Lecture 6

Hierarchical Clustering

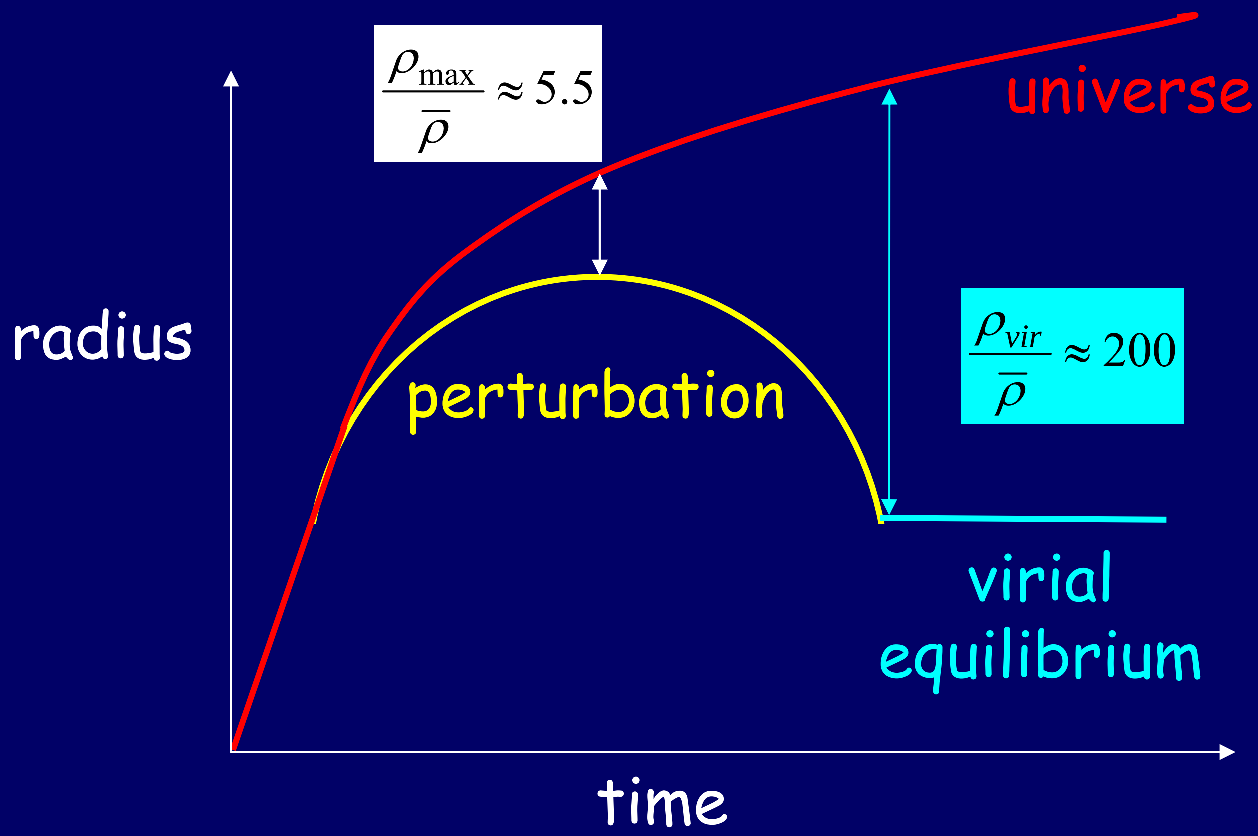
Press Schechter: Halo Distribution

Extended PS: Merging Tree

Biasing: Galaxies/Subhalos in Halos

HOD: Halo Occupation Distribution

Spherical Collapse



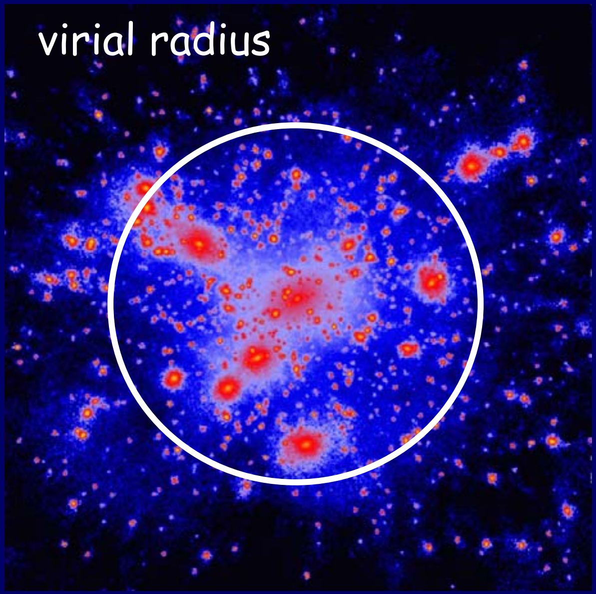
virial equilibrium: $E = -\frac{1}{2}\frac{GM}{R_{vir}} = -\frac{GM}{R_{\max}}$

Virial Scaling Relations

Virial equilibrium: $V^2 = \frac{GM}{R}$

Spherical collapse: $\frac{M}{(4\pi/3)R^3} = \Delta\rho_u = \Delta\rho_{u0} a^{-3}$ $\Delta \approx 200$

$$\rightarrow M \propto V^3 a^{3/2} \propto R^3 a^{-3}$$



Weak dependence on time of formation:

$$D(a)\delta_0(M) \approx 1 \quad \rightarrow \quad a \propto M^{\alpha} \quad \alpha = (n+3)/6 \approx 0.1-0.2$$

$$M \propto V^4 \quad \text{for } n = -2$$

Practical formulae:

$$\rho_u \approx 2.76\times 10^{-30}\, g\, cm^{-3}\, \Omega_{m0.3} h_{0.7}^2 a^{-3}$$

$$M_{11} \approx 6.06\, V_{100}^3 A^{-3/2} \approx 342\, R_{Mpc}^3 A^{-3}$$

$$A \equiv a\, (\Delta_{200}\Omega_{m0.3} h_{0.7}^2)^{-1/3}$$

$$\Delta(a) \approx [18\pi^2 - 82\Omega_\Lambda(a) - 39\Omega_\Lambda(a)^2]/\Omega_m(a) \quad \Delta(a<<1) \approx 178 \quad \Delta_0 \approx 340$$

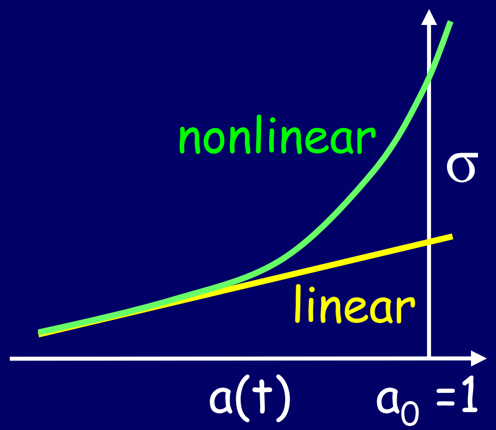
$$\Omega_m(a) = \frac{\Omega_m a^{-3}}{\Omega_\Lambda + \Omega_m a^{-3}} \quad \Omega_m(a) + \Omega_\Lambda(a) = 1$$

Press Schechter Formalism halo mass function n(M,a)

Gaussian random field $P(\delta) = (2\pi\sigma^2)^{-1/2} \exp(-\delta^2 / 2\sigma^2)$

random spheres of mass M

linear-extrapolated δ_{rms} at a: $\sigma(M,a) = \sigma_0(M)\, D(a)$

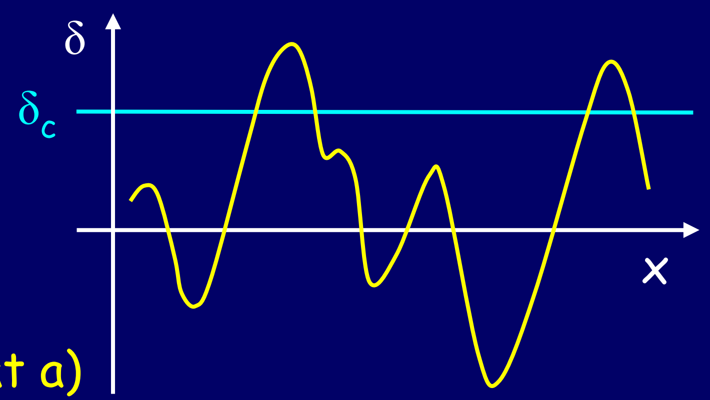


fraction of spheres with $\delta > \delta_c = 1.68$:

$$F(M,a) = \int_{\delta_c}^{\infty} d\delta\, [2\pi\sigma^2(M,a)]^{-1/2} \exp[-\delta^2 / 2\sigma^2(M,a)]$$

$$= (2\pi)^{-1/2} \int_{\delta_c/\sigma(M,a)}^{\infty} dx\, \exp(-x^2/2)$$

$$\nu_c \equiv \frac{\delta_c}{D(a)\,\sigma_0(M)}$$



PS ansaz: F is the mass fraction in halos >M (at a)

derivative of F with respect to M:

$$n(M,a)dM = -\left(\frac{2}{\pi}\right)^{1/2} \frac{\bar{\rho}}{M} \nu_c \frac{d \ln \sigma_0}{d \ln M} \exp\left(-\frac{\nu_c{}^2}{2}\right) \frac{dM}{M}$$

Mo & White 2002

Press Schechter Formalism cont.

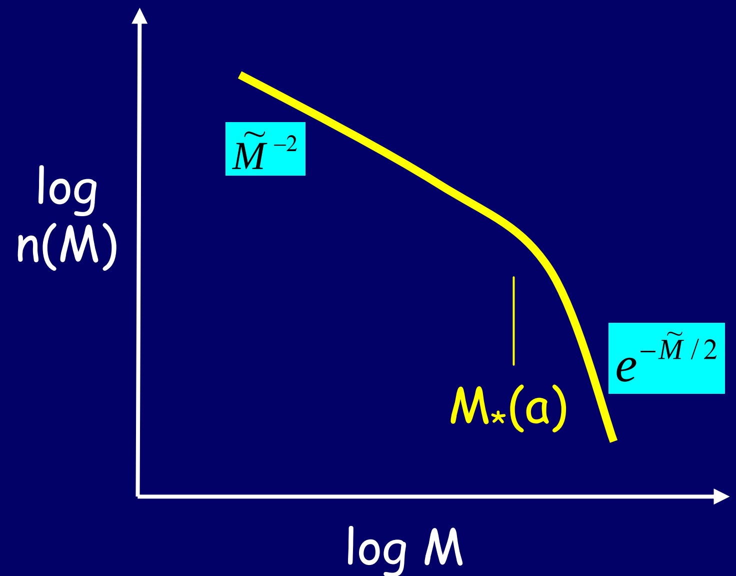
$$n(M,a)dM = -\left(\frac{2}{\pi}\right)^{1/2} \frac{\bar{\rho}}{M} \nu_c \frac{d\ln\sigma_0}{d\ln M} \exp\left(-\frac{\nu_c^{\,2}}{2}\right) \frac{dM}{M}$$

Example: $P_k \propto k^n \rightarrow \sigma_0(M) \propto M^{-\alpha} \rightarrow \nu_c = (M/M_*)^{\alpha}$

$\alpha = (3+n)/6$ $\qquad \dfrac{d\ln\sigma_0}{d\ln M} = \alpha$

$$n(M) \propto \alpha\, \tilde{M}^{\alpha-2} \exp(-\tilde{M}^{2\alpha}/2) \qquad \tilde{M} \equiv M/M_*$$

self-similar evolution, scaled with M_*



$$\nu_c \equiv \frac{\delta_c}{D(a)\,\sigma_0(M)}$$

time $\quad P_k$

$M_*(a)$ defined by $\sigma(M_*, a) \equiv \delta_c$

approximate

$$M_*(a) = M_{*0} D(a)^{1/\alpha} \sim 10^{13} M_{\circ}\, a^5$$

$$\sigma^2(R) = (2\pi)^{-1} \int_0^{\infty} dk\, k^2\, P(k)\, \tilde{W}^2(kR)$$

in a flat universe

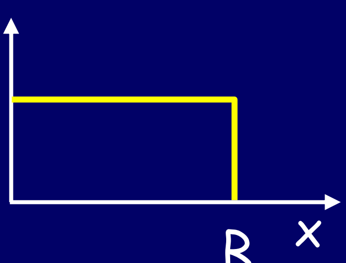
$$D(a) = a\, g(a)/g(1)$$

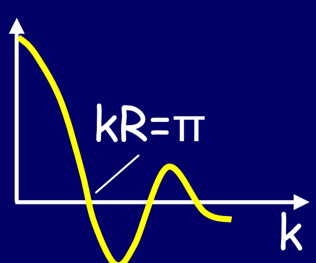
$$g(a) \approx \frac{5}{2}\Omega_m(a)\left(\Omega_m(a)^{4/7} - \Omega_\Lambda(a) + \frac{1+\Omega_m(a)/2}{1+\Omega_\Lambda(a)/70}\right)^{-1}$$

$$\Omega_m(a) = \frac{\Omega_m a^{-3}}{\Omega_\Lambda + \Omega_m a^{-3}}$$

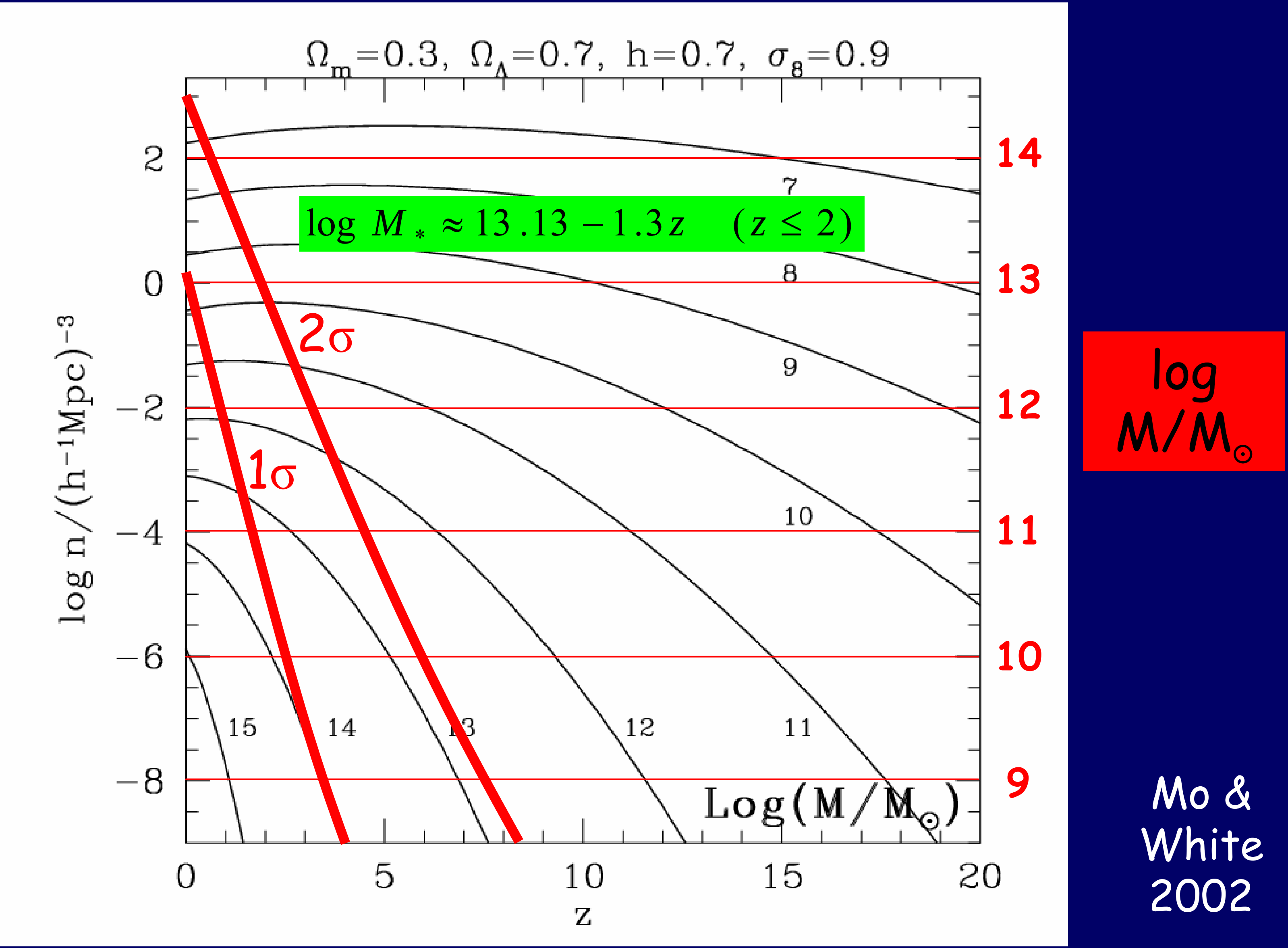
Top Hat

$W_R(x) = \Theta(x/R) \qquad \tilde{W}_R(k) = 3[\sin(kR) - kR\cos(kR)]/(kR)^3$





Press-Schechter in ΛCDM



Mo & White 2002

Press-Schechter by Excersion Set: n(M,a)

Bond et al. 91
Lacey & Cole 93
White 9410043

Linear $\delta(x)$ at some fiducial a when D(a)=1

Top-hat smoothing in k-space:
varying smoothing scale $k_c \sim 1/R_c$

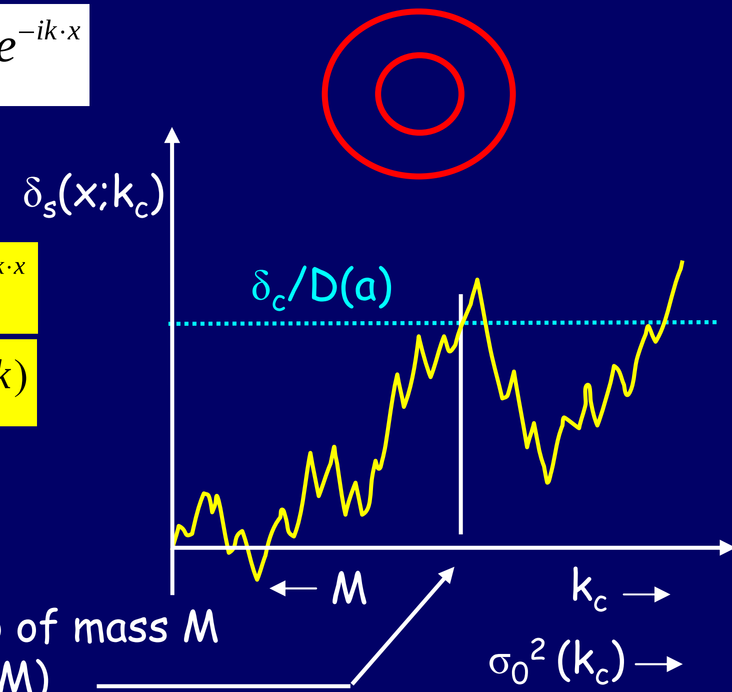
$$\delta_s(x;k_c) = \int_{k<k_c} d^3k \; \delta_k \; e^{-ik\cdot x}$$

At a fixed point x. As k_c varies, δ_s executes a random walk:

step: $$\Delta\delta_s = \delta_s(x;k_c+\Delta k_c) - \delta_s(x;k_c) = \int_{k_c<k<k_c+\Delta k_c} d^3k \; \delta_k e^{-ik\cdot x}$$

variance: $$\Delta\sigma_0^{\,2} = \sigma_0^{\,2}(k_c+\Delta k_c) - \sigma_0^{\,2}(k_c)$$ $$\sigma_0^{\,2}(k_c) = \int_{k<k_c} d^3k \; P(k)$$

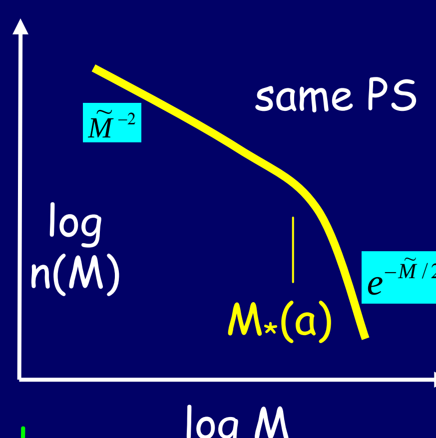
$\Delta\delta_s$ is a Gaussian random variable,
independent of δ_s: Markov random walk.



PS ansatz: mass element initially at x belongs to halo of mass M at a if the random walk first crosses $\delta_c/D(a)$ at σ_0^2 (M)

Fraction of mass in halos >M =
fraction of trajectories $\delta_s(x;k_c)$ which first cross δ_c/D at $k_c<k(M)$

Solution: $$n(M,a)dM = -\left(\frac{2}{\pi}\right)^{1/2} \frac{\bar{\rho}}{M} \nu_c \frac{d\ln\sigma_0}{d\ln M} \exp\left(-\frac{\nu_c}{2}\right) \frac{dM}{M}$$ $$\nu_c \equiv \frac{\delta_c}{D(a)\,\sigma_0(M)}$$



Markov: Past of x (right) independent of its future (left), so the history of halos of mass M is superclusters and in voids are statistically identical, i.e. their galaxy populations should be identical.

Proof:

Fraction of mass in halos >M =
fraction of trajectories $\delta_s(x;k_c)$ which first cross δ_c/D at $k_c<k(M)$

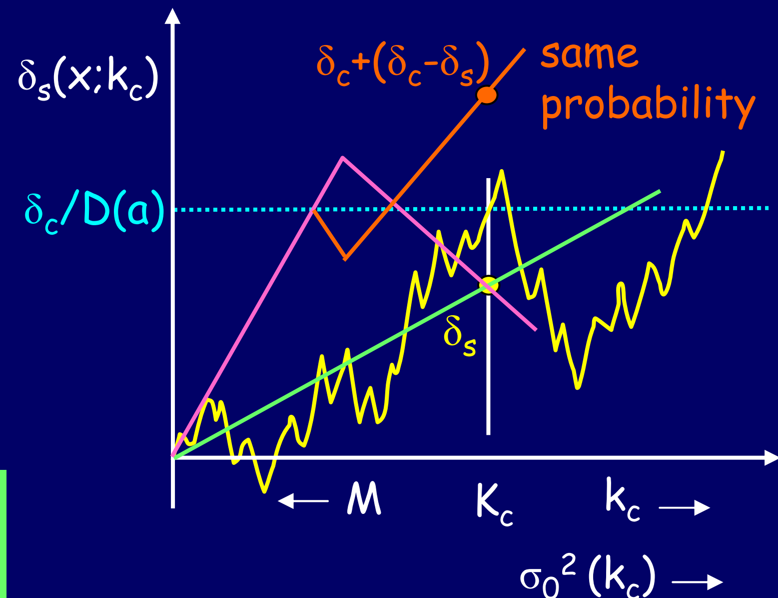
For a given a and $k_c=K_c$ δ_s is Gaussian:

$$P(\delta_s) = [2\pi D(a)\sigma_0(K_c)]^{-1/2} \exp(-\delta_s^{\,2}/2D^2\sigma_0^{\,2})$$

#(points $\delta_s<\delta_c$ for all $k_c<K_c$) =

#($\delta_s<\delta_c$ for $k_c=K_c$)

– #($\delta_s<\delta_c$ for $k_c=K_c$ but $\delta_s>\delta_c$ at some $k_c<K_c$)

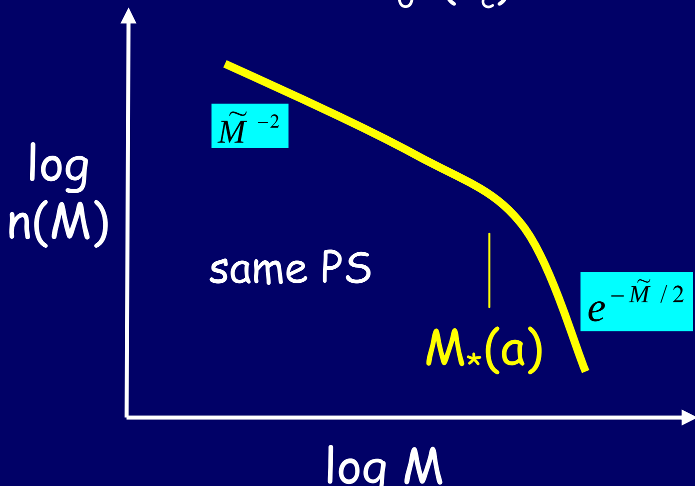


$$P_{first}(\delta_s) = \frac{1}{(2\pi\sigma^2)^{1/2}}\left[\exp\left(\frac{-\delta_s^{\,2}}{2\sigma^2}\right) - \exp\left(\frac{-(2\delta_c-\delta_s)^2}{2\sigma^2}\right)\right]$$

$$F(>K_c) = \int_{-\infty}^{\delta_c} P(\delta_s)\,d\delta_s = \int_{-\infty}^{\delta_c/\sigma}\frac{dx}{(2\pi)^{1/2}}e^{-x^2/2} - \int_{\delta_c/\sigma}^{\infty}\frac{dx}{(2\pi)^{1/2}}e^{-x^2/2}$$

Differentiate with respect to K_c (or M) :

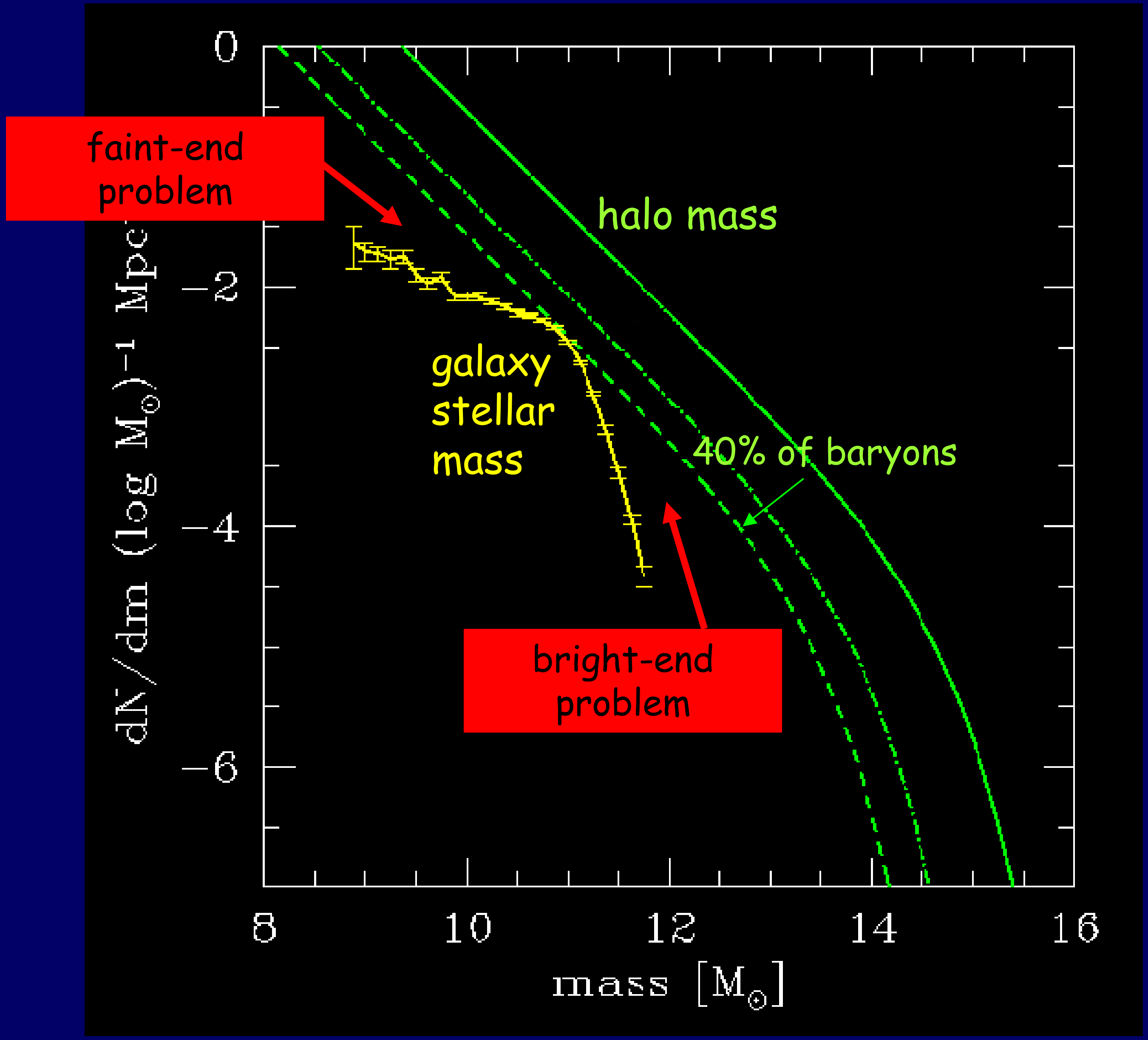
$$n(M,a)dM = -\left(\frac{2}{\pi}\right)^{1/2}\frac{\bar{\rho}}{M}\nu_c\frac{d\ln\sigma_0}{d\ln M}\exp\left(-\frac{\nu_c}{2}\right)\frac{dM}{M} \qquad \nu_c \equiv \frac{\delta_c}{D(a)\,\sigma_0(M)}$$



$$n(M) \propto \alpha\,\tilde{M}^{\alpha-2}\exp(-\tilde{M}^{2\alpha}/2) \qquad \tilde{M} \equiv M/M_*$$

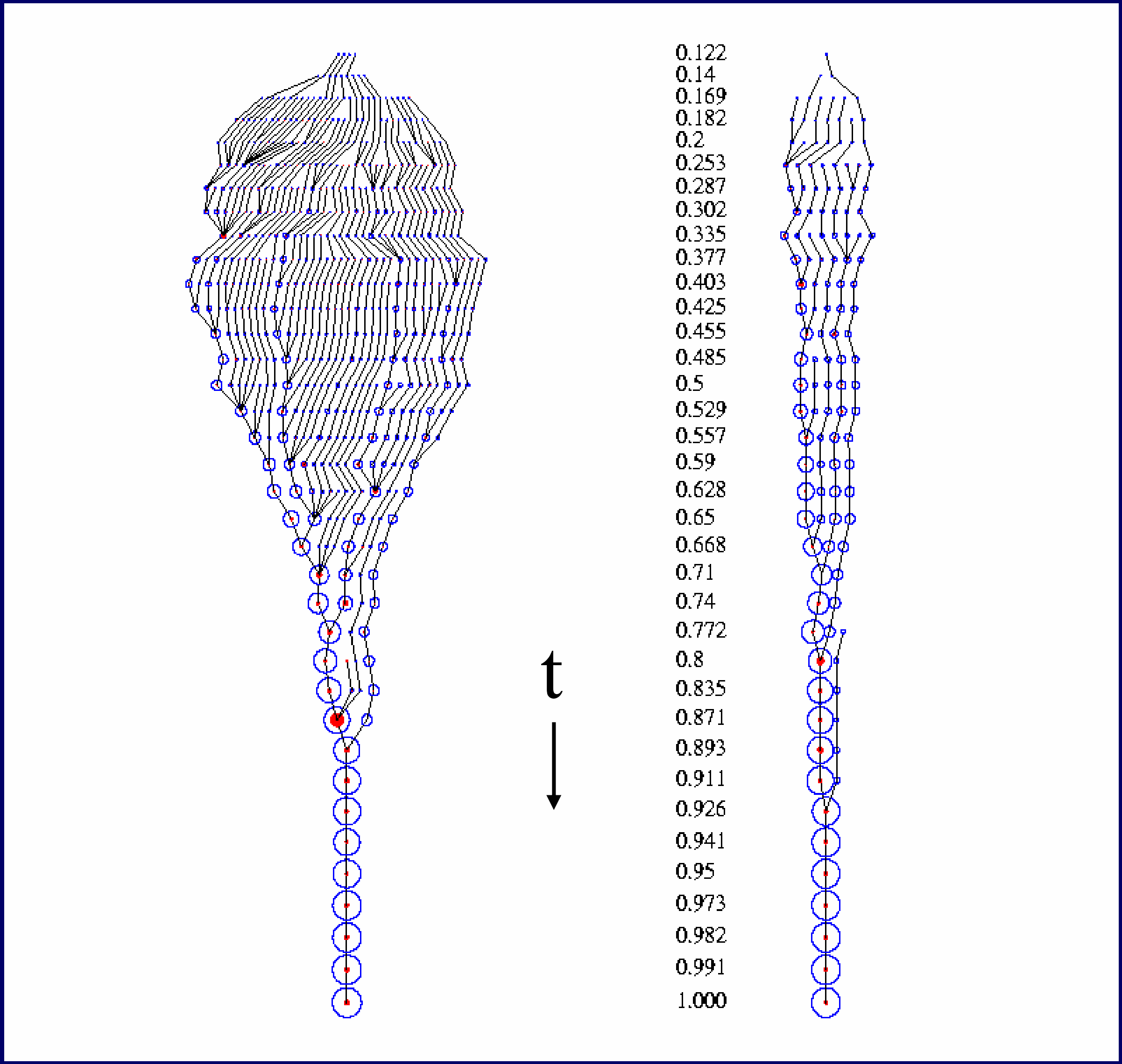
self-similar evolution, scaled with M_*

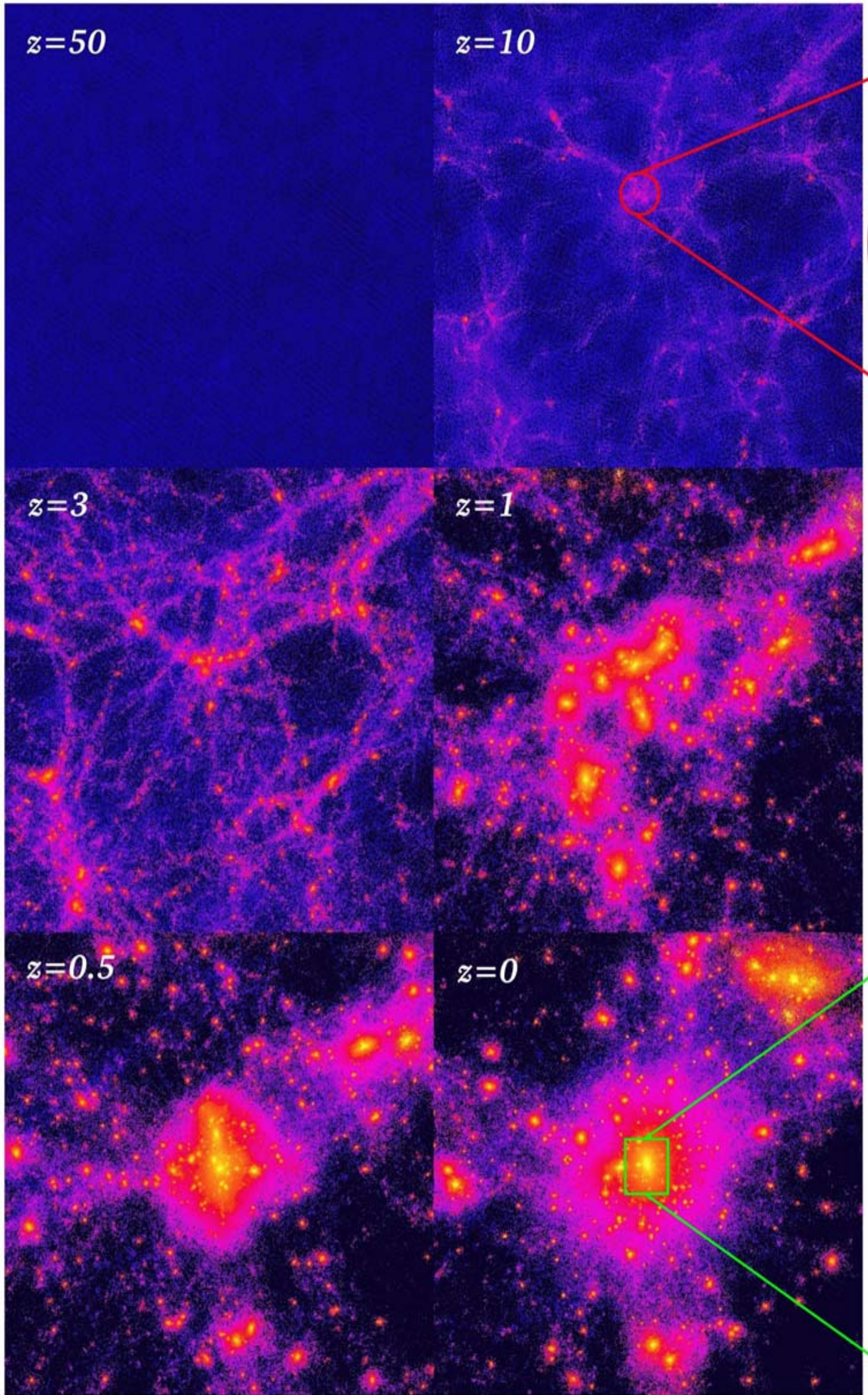
Mass versus Light Distribution

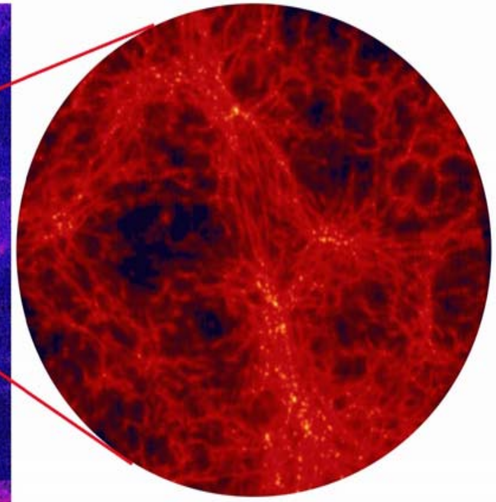


Conditional Merger Tree: Extended Press-Schechter

Merger Tree: conditional probability



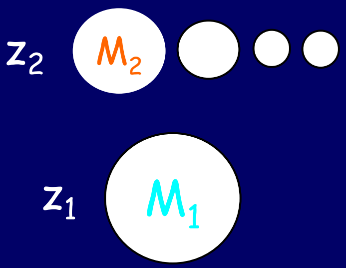




spherical collapse
or mergers

Extended Press-Schechter (EPS): Merger Tree

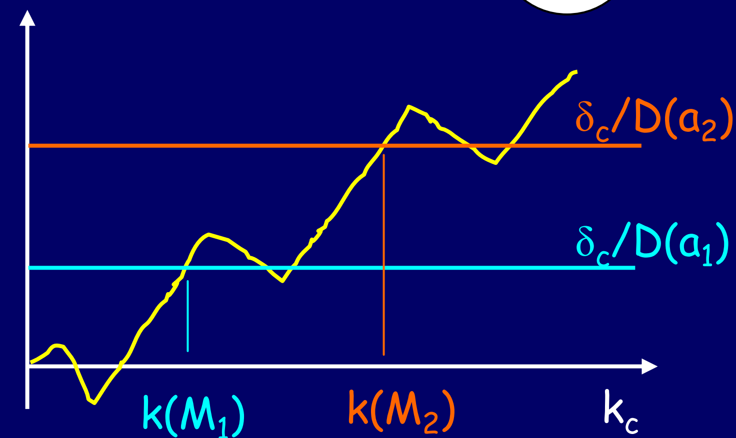
Given that a mass element belongs to halo M_1 at z_1,
what is the probability that it belonged to halo M_2 ($<M_1$) at z_2 ($>z_1$)?



Equivalent:

Given that $\delta_s(x;k_c)$ first crossed $\delta_c/D(a_1)$ at $k_c=k(M_1)$,

what is the probability that it first crossed $\delta_c/D(a_2)$ at $k_c=k(M_2)$.

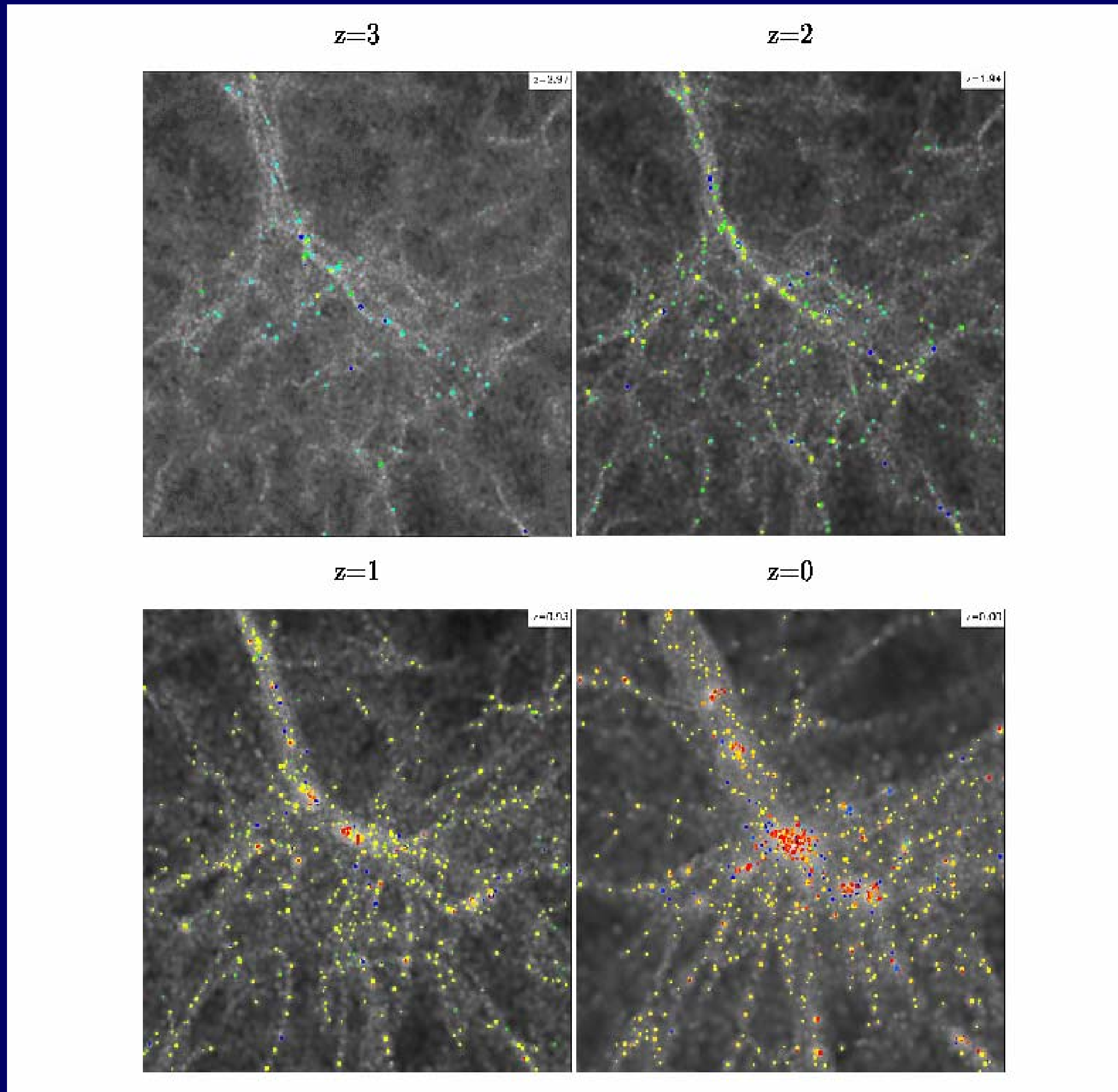


The same problem as before but with the origin shifted:

$$n(M_2, z_2 \mid M_1, z_1)\, dM_2 = -\left(\frac{2}{\pi}\right)^{1/2} \frac{M_1}{M_2} \frac{\delta_c (D_2^{-1} - D_1^{-1})}{(\sigma_2^2 - \sigma_1^2)^{1/2}} \frac{d \ln(\sigma_2^2 - \sigma_1^2)^{1/2}}{d \ln M_2} \exp\left(\frac{\delta_c^2 (D_2^{-1} - D_1^{-1})^2}{2(\sigma_2^2 - \sigma_1^2)}\right) \frac{dM_2}{M_2}$$

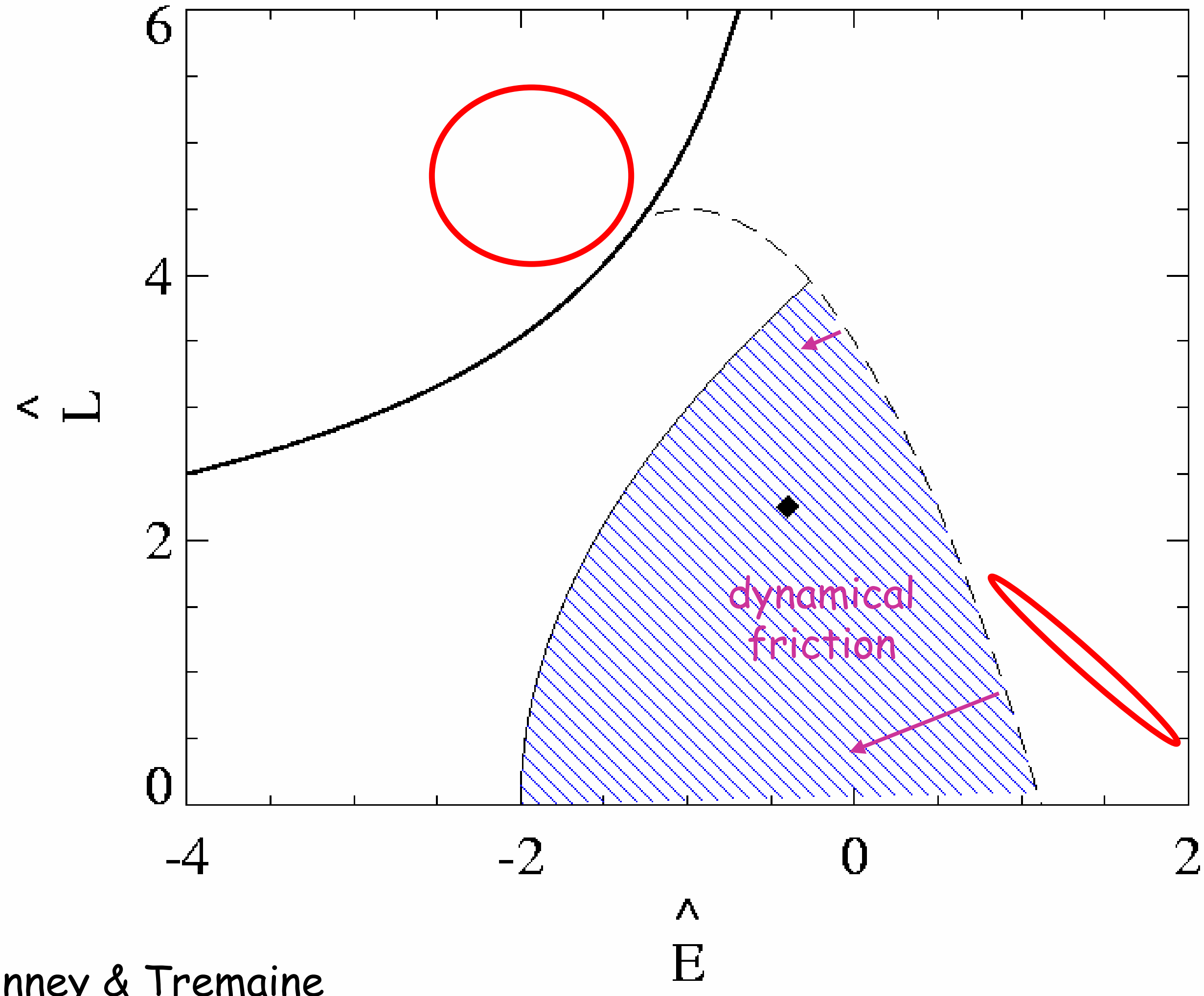
- # of bright E galaxies in a cluster: $M=10^{15}$ today, how many 10^{12} progenitors at $z=2$?
- descendents of LGBs: massive halos at $z=3$ have $n=10^{-2}\,\mathrm{Mpc}^{-1}$, what mass halos do they inhibit today?
- When did the most massive progenitor include half its current mass?
- How often do two 10^{12} halos merge?
- Infall rate of spirals into clusters: How often does a 10^{15} halo accrete a 10^{12} halo?

Formation of galaxies in a cluster



GIF

Orbits that lead to Mergers



Binney & Tremaine

Galaxy/Halo Biasing

Examples:

- cluster clustering
- bright galaxies (LBG)
- clustering of different galaxy types

Biasing: Subhalos in Host Halos (from EPS)

Host halo: a sphere of radius R today, mass M. $\delta = \frac{M}{(4\pi/3)\bar{\rho}R^3} - 1$

comoving initial radius $R_0 = R(1+\delta)^{1/3}$

linear-extrapolated to today $\delta_0(\delta;\Omega,\Lambda)$

Subhalos: average # of subs (m,z) in host (R, δ), using EPS $N_{subs}(m,z\,|\,R_0,\delta_0)$

Average over all δ's at fixed R $\bar{N}_{halos} = n_{halos}(m,z)\,(4\pi/3)R^3$ $\delta_{subs} \equiv N_{subs}/\bar{N}_{halos} - 1$

Obtain from EPS for small subs and proto-host-halo $m \ll M \quad D\delta \ll \delta_c$

$$\delta_{subs} \approx \left(1 + \frac{\nu^2 - 1}{\delta_c / D}\right)\delta_{mass}$$

$$\nu \equiv \frac{\delta_c}{D\sigma_0(m)} \qquad \nu = 1 \text{ for } m = M_*(z)$$

Linear biasing factor: b $\gtreqless$ 1 for m $\gtreqless$ $M_*(z)$

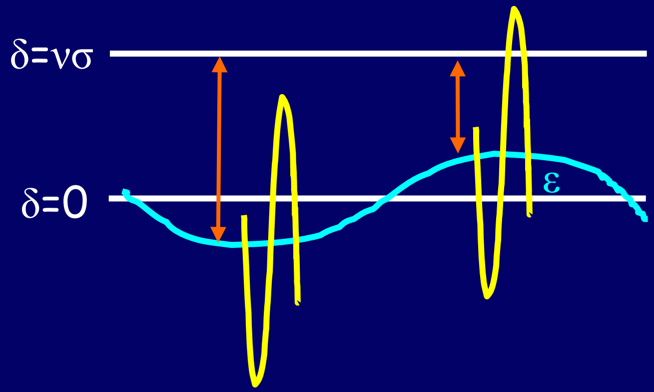
Mo & White

Peak biasing in a Gaussian field

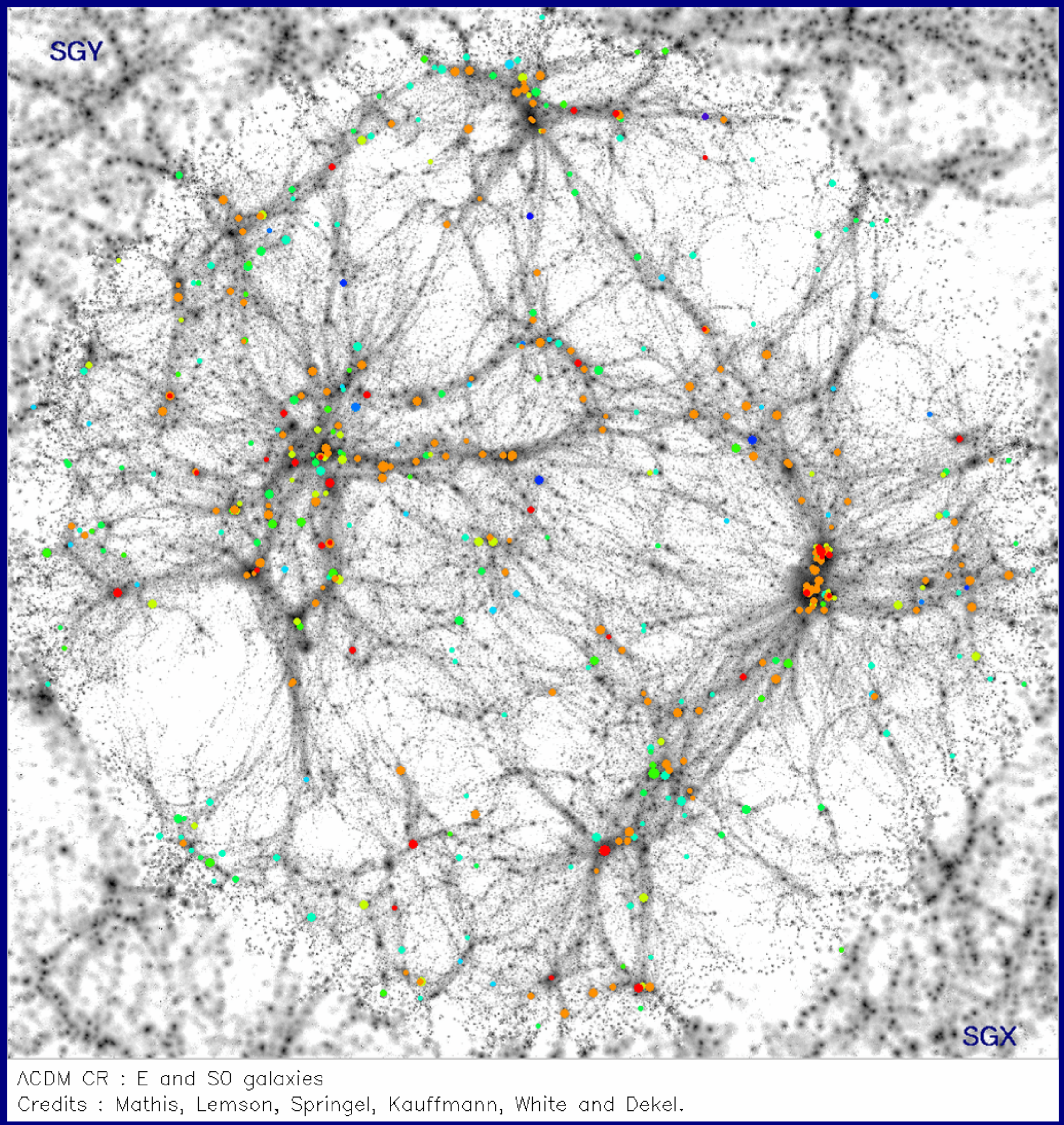
$$P(\delta > \nu\sigma) \propto \exp\left(-\frac{(\nu\sigma \pm \varepsilon)^2}{2\sigma^2}\right)$$

Kaiser 1984,
Bardeen et al. 86

$$\xi_{subs}(r) = (\nu/\sigma)^2\,\xi_{mass}(r)$$

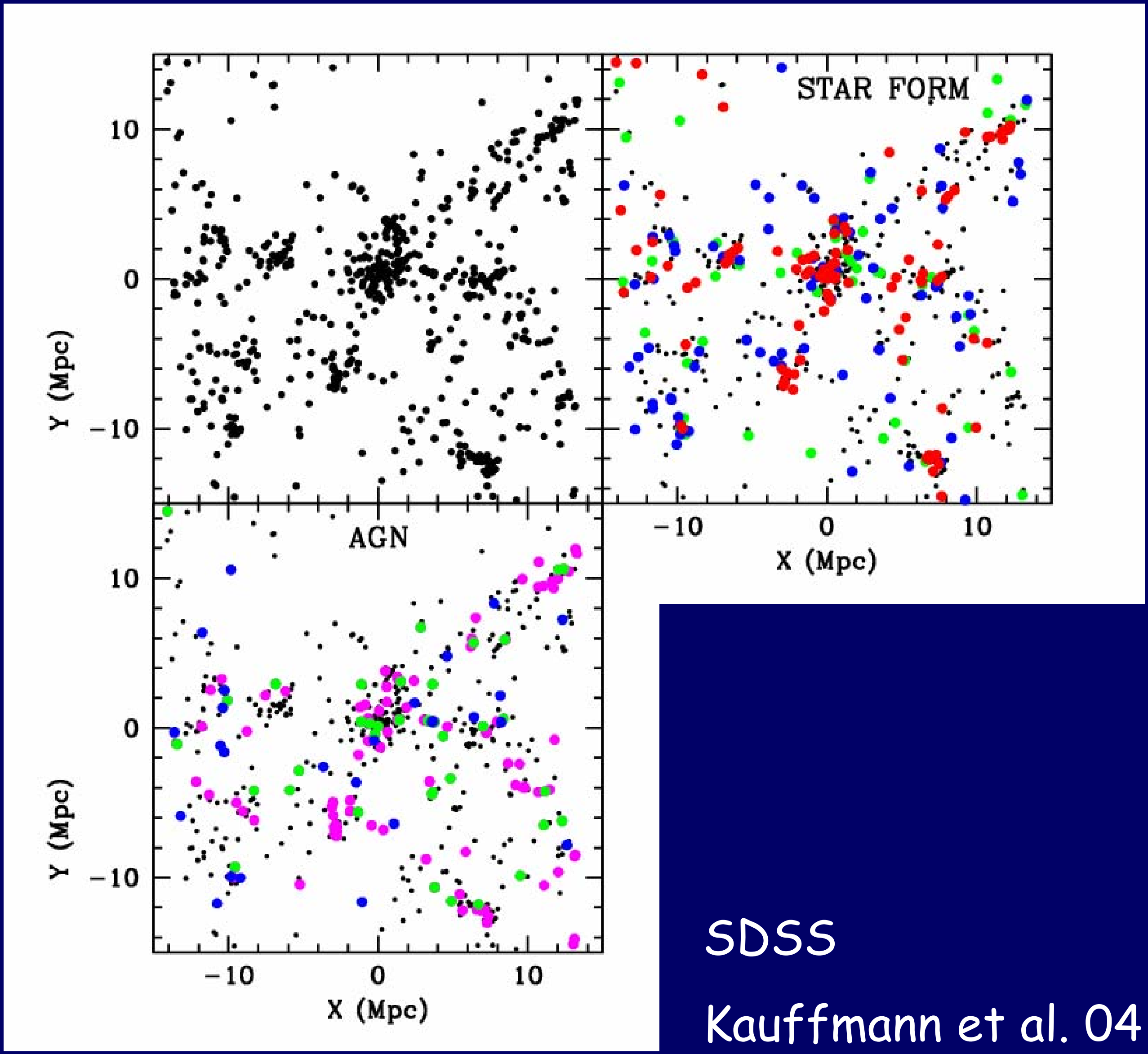


Elliptical galaxies in the local universe: biased with respect to the dark matter



GIF simulation

Massive Ellipticals in Clusters

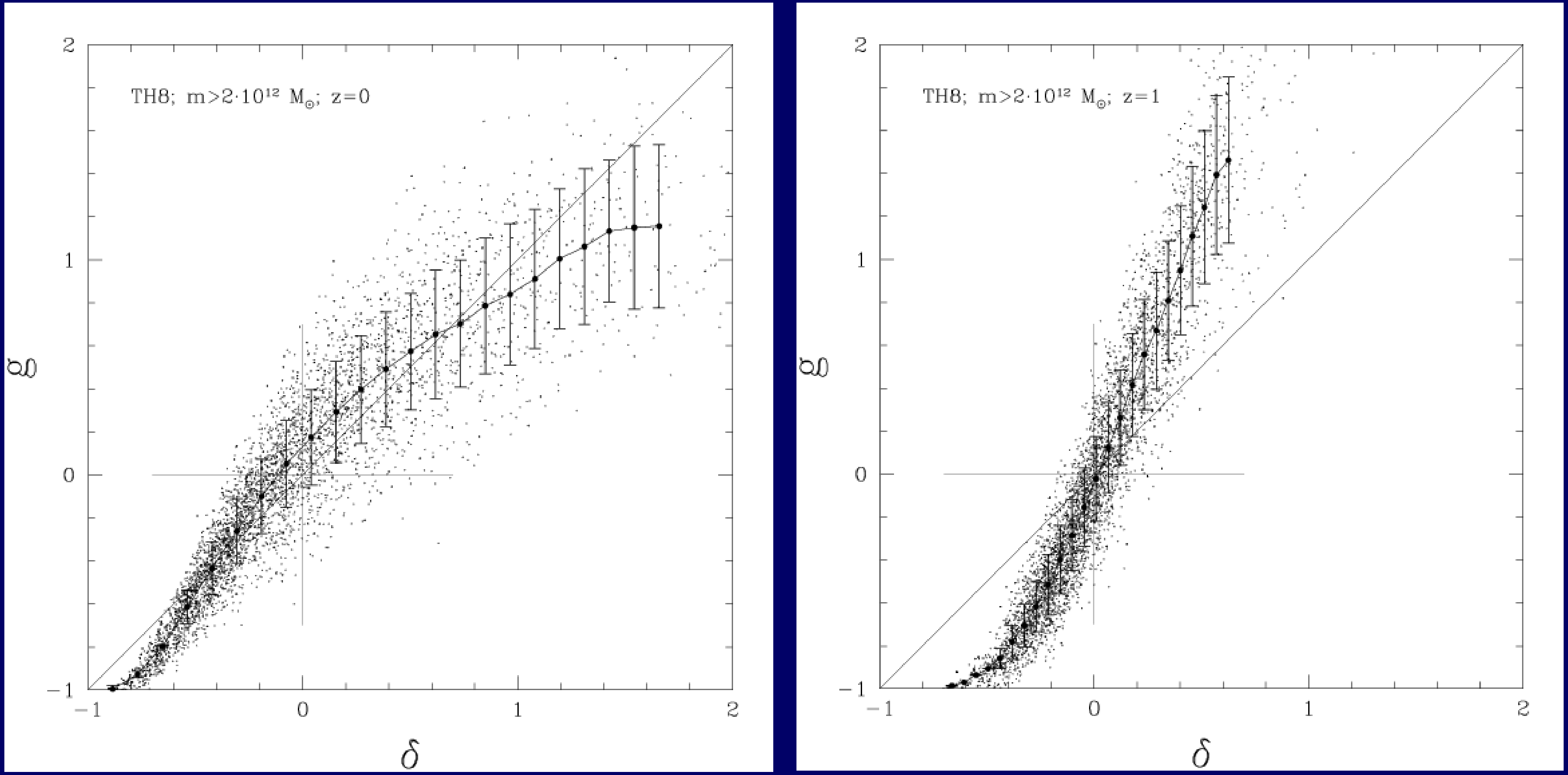


SDSS

Kauffmann et al. 04

Nonlinear Stochastic Biasing

Dekel & Lahav



mean biasing $b(\delta)\delta = \langle g \mid \delta \rangle = \int dg\, P(g \mid \delta)\, g$

$\varepsilon = g - \langle g \mid \delta \rangle$

"linear" biasing $\hat{b} = \langle b(\delta)\delta^2 \rangle / \sigma^2$

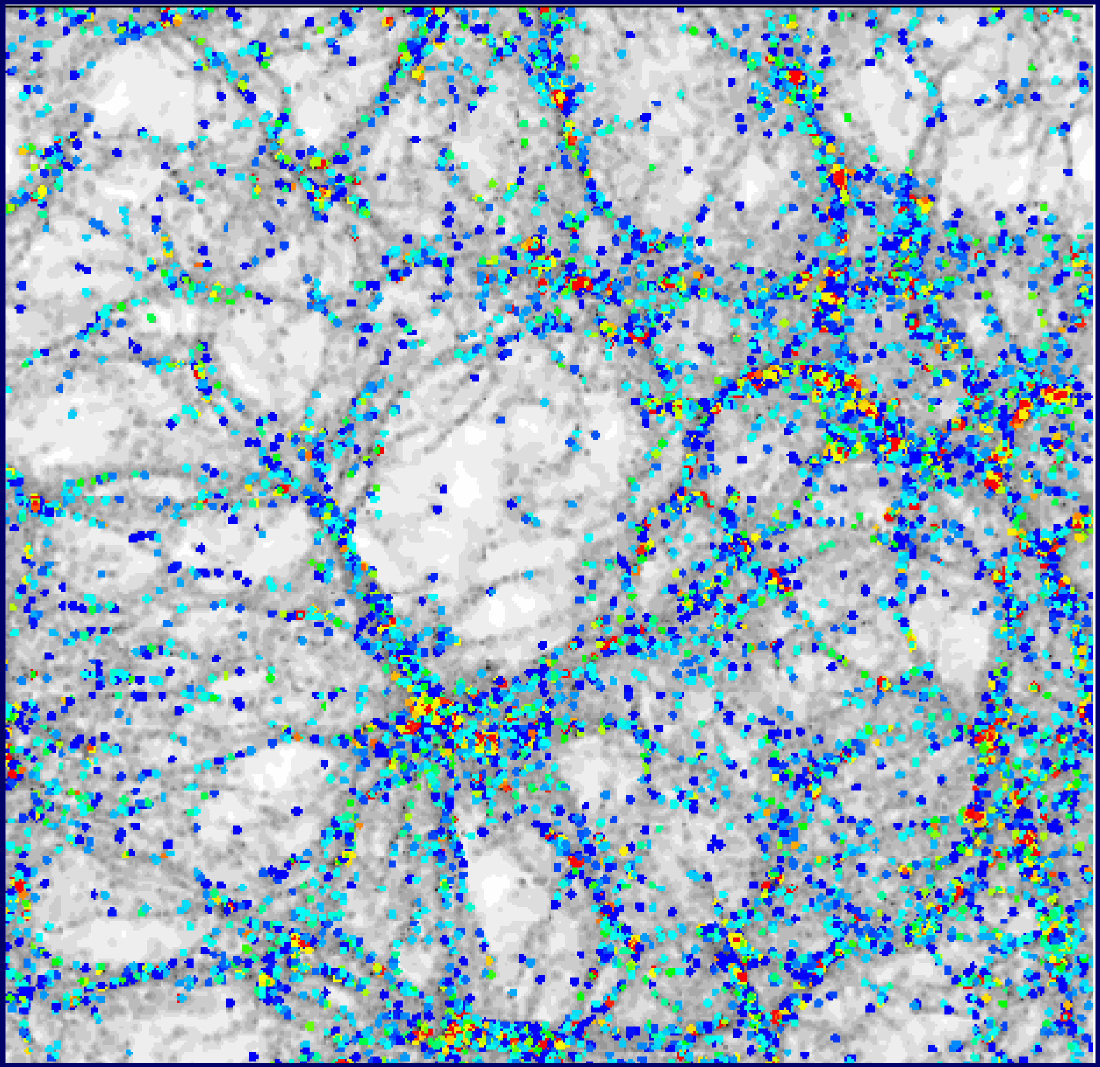
$\sigma_b^2(\delta) = \langle \varepsilon^2 \mid \delta \rangle$

nonlinearity $\tilde{b}^2 = \langle b^2(\delta)\delta^2 \rangle / \sigma^2$

biasing scatter $\sigma_b^2 = \langle \varepsilon^2 \rangle / \sigma^2$

Correlation Function and HOD

Galaxy type correlated with large-scale structure



elliptical

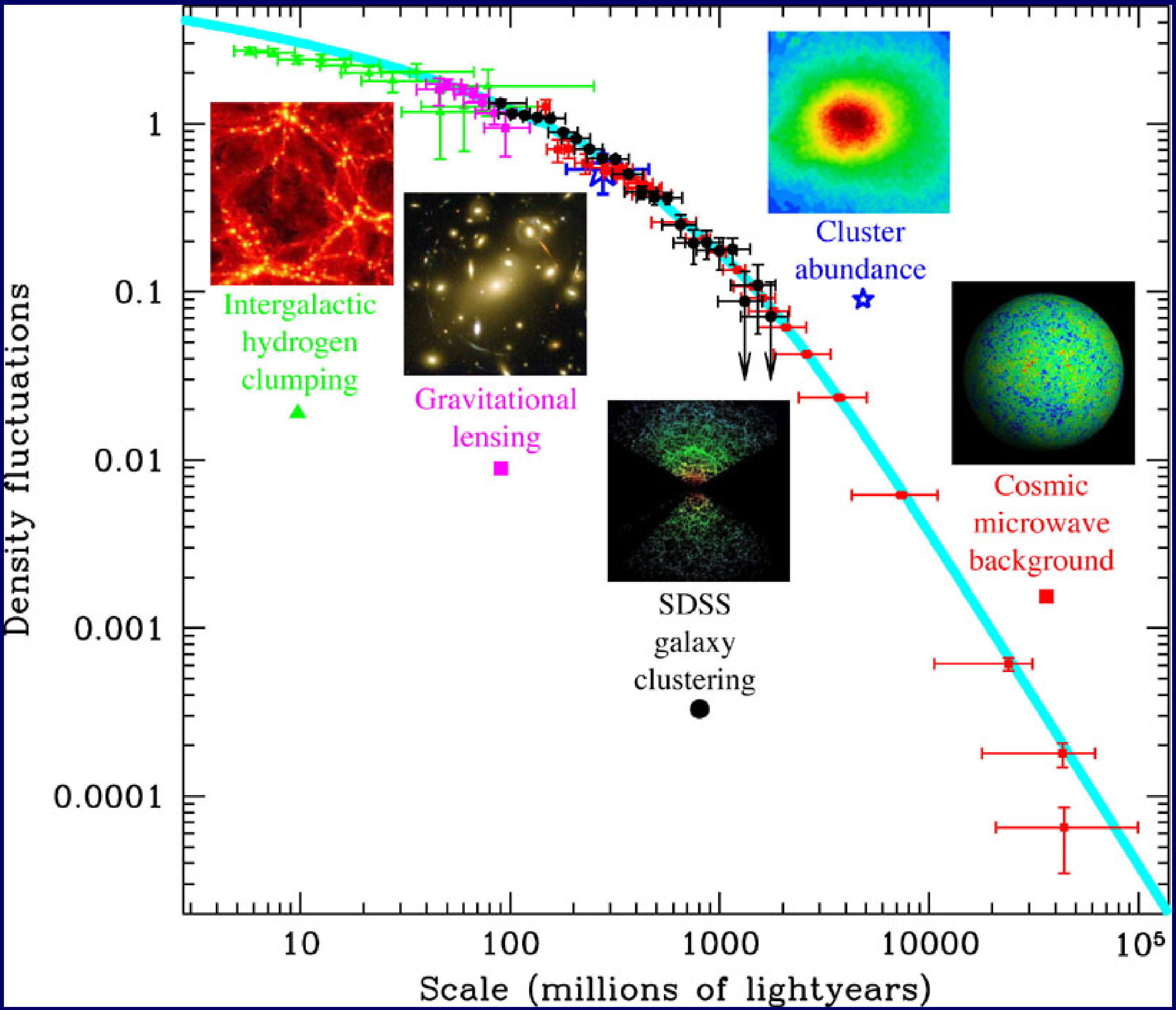
elliptical

bulge+disk

disk

Semi-Analytic Modeling

Power Spectrum



ΛCDM Power Spectrum

$$P(k) \propto k\, T^2(k)$$

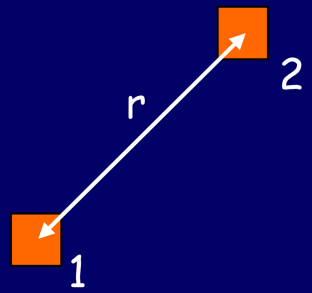
$$T(k) = \frac{\ln(1+2.34q)}{2.34q}\left(1+3.89q+(16.1q)^2+(5.46q)^3+(6.71q)^4\right)^{-1/4} \qquad q = \frac{k}{\Omega_m h^2 Mpc^{-1}}$$

normalization: $\sigma_8 \equiv \sigma_{tophat}(R = 8h^{-1}Mpc)$

Correlation Function

$$\xi(r) \equiv \left\langle \delta(\vec{x})\delta(\vec{x}+\vec{r}) \right\rangle_{\vec{x}} \qquad (1)$$

$$\rightarrow \xi(r) = \left\langle \sum_k \sum_{k'} \tilde{\delta}_k \tilde{\delta}_{k'} e^{i(k'-k)\cdot x} e^{-ik\cdot r} \right\rangle$$



δ real, can replace by complex conjugate $\tilde{\delta}_{k'}(-k') = \tilde{\delta}_{k'}^{\,*}(k')$

all cross terms k≠k' vanish on average because of periodic boundary conditions

isotropy $\left\langle |\delta_k|^2(\vec{k}) \right\rangle = |\delta_k|^2(k)$

$$\xi(r) = \frac{V}{2\pi} \int \left\langle |\delta_k|^2 \right\rangle e^{-ik\cdot r} d^3k$$

angular integration $\int_0^1 \cos(kr\cos\theta)\,\sin\theta\, d\theta = (kr)^{-1} \int_0^{kr} \cos y\, dy$

$$\rightarrow \xi(r) = \frac{V}{2\pi} \int P(k) \frac{\sin kr}{kr} 4\pi k^2 dk$$

example $P(k) \propto k^n \quad \rightarrow \xi(r) \propto r^{-(n+3)} \int_{kr=0}^{\infty} dx\, x^{n+1} \sin x$

Alternative interpretation:

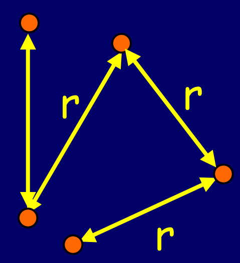
Construct a realization: select $\rho(x_1)$ and $\rho(x_2)$ from an ensemble.

Place a galaxy at volume δV with probability δP=ρ(x)δV $\delta P_{1,2} = \rho(x_1)\delta V_1 \; \rho(x_2)\delta V_2$

$$\delta = (\rho - \langle\rho\rangle)/\langle\rho\rangle \quad \rightarrow^{(1)} \quad \langle \rho(x)\rho(x+r) \rangle = \langle\rho\rangle^2 [1+\xi(r)] \quad (2)$$

$$\langle \delta P \rangle_{ensemble} = \langle \rho(x_1)\rho(x_2) \rangle \, \delta V_1 \delta V_2 =^{(2)} \langle\rho\rangle^2 [1+\xi(r)] \, \delta V_1 \delta V$$

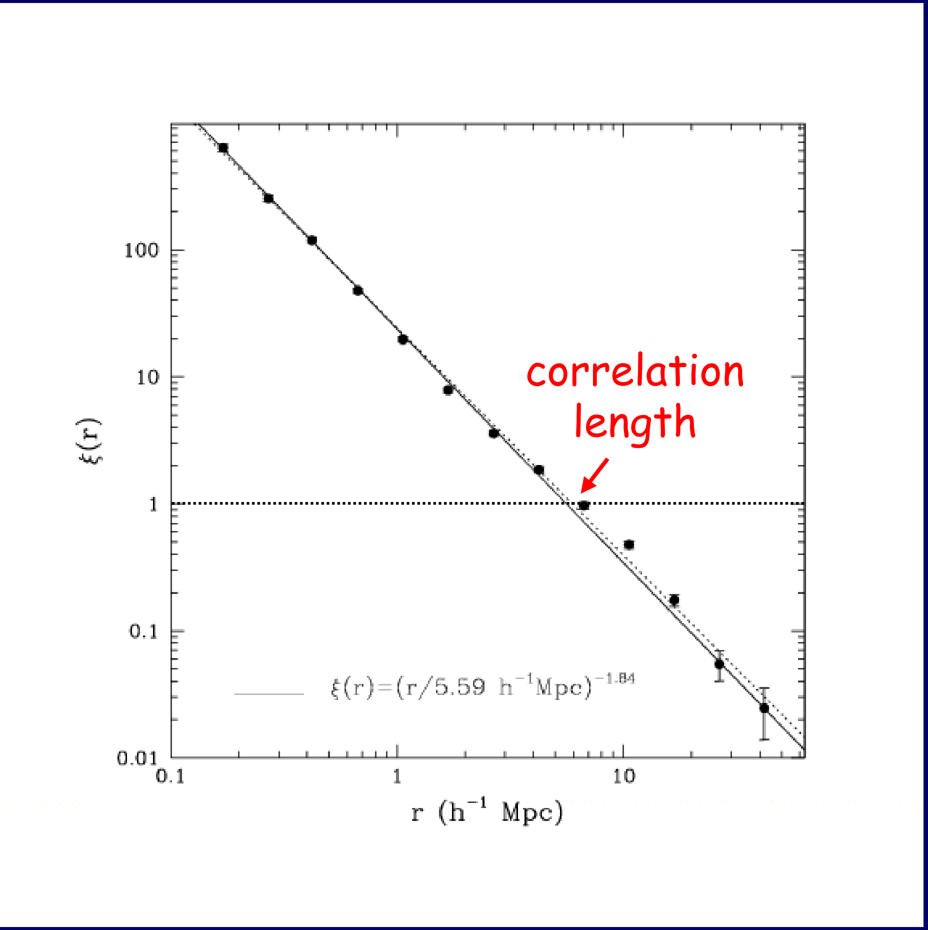
Exess probability over Poisson $1+\xi(r) = \dfrac{\#\, pairs\ (r)}{\#\, Poisson\ pairs\ (r)}$



Galaxy Correlation Function

Crude Description: power law

$$\xi(r) = \left(\frac{r}{r_0}\right)^{-\gamma} \qquad r_0 \approx 5\ h^{-1}Mpc \qquad \gamma \approx 1.8$$

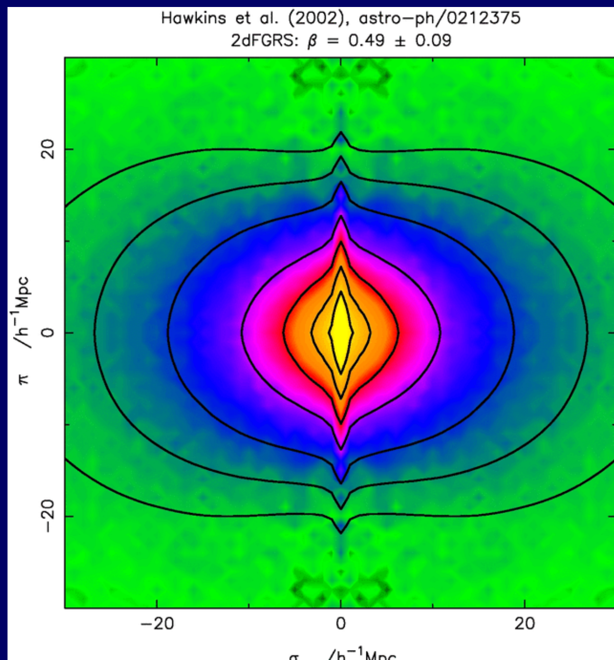


Zehavi et al. 2004
SDSS

Measured Correlation Functions

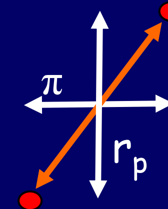
redshift distortions

redshift



angular separation

$\xi(r_p, \pi)$



$$\omega_p(r_p) = 2\int_0^\infty d\pi\ \xi(r_p, \pi)$$

$$\omega_p(r_p) = 2\int_0^\infty dy\ \xi[(r_p^2 + y^2)^{1/2}]$$

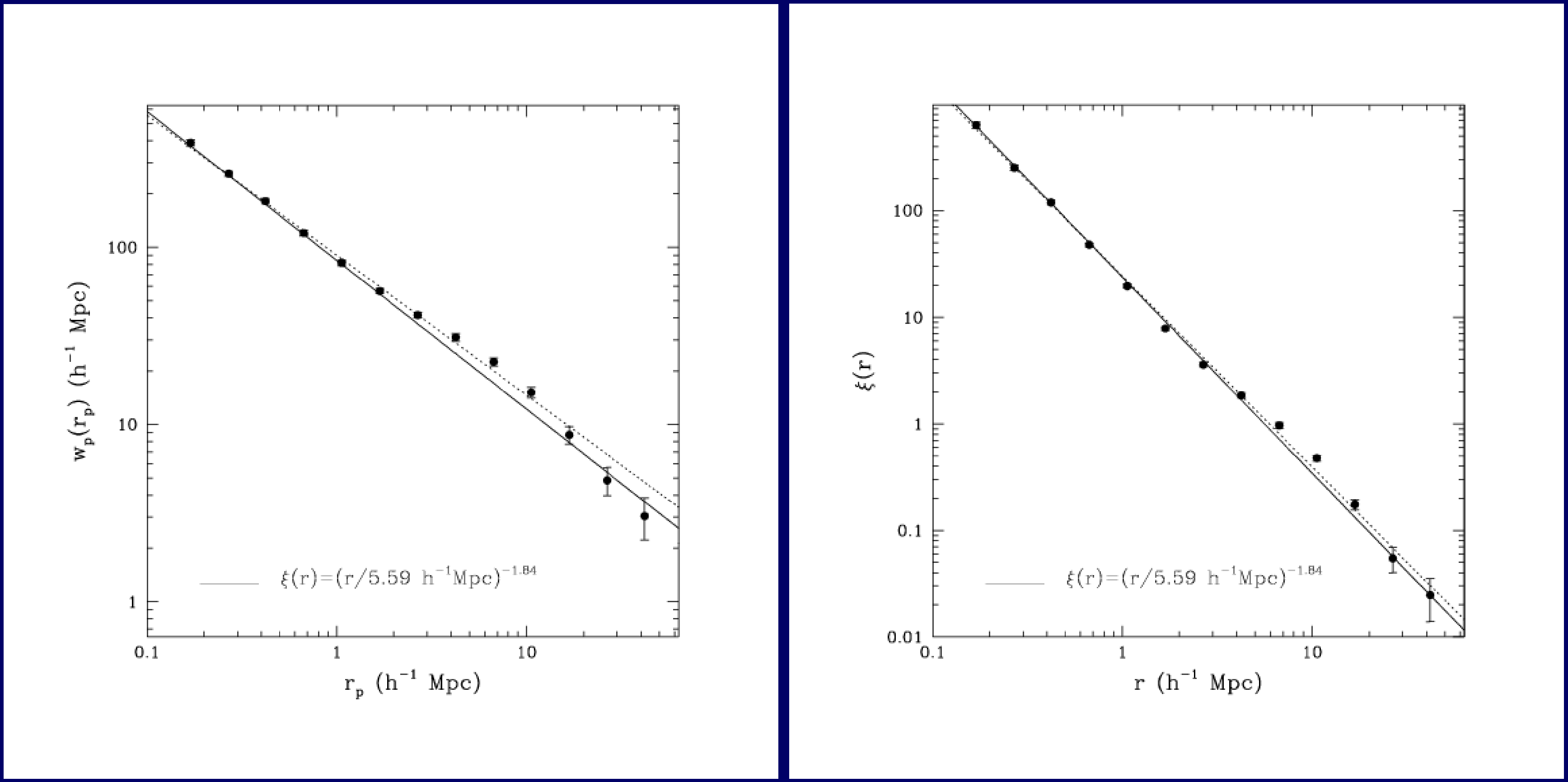
$$= 2\int_{r_p}^\infty r\, dr\ \xi(r)\,(r^2 - r_p^2)^{-1/2}$$

$$\xi(r) = -\pi^{-1}\int_r^\infty \omega_p(r_p)\,(r_p^2 - r^2)^{-1/2}\, dr_p$$

Davis & Peebles 83

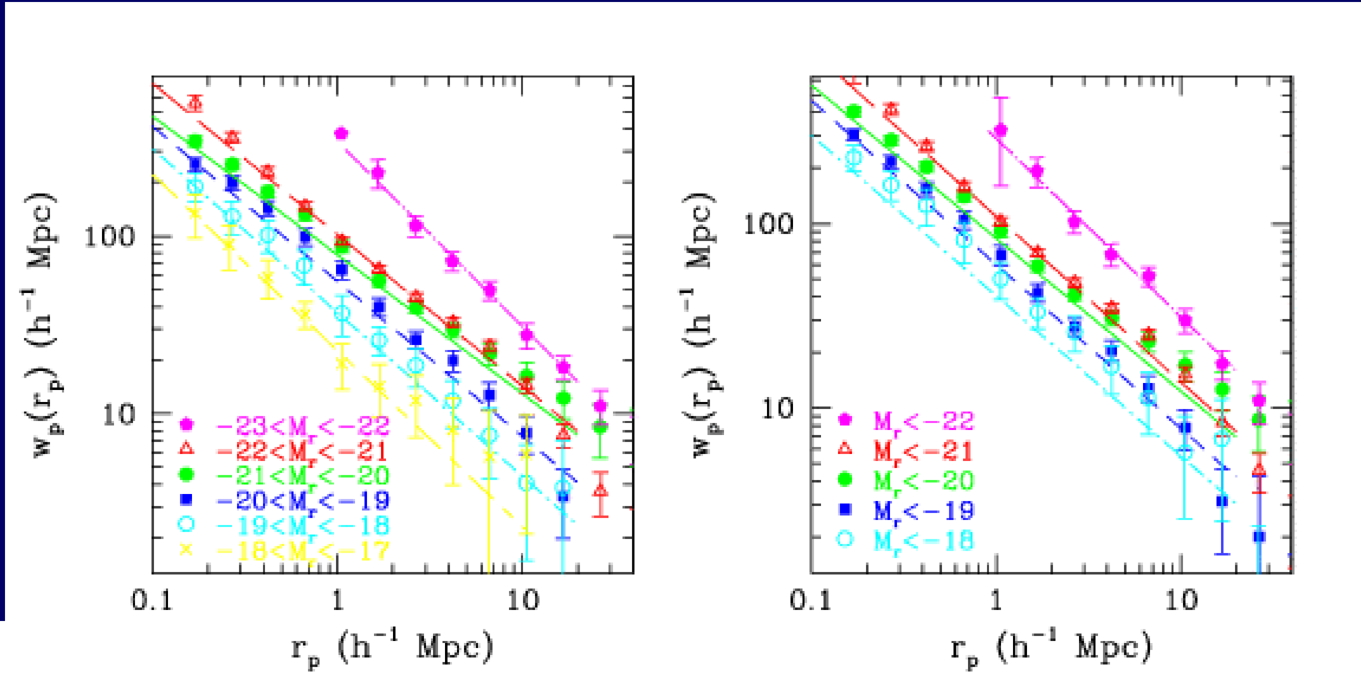
$$v_{obs} = cz = Hr + v_{pec}$$

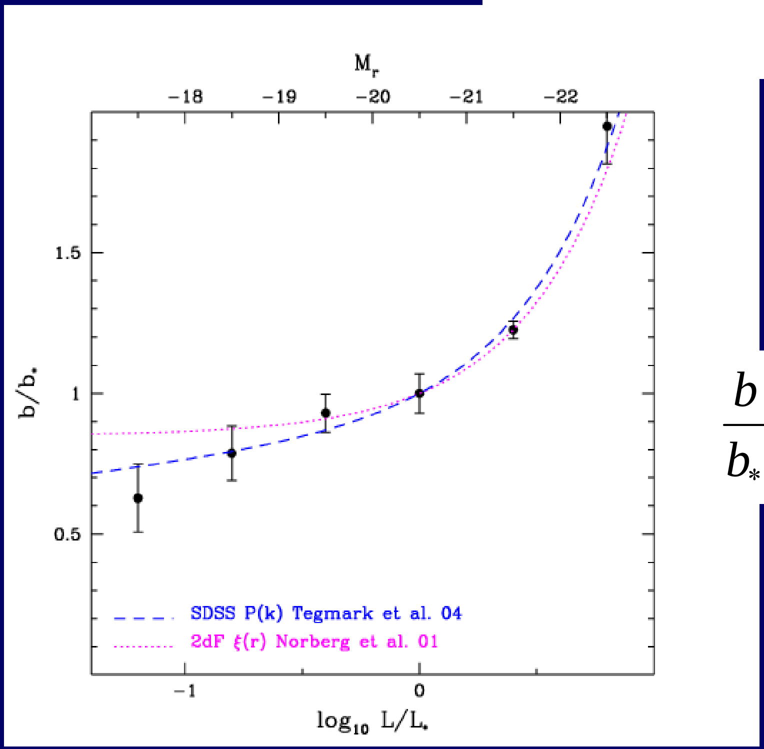
Galaxy Correlation Function



Zehavi et al. 04 SDSS

Biasing: Luminosity



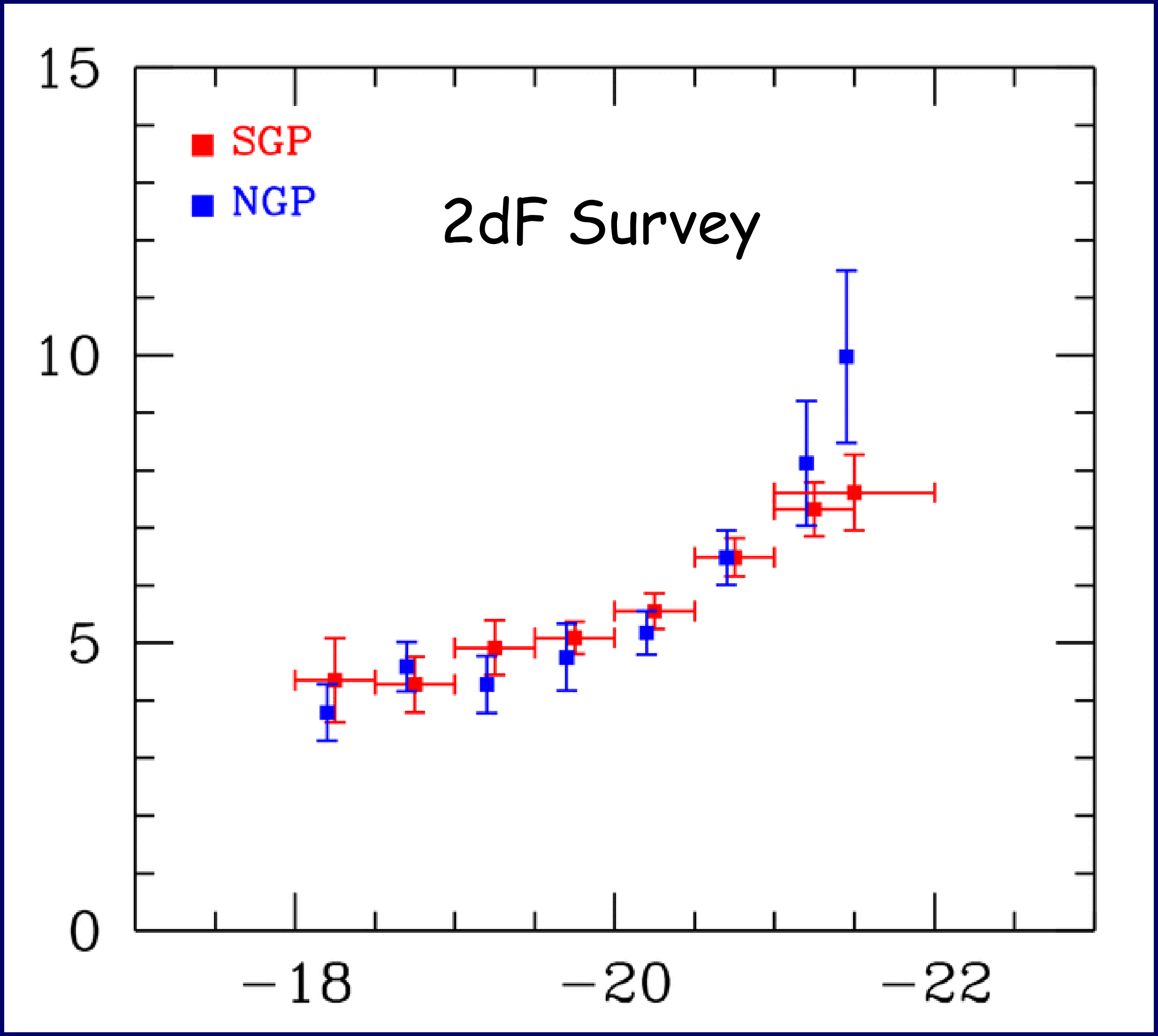


Zehavi et al. 04 SDSS

$$\frac{b}{b_*} = \frac{\omega_p(L)}{\omega_p(L_*)} \; at \; r_p = 2.7h^{-1}Mpc$$

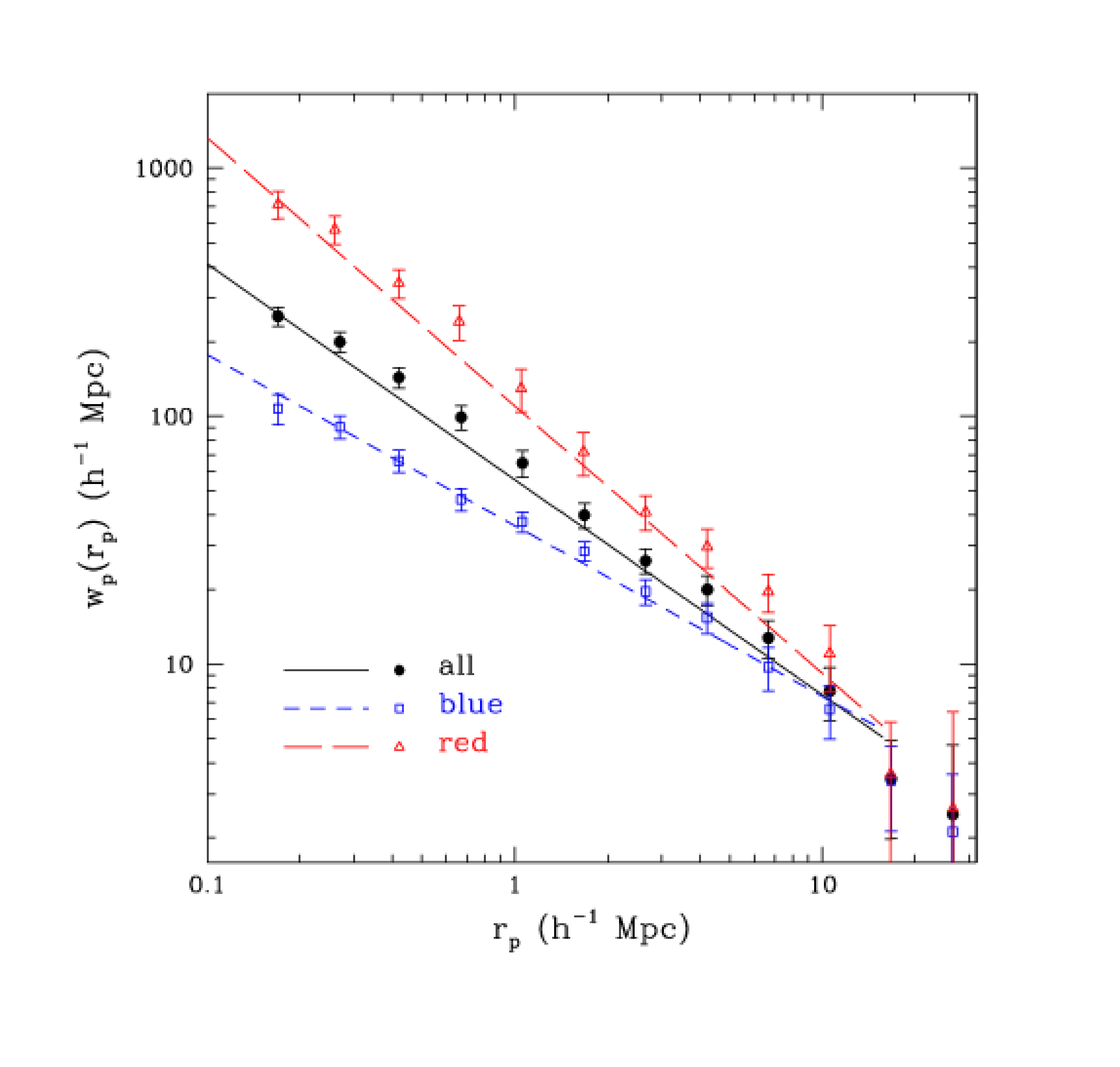
Luminosity Dependence of Galaxy Clustering

Correlation length (Mpc/h)

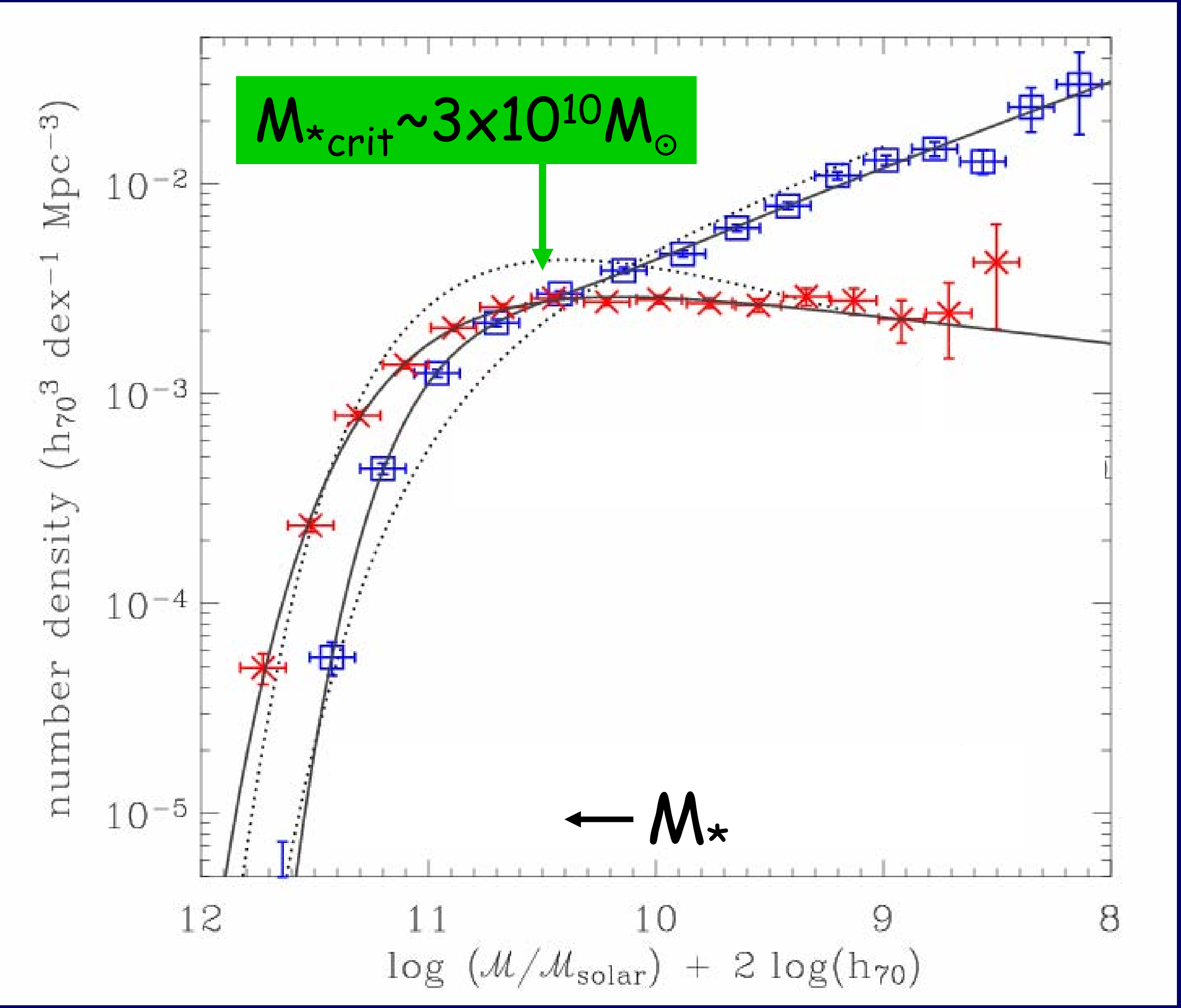


Luminosity ⟶

Biasing: color



Luminosity function: Early vs Late type

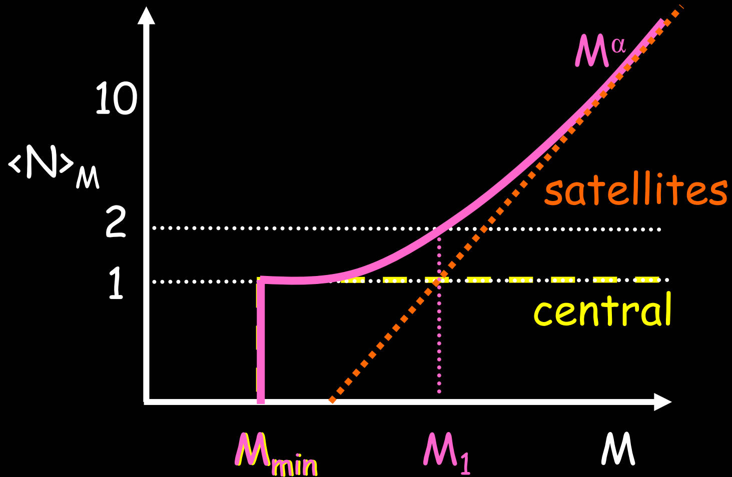


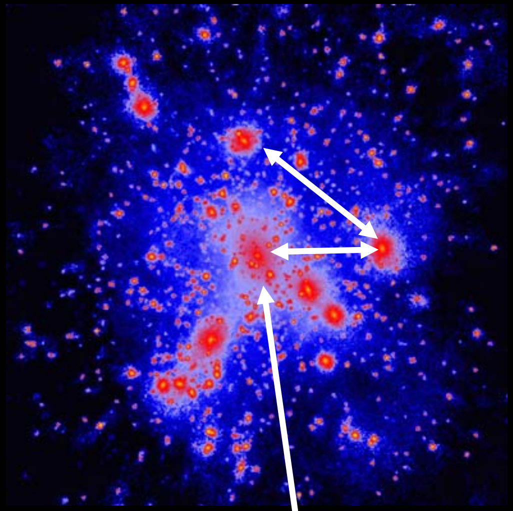
SDSS
Baldry et al. 04

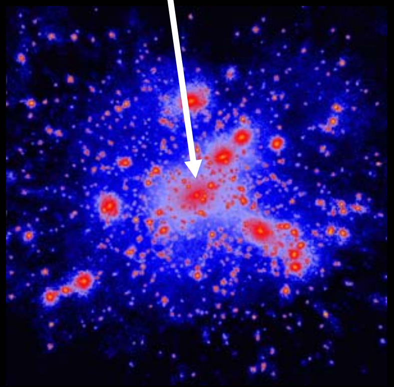
HOD model of Clustering

HOD = Halo Occupation Distribution

Galaxies $m > m_{min}$ in a halo M: conditional probability P(N|M)







Correlation Function

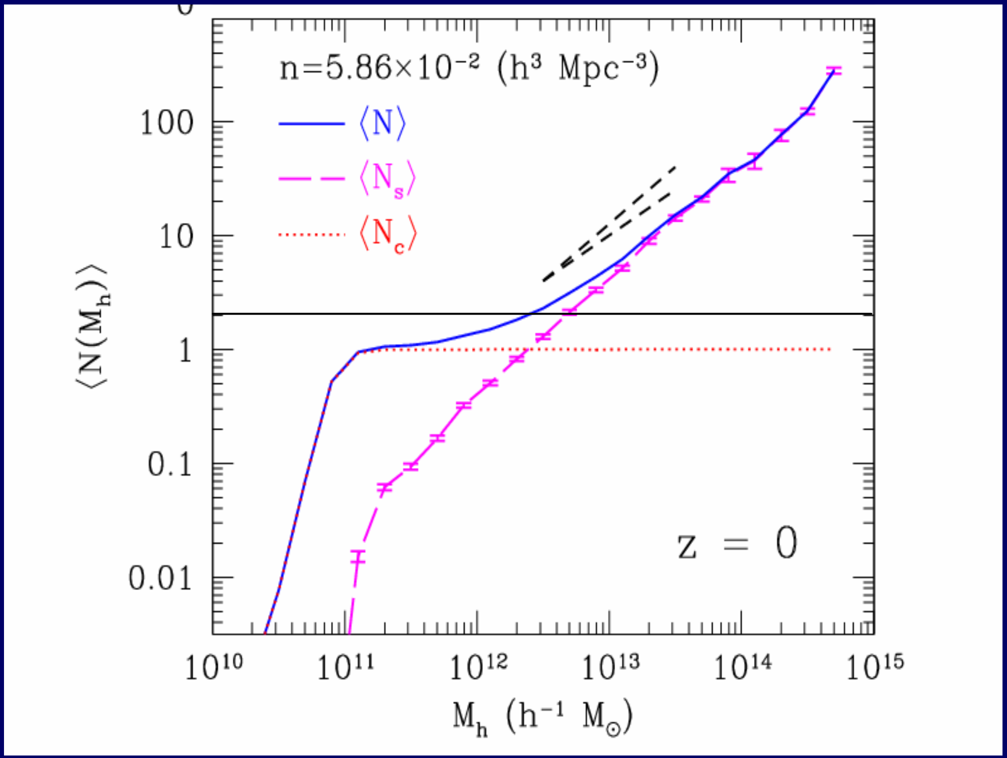
$$\xi(r) = 1 + \xi_{1h}(r) + \xi_{2h}(r)$$

$$1 + \xi_{1h}(r) = \frac{1}{2\pi r^2 \bar{n}_g^2} \int_0^\infty \frac{1}{2R(M)} \frac{dn}{dM} dM \; \frac{1}{2} \langle N(N-1) \rangle_M \; f\left(\frac{r}{2R(M)}\right)$$

\# Poisson pairs | n(M) | av. # pairs in halo | # pairs (r) from universal ρ(r)

$$\xi_{2h}(r) = \left\langle n(M) \langle N \rangle_M \; \xi_{halos\ M}(r) \right\rangle$$

Dark-Matter Halo Occupation Distribution



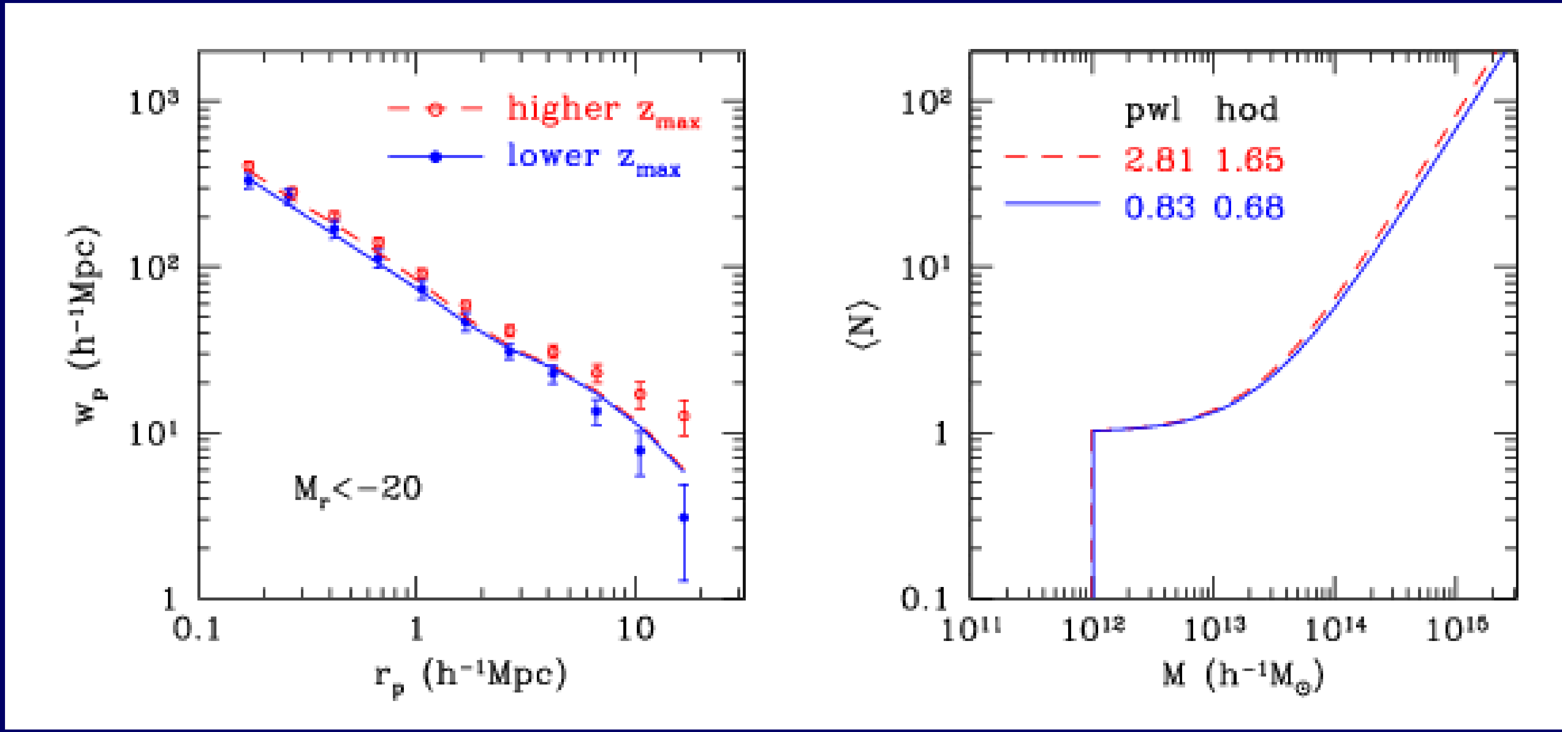
Kravtsov et al. 04,
N-body simulations

$M \sim M_*(t) \rightarrow$ group   at $z=0 \sim 10^{13} M_\odot$ at $z=1 \sim 10^{12} M_\odot$

$M \ll M_*(t) \rightarrow$ early formation,
satellites decay by dynamical friction

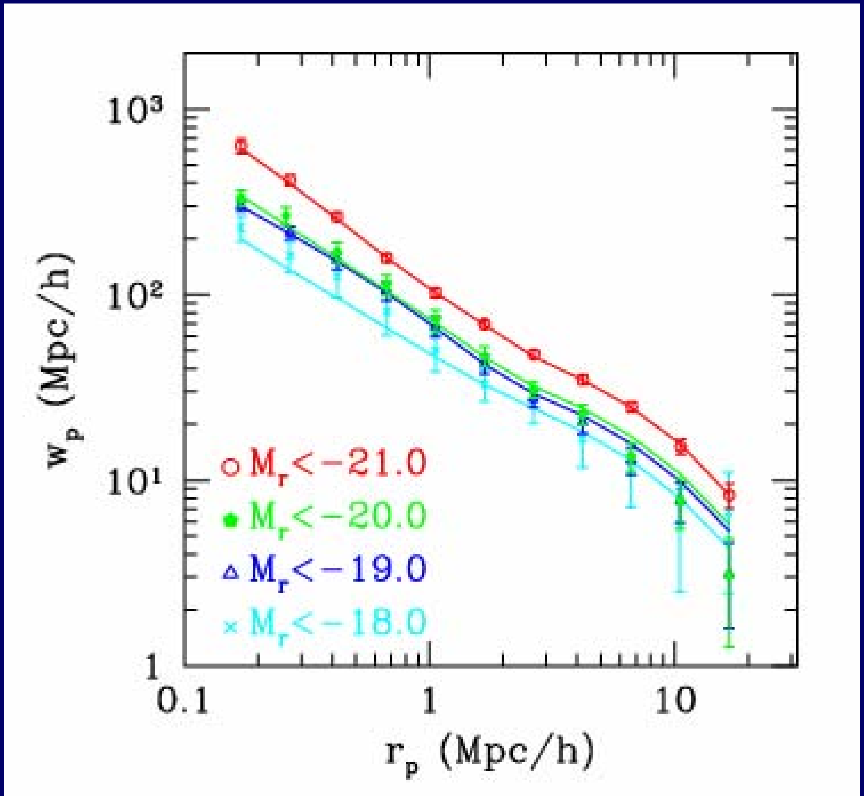
$$\frac{m_{sat}}{M_{halo}} < (0.01-0.1)\left(\frac{M_{halo}}{M_{*0}}\right)^{0.3}$$

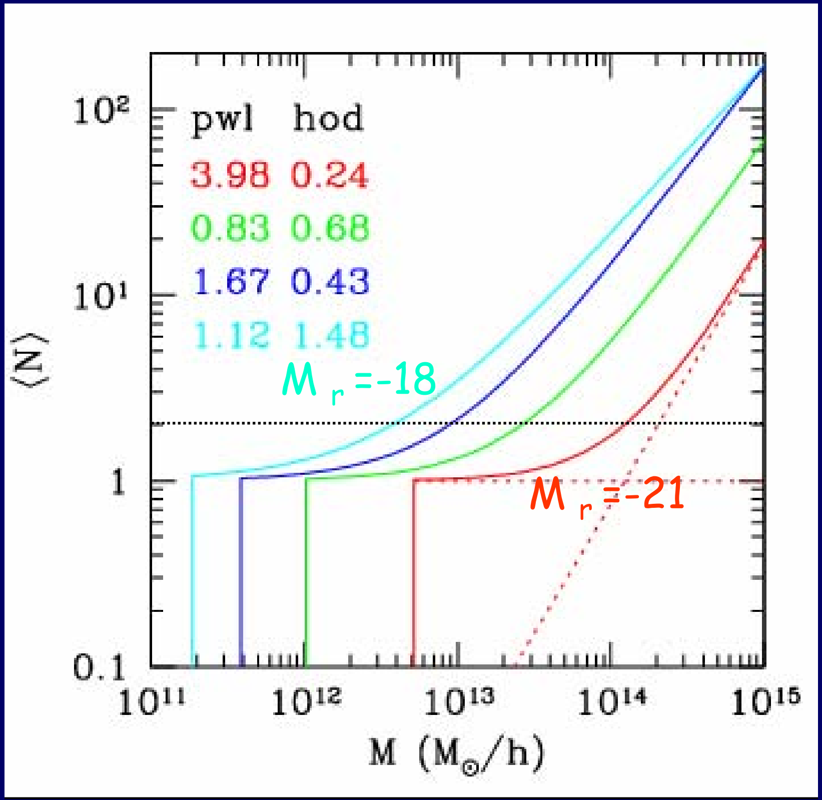
HOD from Correlation Function



Zehavi et al. 04 SDSS

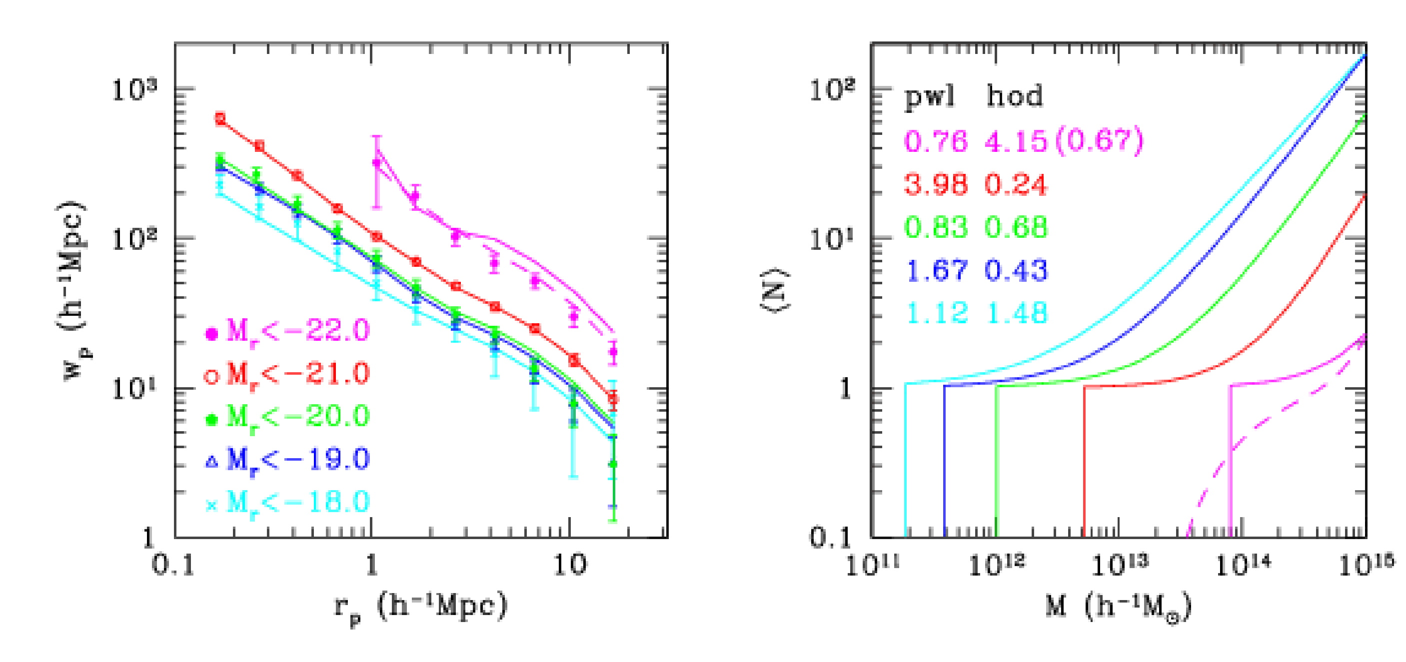
Biasing: Luminosity





Zehavi et al. 04, SDSS

Biasing: Luminosity



Biasing: color