A special lecture series on Galaxy Formation

by Avishai Dekel (Chaire Internationale Blaise Pascal)

for graduate students and researchers; IAP/OP Wednesdays 17:00-19:00

Octobre 20	1. the standard cosmology
	2. linear growth of fluctuations by gravitational instability
Novembre 17	3. statistics of density fluctuations: the CDM scenario
	4. nonlinear growth: spherical model, filamentary structure
Decembre 8	5. numerical simulations of structure formation
	6. hierarchical clustering: Press-Schechter formalism, biasing
Decembre 15	7. dark-matter halos: density profile, cusp/core problem
	8. halo substructure: dynamical friction, tidal effects, HOD
Janvier 5	9. angular momentum problem: tidal torques, disk formation
	10. the origin of galaxy scaling relations and their scatter
Janvier 12	11. semi-analytic modeling: cooling, star formartion, mergers
	12. feedback processes: supernova, AGN and black holes
Fevrier 9	13. cold flows versus shock heating
	14. origin of bi-modality in galaxies
Fevrier 16	15. dwarf galaxies and the “fundamental line”
	16. dark-dark halos: effect of cosmological photoionization

ΛCDM Power Spectrum

$$P(k) \propto k\,T^2(k)$$

$$T(k) = \frac{\ln(1+2.34q)}{2.34q}\left(1+3.89q+(16.1q)^2+(5.46q)^3+(6.71q)^4\right)^{-1/4} \qquad q = \frac{k}{\Omega_m h^2 Mpc^{-1}}$$

normalization: $\sigma_8 \equiv \sigma_{tophat}(R = 8h^{-1}Mpc)$

Lecture 4

Non-linear Growth of Structure

Spherical Collapse,

Virial Theorem,

Zel'dovich Approximation,

N-body Simulations

Filamentary Structure: Zel'dovich Approximation

Approximate the displacement from initial position

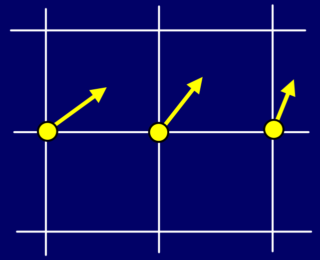
$$x(q,t) = q + D(t)\,\psi(q), \quad \psi = -\nabla\phi$$

Velocity & acceleration along displacement → trajectories straight lines

$$\dot{x} = \dot{D}\psi, \quad \ddot{x} = \ddot{D}\psi \propto \dot{x}$$

as in linear

central force → potential flow



In physical coordinates

$$r = ax, \quad v = \dot{r} = \dot{a}x + a\dot{x} = Hr + v_{pec}$$

Density (Lagrangian):

continuity $\rho(x,t)\,d^3x = \rho_q d^3q$

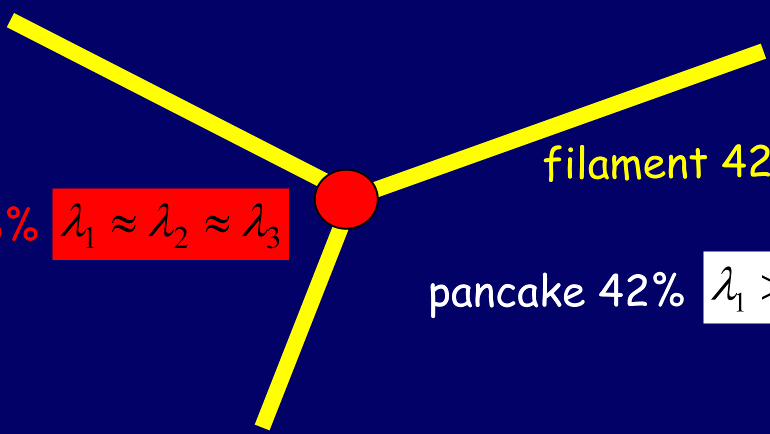
$$\rightarrow \rho(x,t) = \frac{\rho_q}{\| \partial\vec{x} / \partial\vec{q} \|} = \frac{\rho_q}{(1-D(t)\lambda_1)(1-D(t)\lambda_2)(1-D(t)\lambda_3)}$$

$$\lambda_i \equiv \frac{\partial^2\phi}{\partial^2 q_i}, \quad \lambda_1 \geq \lambda_2 \geq \lambda_3$$

Jacobian

→ caustics

deformation tensor eigenvalues



filament 42% $\lambda_1 \approx \lambda_2 >> \lambda_3$

cluster 8% $\lambda_1 \approx \lambda_2 \approx \lambda_3$

pancake 42% $\lambda_1 >> \lambda_2 \approx \lambda_3$

Zel'dovich Approximation cont'd

$$\rho(x,t) = \frac{\rho_q}{(1-D(t)\lambda_1)(1-D(t)\lambda_2)(1-D(t)\lambda_3)}$$

$$\lambda_i \equiv \frac{\partial^2 \phi}{\partial^2 q_i}, \quad \lambda_1 \geq \lambda_2 \geq \lambda_3$$

$$\delta = \frac{\rho}{\rho_q} - 1 = -D(\lambda_1 + \lambda_2 + \lambda_3) + D^2(\lambda_1\lambda_2 + \ldots) + D^3(\lambda_1\lambda_2\lambda_3) + \ldots$$

linear $$\delta = -D(\lambda_1 + \lambda_2 + \lambda_3) = -D\,\nabla\cdot\psi = -D\,\nabla\cdot\frac{\dot{x}}{\dot{D}} = -\frac{D}{\dot{D}}\nabla\cdot v = -\frac{1}{Hf(\Omega)}\nabla\cdot v$$

→ D is the growing mode of GI obeying $\ddot{D} + 2H\dot{D} = 4\pi G\rho D$

Error:

plug density in Poisson eq. $\delta_{Poisson} \propto \nabla^2\phi_{grav} \propto -\nabla\psi = -(\lambda_1 + \lambda_2 + \lambda_3) \propto \delta_{linear}$

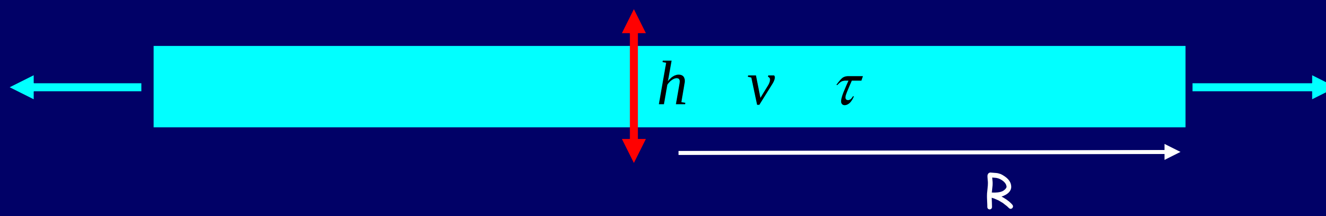
→ error is 2nd +3rd terms $$\frac{\Delta\rho}{\rho} = -(D\lambda_1)^2\left(\frac{\lambda_2}{\lambda_1} + \frac{\lambda_3}{\lambda_1} + \frac{\lambda_2\lambda_3}{\lambda_1^{\,2}}\right) + 2(D\lambda_1)^3\frac{\lambda_2\lambda_3}{\lambda_1^{\,2}}$$

error small in linear regime $D\lambda_1 << 1$

or pancakes $\lambda_1 >> \lambda_2, \lambda_3$

error big in spherical collapse $\lambda_1 \sim \lambda_2 \sim \lambda_3$

Non-dissipative Pancakes: why flat?



oscilation time << expansion time $\tau << H^{-1}$

adiabatic invariant $const. \sim \int_0^{\tau} \dot{x}^2 dt \sim v^2 \tau \sim h\, v$

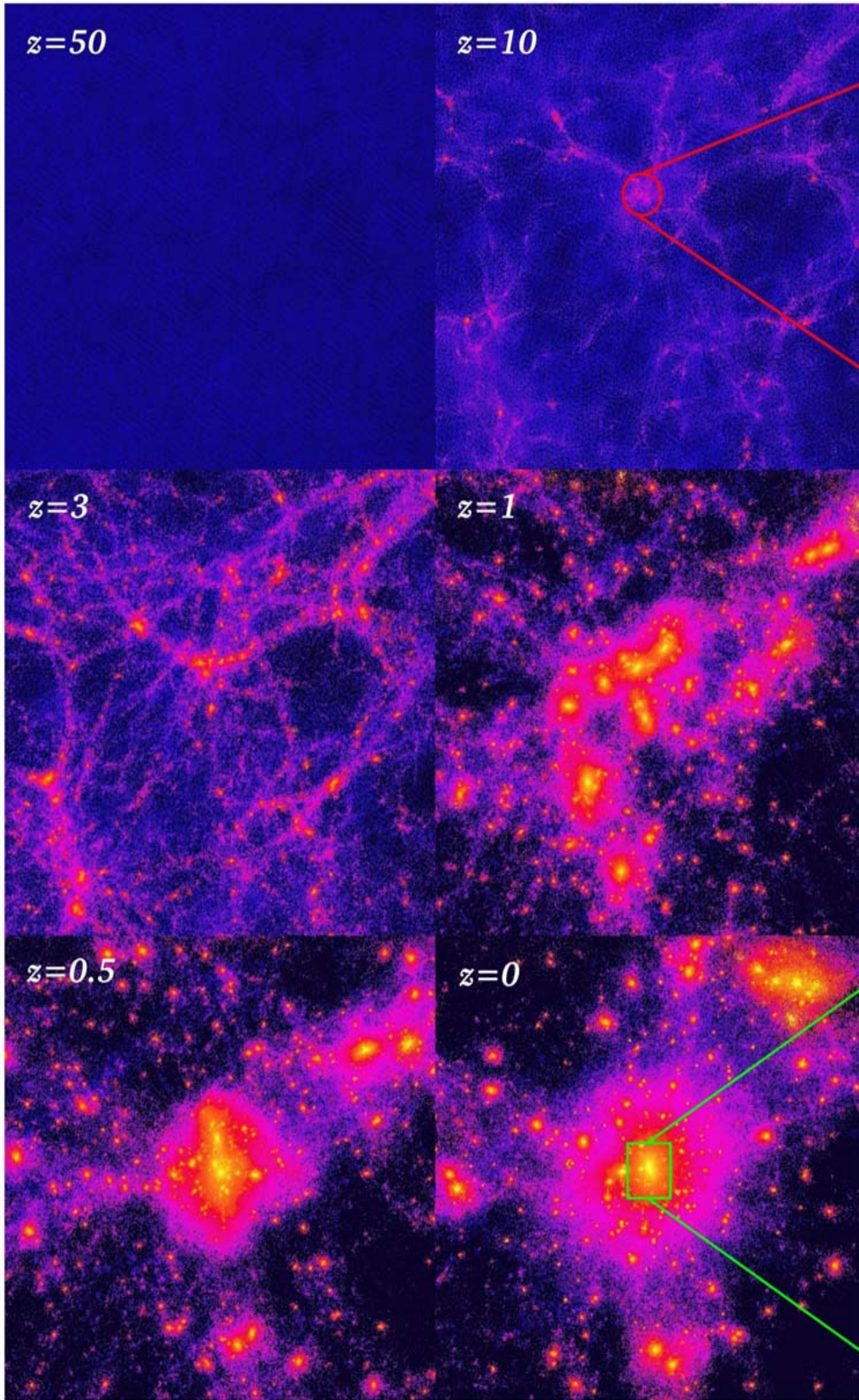
$$v \sim f\tau \propto f v^{-2} \rightarrow v \propto f^{1/3} \propto R^{-2/3}$$

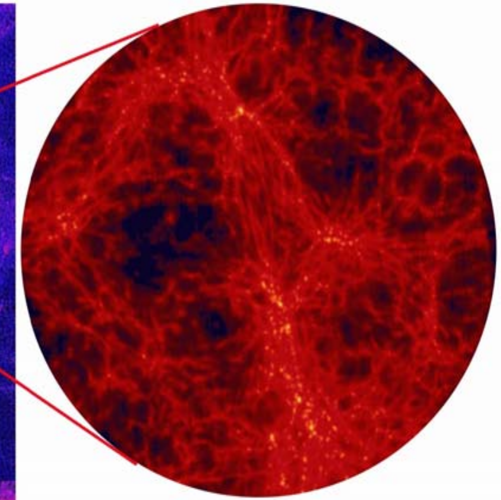
$$f \propto \Sigma \propto R^{-2}$$

$$h \propto v^{-1} \propto R^{2/3}$$

$$\frac{h}{R} \propto R^{-1/3} \propto a^{-1/3}$$

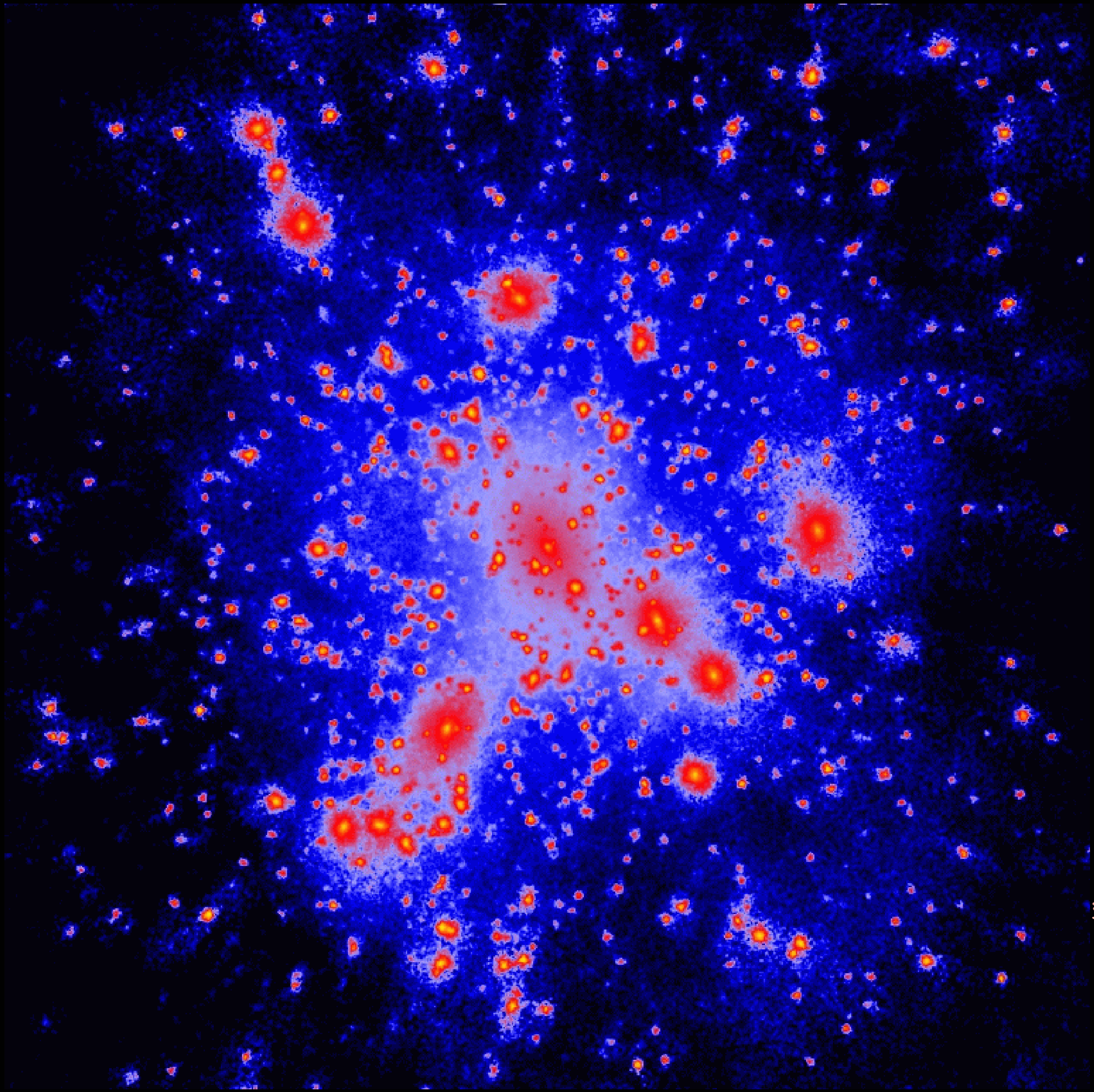
pancake becomes flatter in time





spherical collapse
or mergers

N-body simulation of Halo Formation



Top-Hat Model (Λ=0, matter era)

a bound sphere (k=1) in EdS universe (k=0)

$$\dot{a}^2 = \frac{2a^*}{a} - k \qquad a^* \equiv (4\pi/3)G\rho a^3 = const.$$

conformal time

$$d\eta \equiv \frac{dt}{a(t)}$$

$$a = (a^*/2)\eta^2 \qquad t = (a^*/6)\eta^3$$

$$a_p = a_p^*(1-\cos\eta_p) \quad t = a_p^*(\eta_p - \sin\eta_p)$$

$$t_p = t \;\rightarrow \eta^3(\eta_p) = \frac{6a_p^*}{a^*}(\eta_p - \sin\eta_p) \rightarrow a(\eta_p) = \frac{1}{2}\left(\frac{6a_p^*}{a^*}(\eta_p - \sin\eta_p)\right)^{2/3}$$

overdensity:

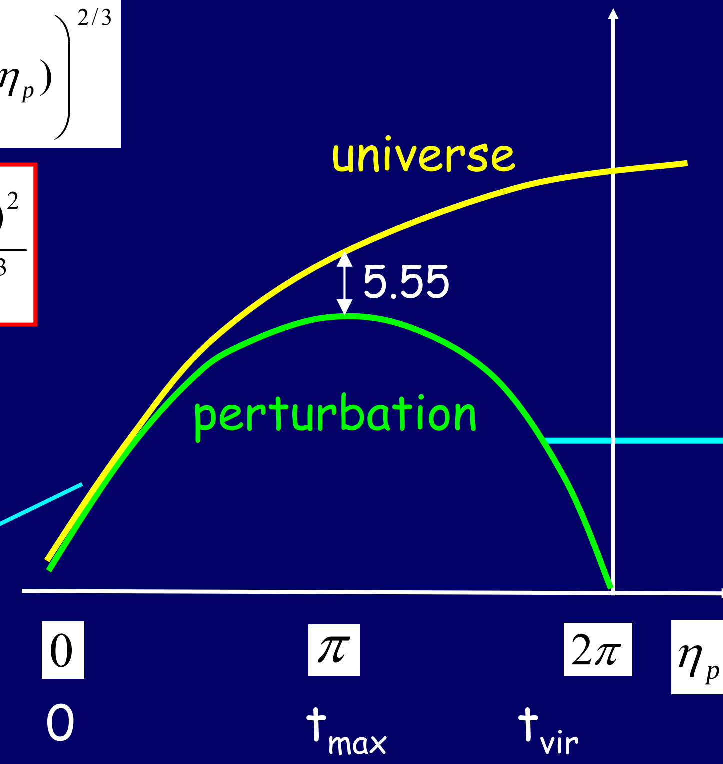
$$a^* \propto \rho a^3 \qquad a_p^* \propto \rho_p a_p^3 \;\rightarrow\; \frac{\rho_p}{\rho} = \frac{a_p^*}{a^*}\left(\frac{a}{a_p}\right)^3 = \frac{9(\eta_p - \sin\eta_p)^2}{2(1-\cos\eta_p)^3}$$

linear perturbation $\delta\rho/\rho << 1 \quad \eta_p << 1$

Taylor $\cos\eta \approx 1 - \frac{1}{2}\eta^2 + \frac{1}{24}\eta^4 \quad \sin\eta \approx \eta - \frac{1}{6}\eta^3 + \frac{1}{120}\eta^5$

$$\frac{\delta\rho}{\rho} \equiv \frac{\rho_p - \rho}{\rho} \approx 0.15\eta_p^2 \propto a \propto t^{2/3}$$

$$\delta \propto a$$



turnaround $\eta_p = \pi$

$$\frac{\rho_p}{\rho} = \frac{9\pi^2}{16} \approx 5.55$$

linear equivalent to collapse

$$\delta_{2\pi} = \delta(\eta_p << 1)\left(\frac{t(\eta_p = 2\pi)}{t(\eta_p << 1)}\right)^{2/3} = 0.15\eta_p^2\left(\frac{2\pi}{\eta_p^3/6}\right)^{2/3} \approx 1.68 \equiv \delta_c$$

- **Collapse to Virial Equilibrium**

$$E_{max} \simeq -\frac{GM^2}{R_{max}} \; (E_k \simeq 0) \qquad E_{vir} \simeq \frac{1}{2}E_{grav} \simeq -\frac{1}{2}\frac{GM^2}{R_{vir}}$$

$$\text{E conserved} \quad \rightarrow \quad \boxed{\frac{R_{vir}}{R_{max}} \simeq \frac{1}{2}} \quad \rightarrow \quad \frac{\rho_{vir}}{\rho_{max}} \simeq 8$$

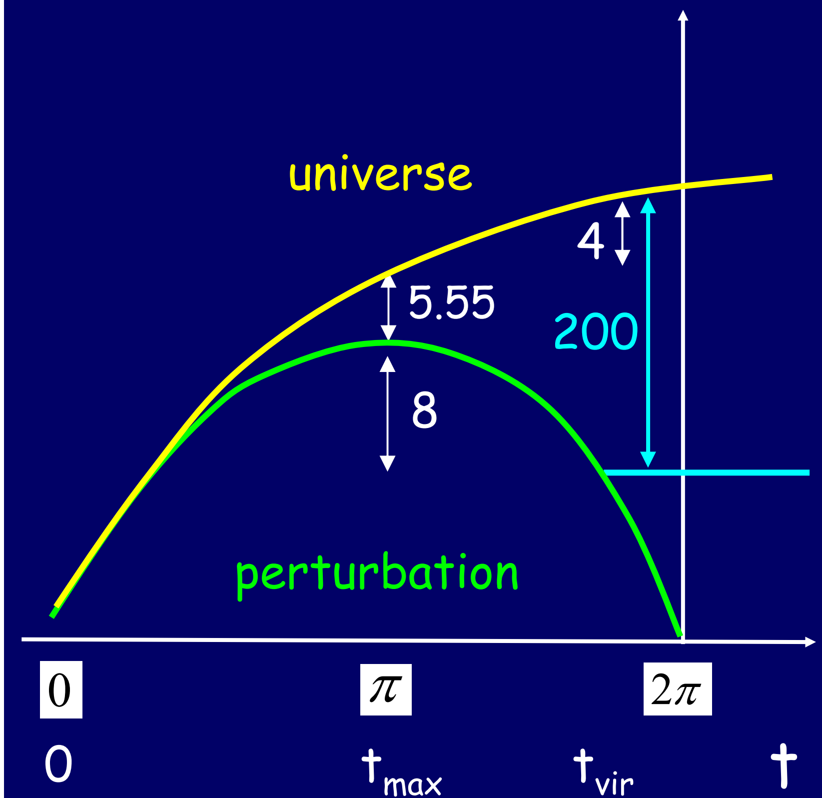
- **Virial density:**

$$\frac{\rho_{vir}}{\rho_{univ}} \simeq 5.55 \times 8 \times \left(\frac{a_{vir}}{a_{max}}\right)^3$$

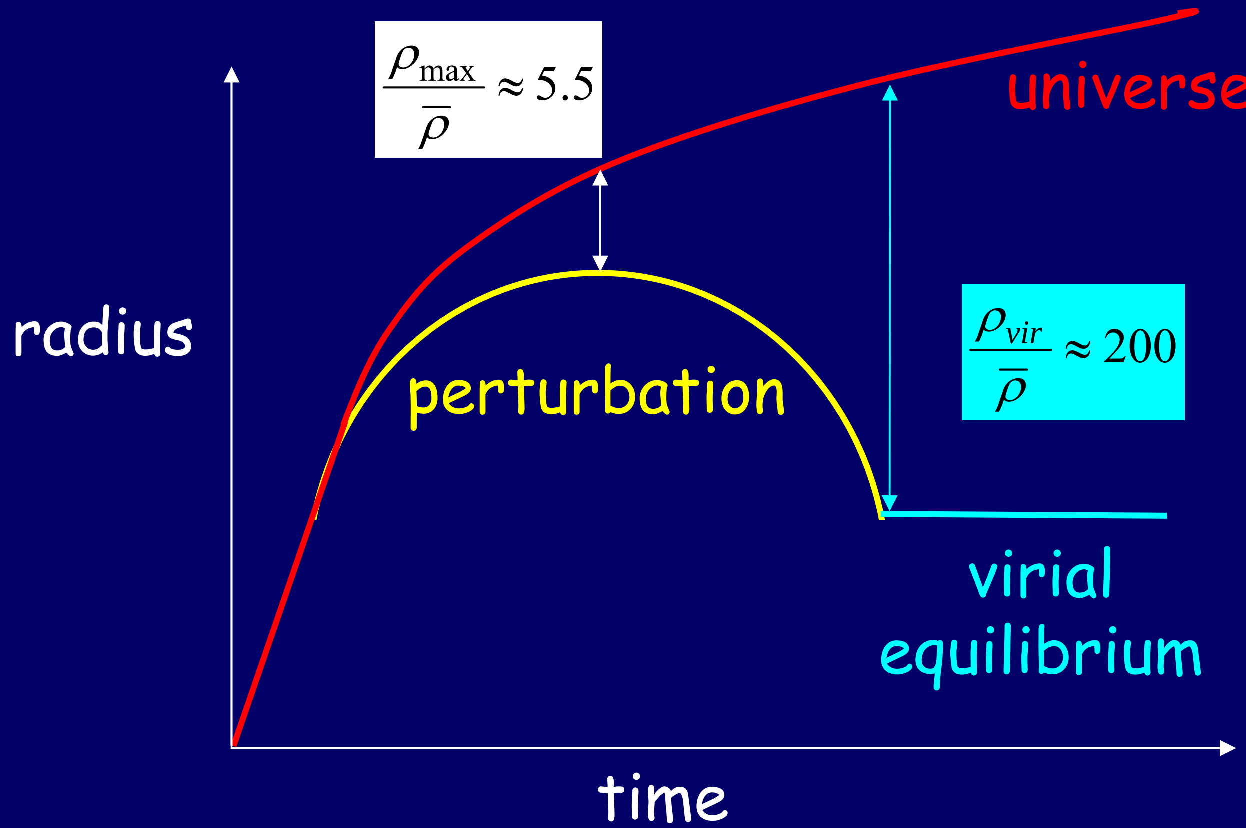
Assume virialization at collapse, $\eta_p \simeq 2\pi$,

$$\frac{t_{col}}{t_{max}} \simeq \frac{2\pi}{\pi} = 2 \quad \rightarrow \frac{a_{vir}}{a_{max}} = \left(\frac{t_{col}}{t_{max}}\right)^{2/3} \simeq 2^{2/3}$$

$$\rightarrow \boxed{\frac{\rho_{vir}}{\rho_{univ}} \simeq 5.55 \times 8 \times 4 \simeq 178 \sim 200}$$



Spherical Collapse



virial equilibrium: $E = -\frac{1}{2}\frac{GM}{R_{vir}} = -\frac{GM}{R_{max}}$

Lecture 6

Hierarchical Clustering

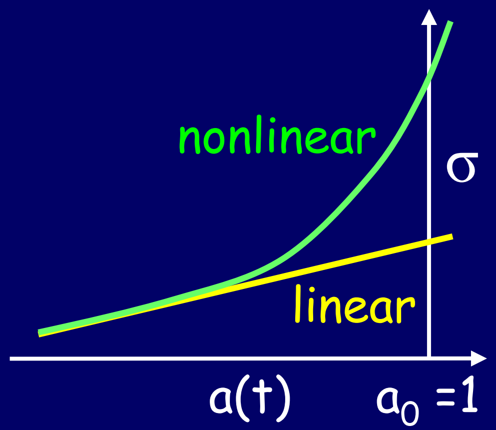
Press Schechter Formalism

Press Schechter Formalism   halo mass function n(M,a)

Gaussian random field $P(\delta) = (2\pi\sigma^2)^{-1/2} \exp(-\delta^2 / 2\sigma^2)$

random spheres of mass M

linear-extrapolated δ_{rms} at a: $\sigma(M,a) = \sigma_0(M)\, D(a)$

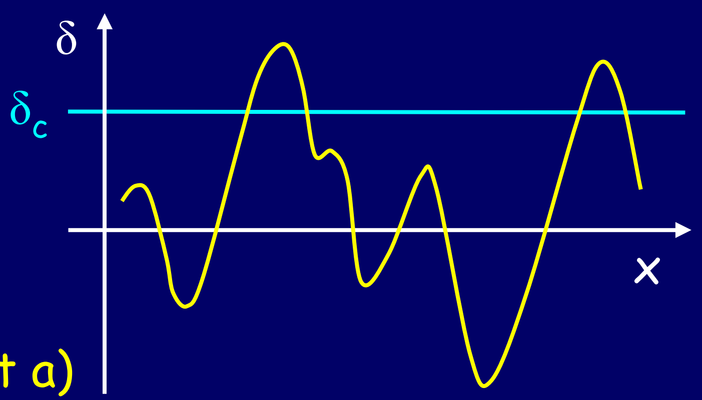


fraction of spheres with $\delta > \delta_c = 1.68$:

$$F(M,a) = \int_{\delta_c}^{\infty} d\delta\, [2\pi\sigma^2(M,a)]^{-1/2} \exp[-\delta^2 / 2\sigma^2(M,a)]$$

$$= (2\pi)^{-1/2} \int_{\delta_c/\sigma(M,a)}^{\infty} dx\, \exp(-x^2/2)$$

$$\nu_c \equiv \frac{\delta_c}{D(a)\, \sigma_0(M)}$$



PS ansaz: F is the mass fraction in halos >M (at a)

derivative of F with respect to M:

$$n(M,a)dM = \left(\frac{2}{\pi}\right)^{1/2} \frac{\bar{\rho}}{M} \nu_c \frac{d \ln \sigma_0}{d \ln M} \exp(-\nu_c{}^2/2) \frac{dM}{M}$$

Mo & White 2002

Press Schechter Formalism cont.

$$n(M,a)dM = \left(\frac{2}{\pi}\right)^{1/2} \frac{\bar{\rho}}{M} \nu_c \frac{d\ln\sigma_0}{d\ln M} \exp(-\nu_c^{\,2}/2) \frac{dM}{M}$$

Example: $P_k \propto k^n \rightarrow \sigma_0(M) \propto M^{-\alpha} \rightarrow \nu_c = (M/M_*)^{\alpha}$

$\alpha = (3+n)/6$ $\quad \dfrac{d\ln\sigma_0}{d\ln M} = \alpha$

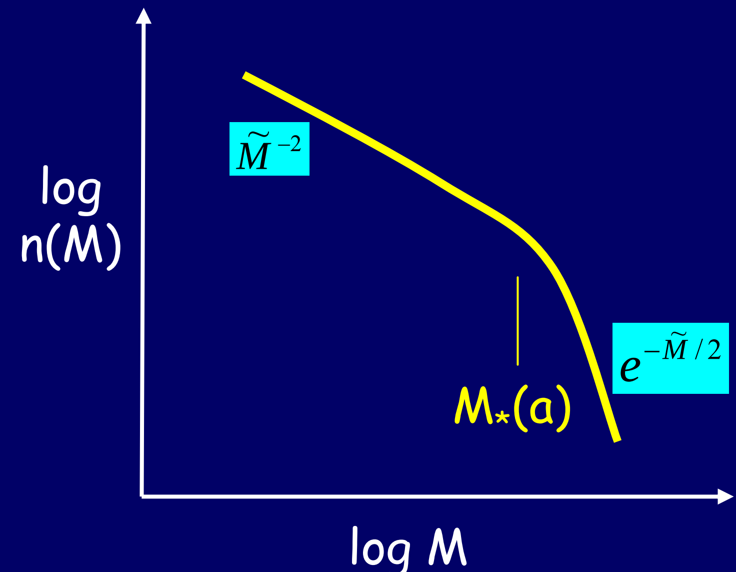
$$n(M) \propto \alpha\, \tilde{M}^{\alpha-2} \exp(-\tilde{M}^{2\alpha}/2) \qquad \tilde{M} \equiv M/M_*$$

self-similar evolution, scaled with M_*

$$\nu_c \equiv \frac{\delta_c}{D(a)\,\sigma_0(M)}$$

time $\quad P_k$

$M_*(a)$ defined by $\sigma(M_*, a) \equiv \delta_c$



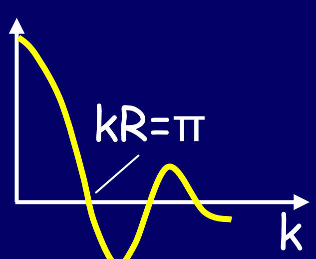
approximate

$$M_*(a) = M_{*0} D(a)^{1/\alpha} \sim 10^{13} M_\circ\, a^5$$

$$\sigma^2(R) = (2\pi)^{-1} \int_0^\infty dk\, k^2\, P(k) \tilde{W}^2(kR)$$

Top Hat

$$W_R(x) = \Theta(x/R) \qquad \tilde{W}_R(k) = 3[\sin(kR) - kR\cos(kR)]/(kR)^3$$



in a flat universe

$$D(a) = a\, g(a)/g(1)$$

$$g(a) \approx \frac{5}{2}\Omega_m(a)\left(\Omega_m(a)^{4/7} - \Omega_\Lambda(a) + \frac{1+\Omega_m(a)/2}{1+\Omega_\Lambda(a)/70}\right)^{-1}$$

$$\Omega_m(a) = \frac{\Omega_m a^{-3}}{\Omega_\Lambda + \Omega_m a^{-3}}$$

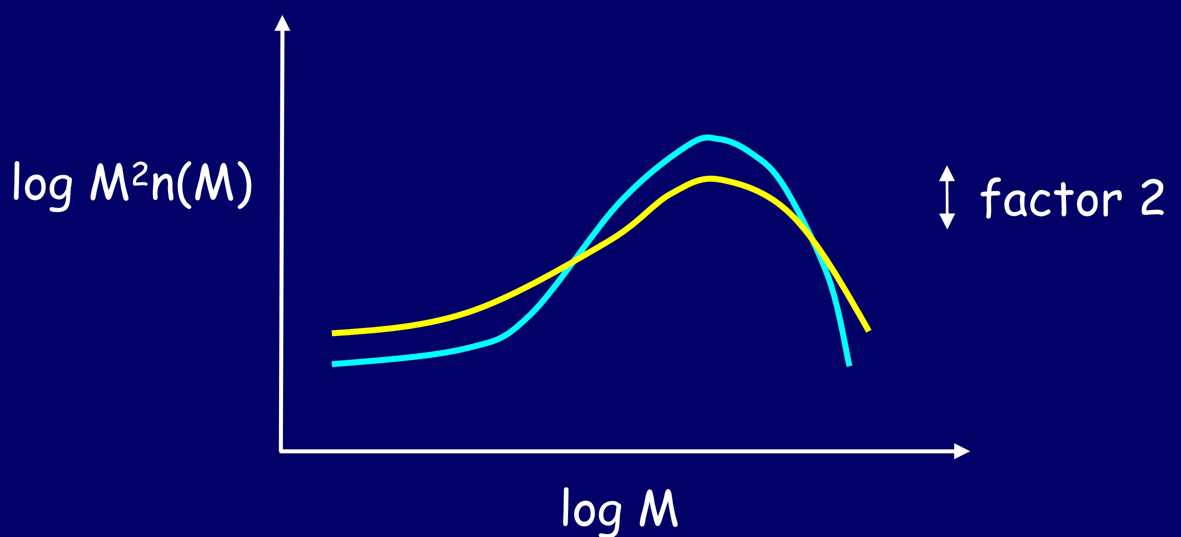
Press Schechter cont.

Better fit using ellipsoidal collapse (Sheth & Tormen 2002)

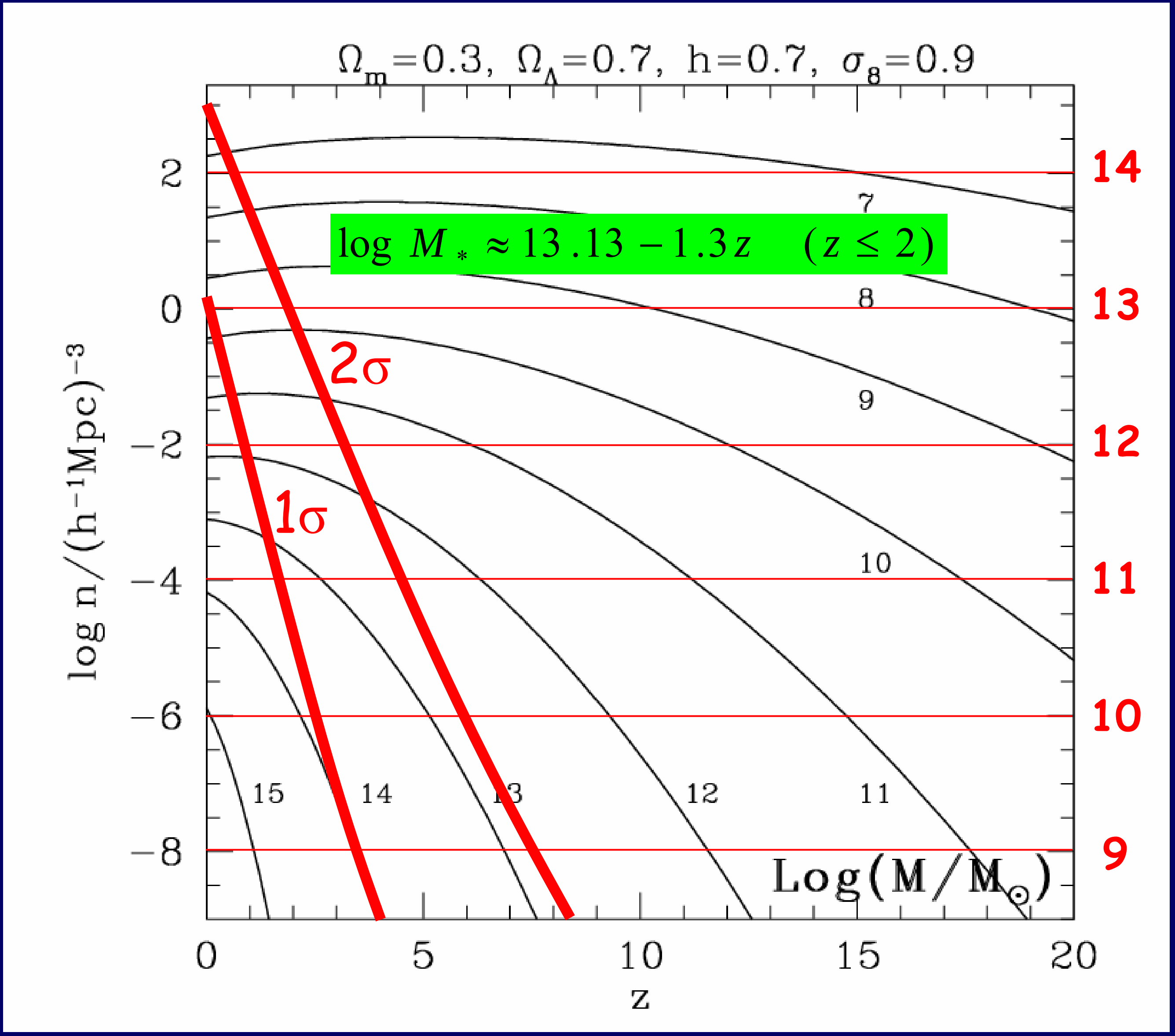
$$F(>M,a) \approx 0.4(1+0.4/\nu^{0.4})\ \mathrm{erfc}(0.85\nu/2^{1/2})$$

$$1\sigma, 2\sigma, 3\sigma \quad 22\%, 4.7\%, 0.54\%$$

Comparison of PS to N-body simulations



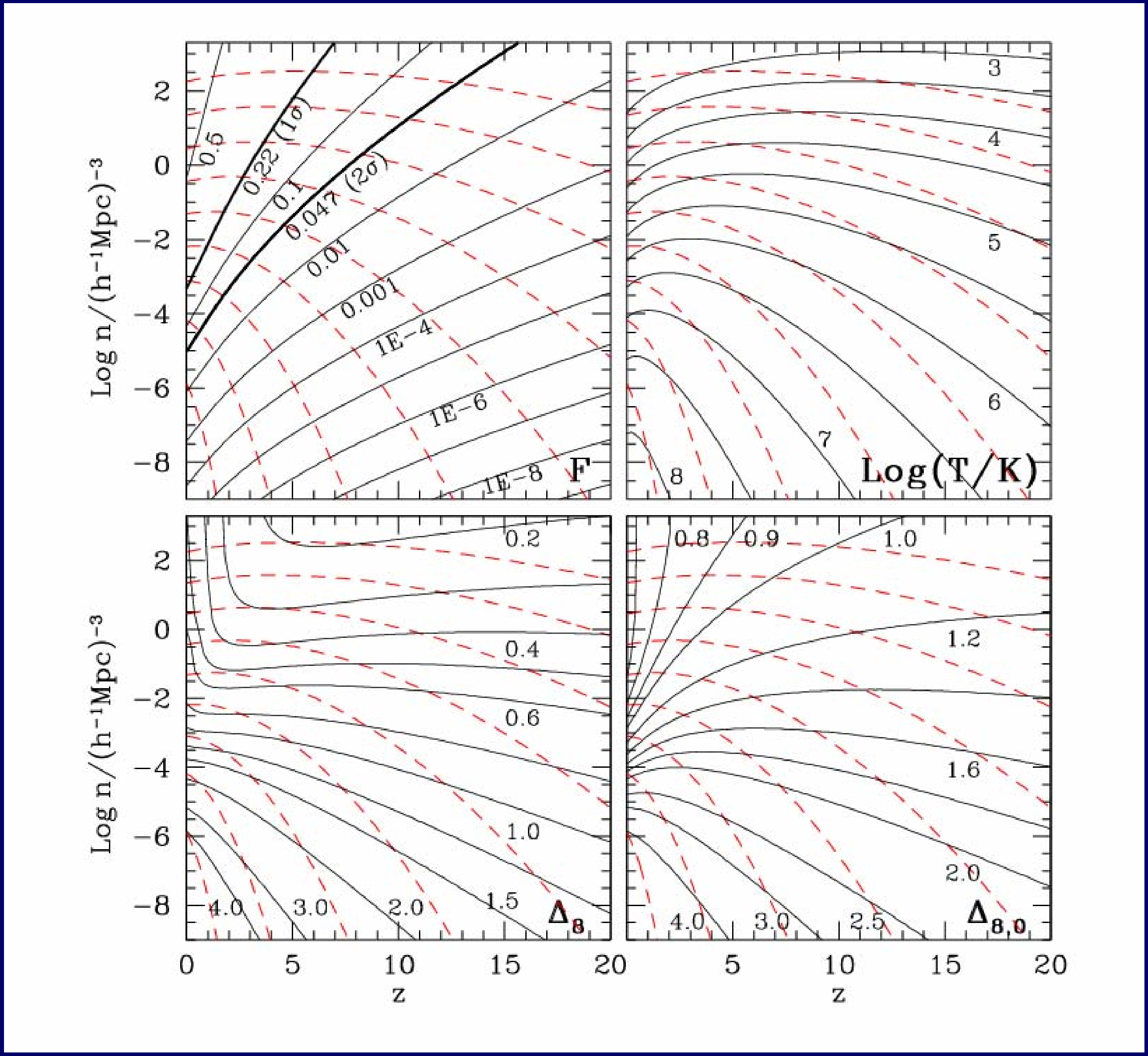
Press-Schechter in ΛCDM



log M/M☉

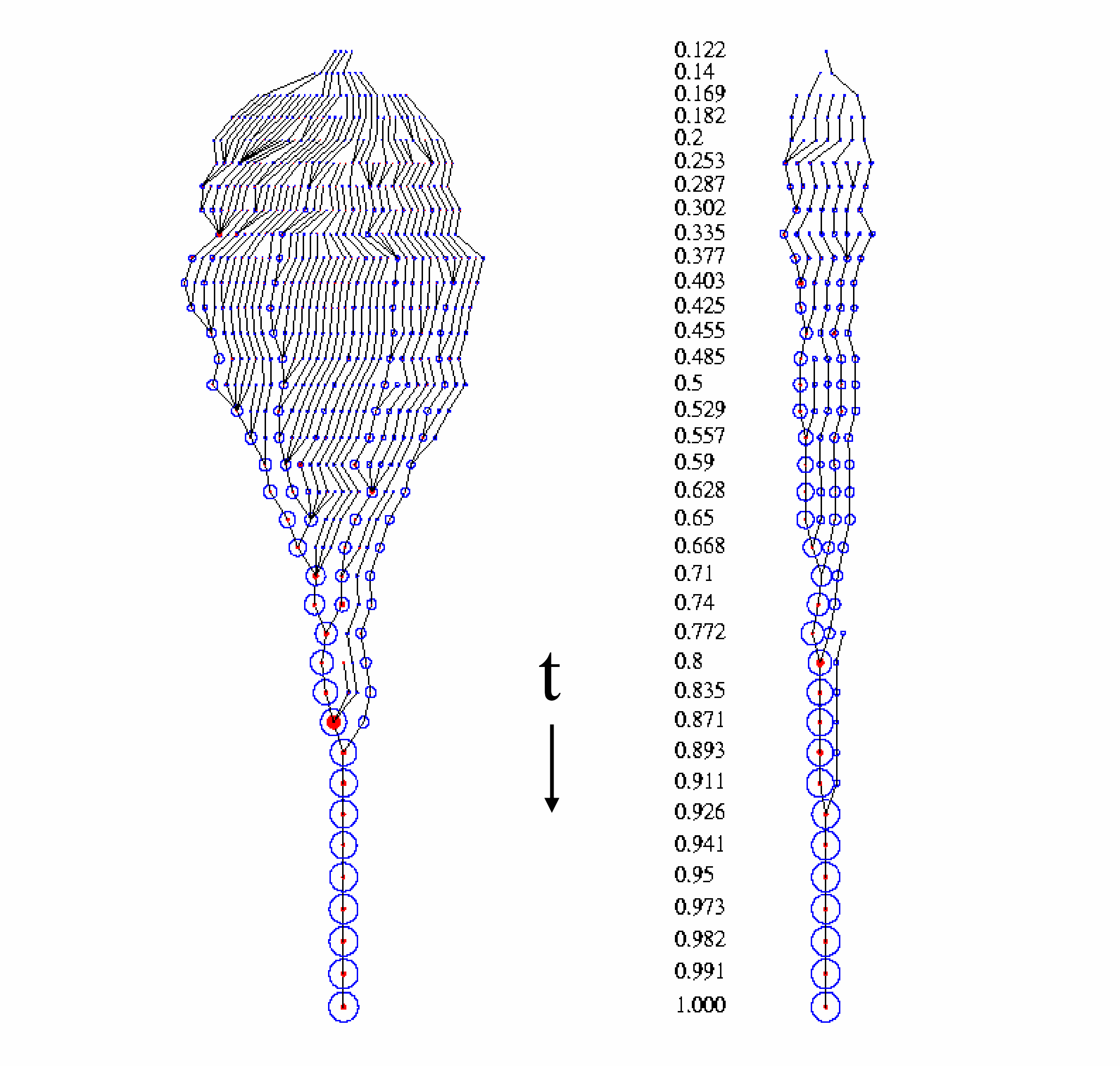
Mo & White 2002

Press-Schechter



Mo & White 2002

Merger Tree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