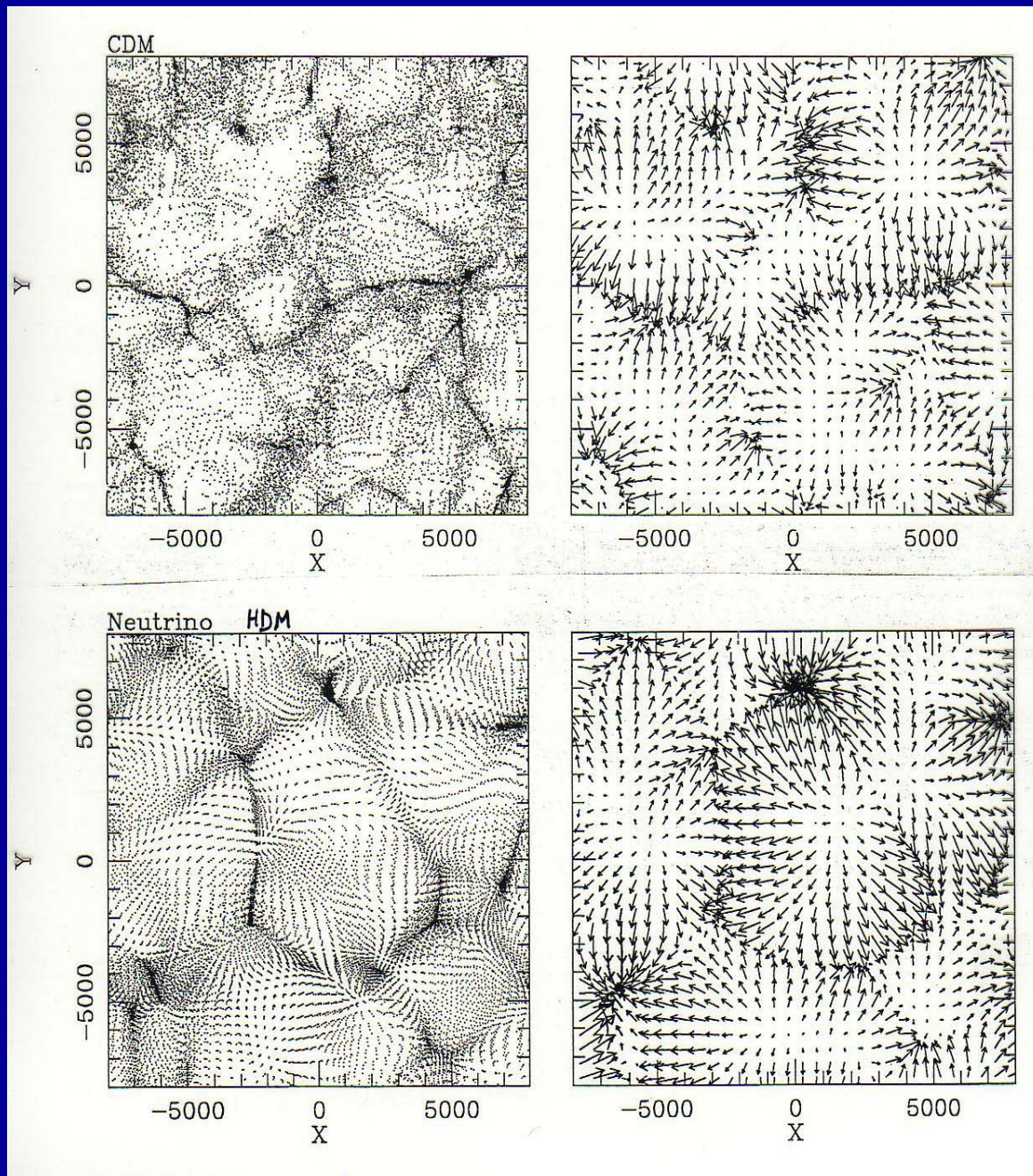
# Micro-Macro Connection

Cold Dark Matter

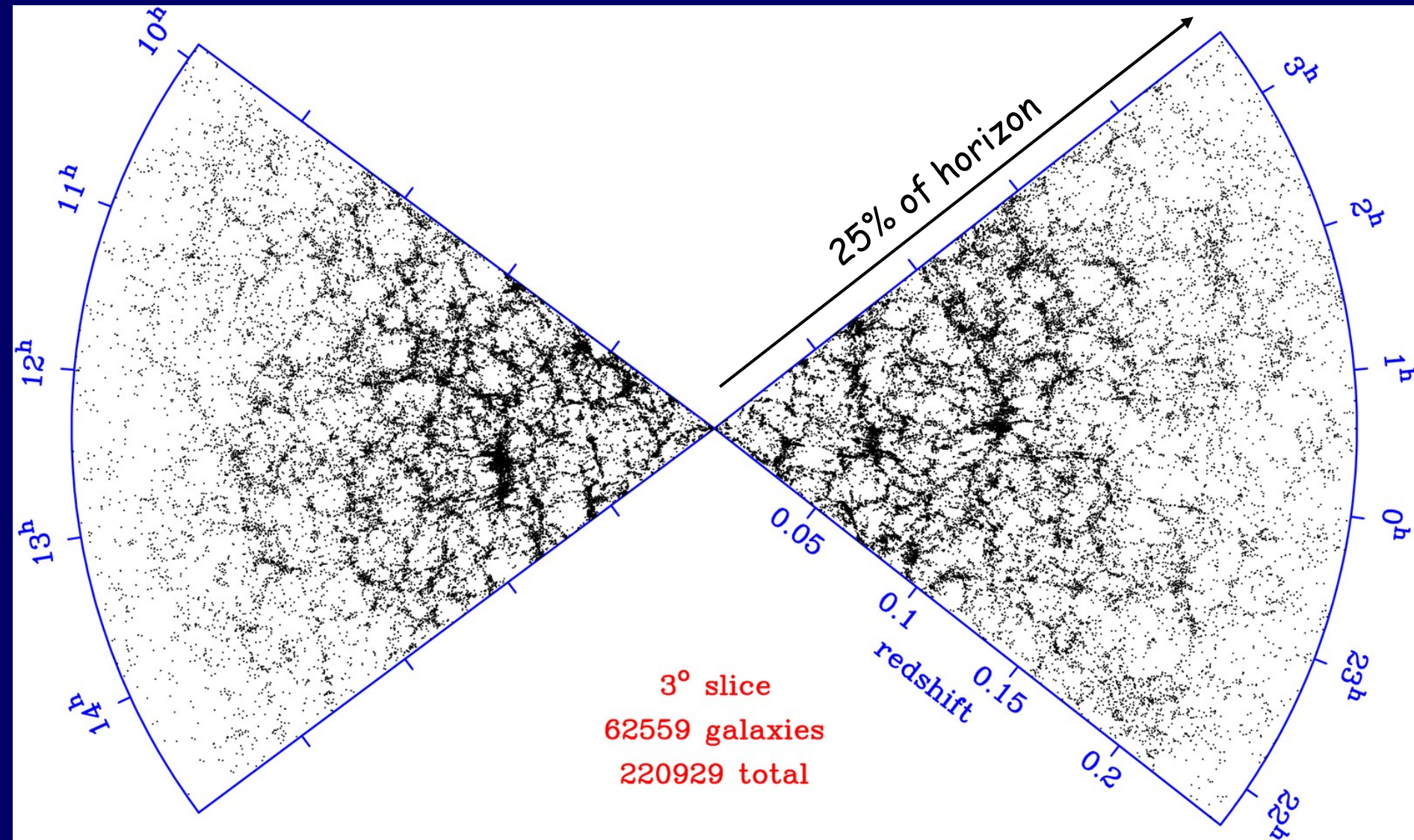


Hot Dark Matter

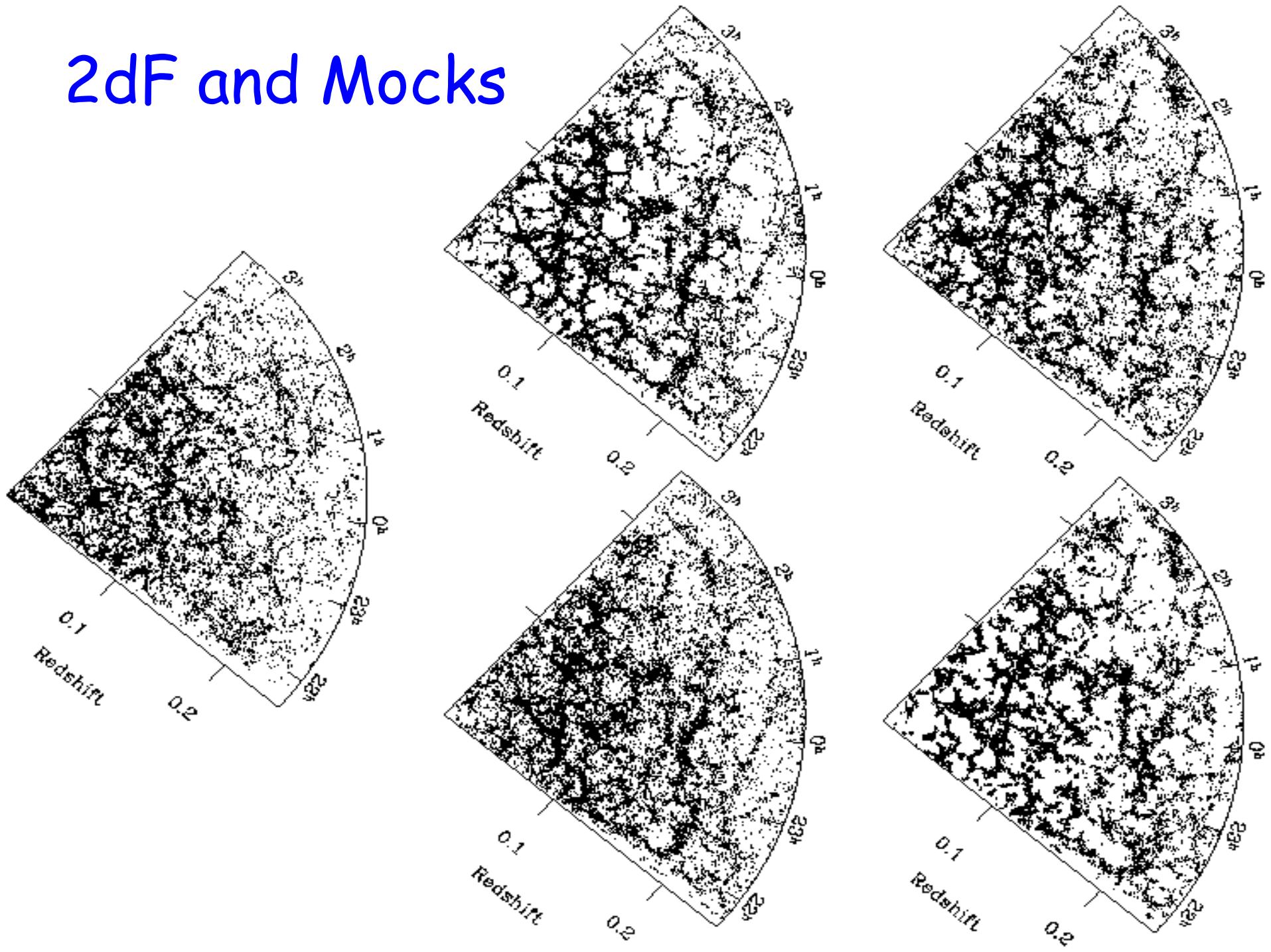
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# 2dF redshift survey



# 2dF and Mocks



# Power Spectrum

