

Galaxy Formation

par Avishai Dekel

The Hebrew University of Jerusalem
& Chaire Internationale de Recherche Blaise Pascal, Paris

**Une série de cours (en anglais) pour les étudiants
en thèse (et M2) et les chercheurs**

les mercredis, de 17h00 à 19h00 à l'amphithéâtre de l'IAP
98bis Bd Arago - Paris 14^{ème} - M° St Jacques ou Denfert-Rochereau

20 octobre

1. The standard cosmology
2. Linear growth of fluctuations by gravitational instability

A special lecture series on Galaxy Formation

by Avishai Dekel (Chaire Internationale Blaise Pascal)

for graduate students and researchers; IAP/OP Wednesdays 17:00-19:00

- | | |
|-------------|--|
| Octobre 20 | 1. the standard cosmology |
| | 2. linear growth of fluctuations by gravitational instability |
| Novembre 17 | 3. statistics of density fluctuations: the CDM scenario |
| | 4. nonlinear growth: spherical model, filamentary structure |
| Decembre 8 | 5. numerical simulations of structure formation |
| | 6. hierarchical clustering: Press-Schechter formalism, biasing |
| Decembre 15 | 7. dark-matter halos: density profile, cusp/core problem |
| | 8. halo substructure: dynamical friction, tidal effects, HOD |
| Janvier 5 | 9. angular momentum problem: tidal torques, disk formation |
| | 10. the origin of galaxy scaling relations and their scatter |
| Janvier 12 | 11. semi-analytic modeling: cooling, star formation, mergers |
| | 12. feedback processes: supernova, AGN and black holes |
| Fevrier 9 | 13. cold flows versus shock heating |
| | 14. origin of bi-modality in galaxies |
| Fevrier 16 | 15. dwarf galaxies and the "fundamental line" |
| | 16. dark-dark halos: effect of cosmological photoionization |

Lecture 7 (& part of 8)

Structure of Dark-Matter Halos

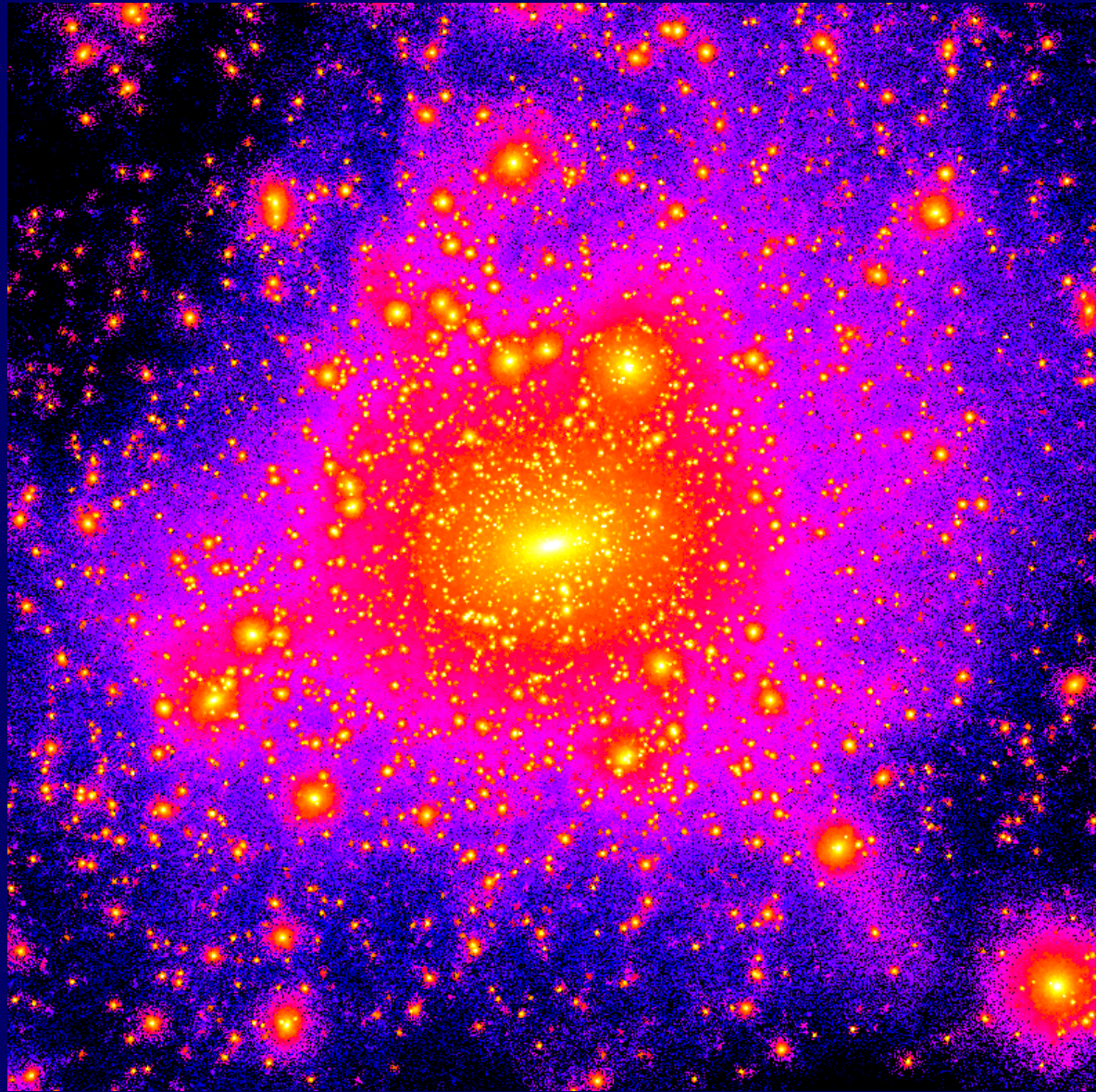
Universal Halo profile

The cusp/core problem

Dynamical friction

Tidal effects

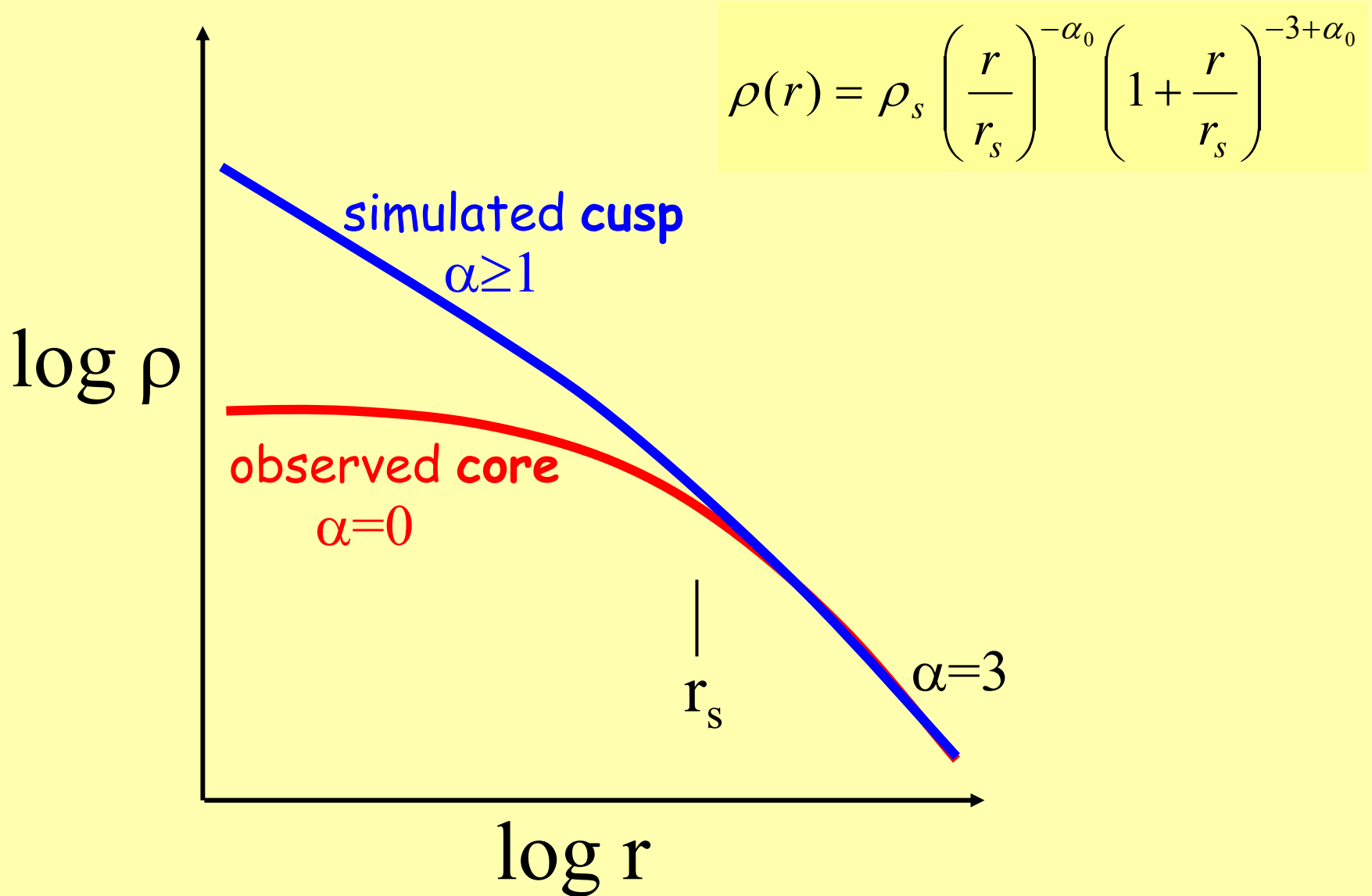
Origin of the cusp in hierarchical clustering



CDM halos (simulations)

- Density profiles are **universal**
shape independent of mass and cosmology.
- Density profiles are **cuspy**
density increases inward down to the innermost resolved radius. Asymptotic power-law near the center?
- Halos are **clumpy**
~10% of the mass is in self-bound clumps ---
the surviving cores of accreted satellites.

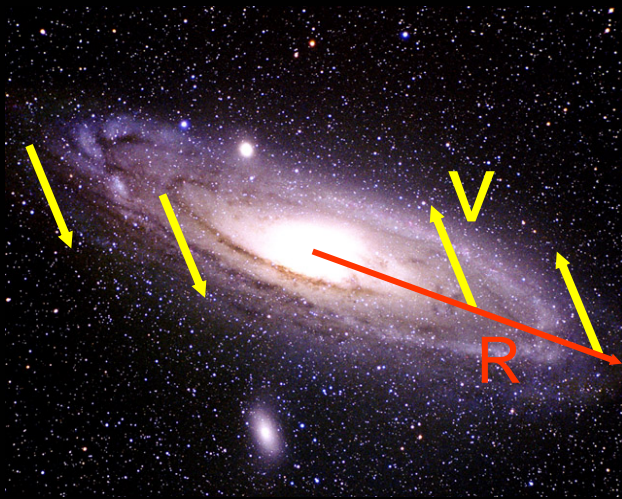
The dark-halo cusp/core problem



Universal Profile

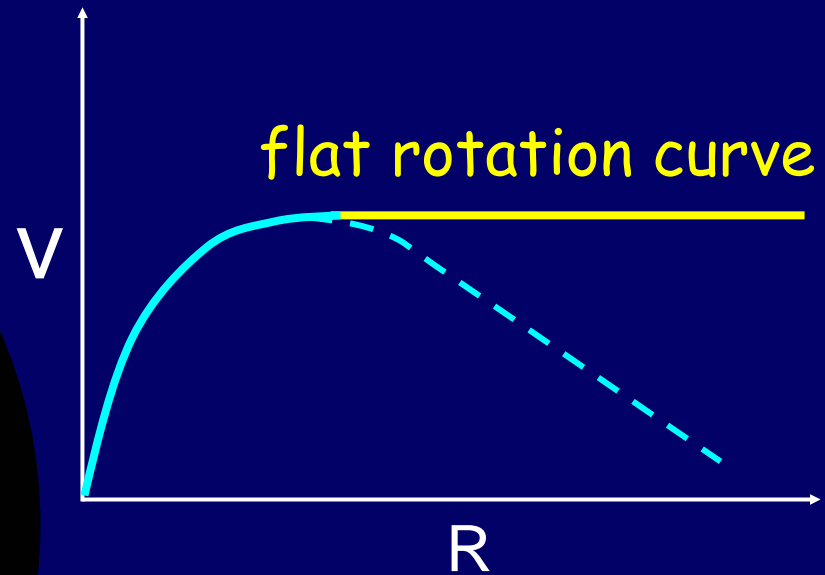
Dark Halos

dark halo



3,000 light years

30,000 ly



$$V^2 = \frac{GM(R)}{R}$$
$$\rightarrow M(R) \propto R$$

Isothermal Sphere

Hydrostatic equilibrium:

$$\frac{GM(r)\rho(r)}{r^2} = -\frac{dP}{dr} = \frac{\alpha\sigma^2\rho(r)}{r}$$

$$\rho(r) = \rho_0 r^{-\alpha} \rightarrow M(r) = \frac{4\pi}{(3-\alpha)} \rho_0 r^{(3-\alpha)}$$

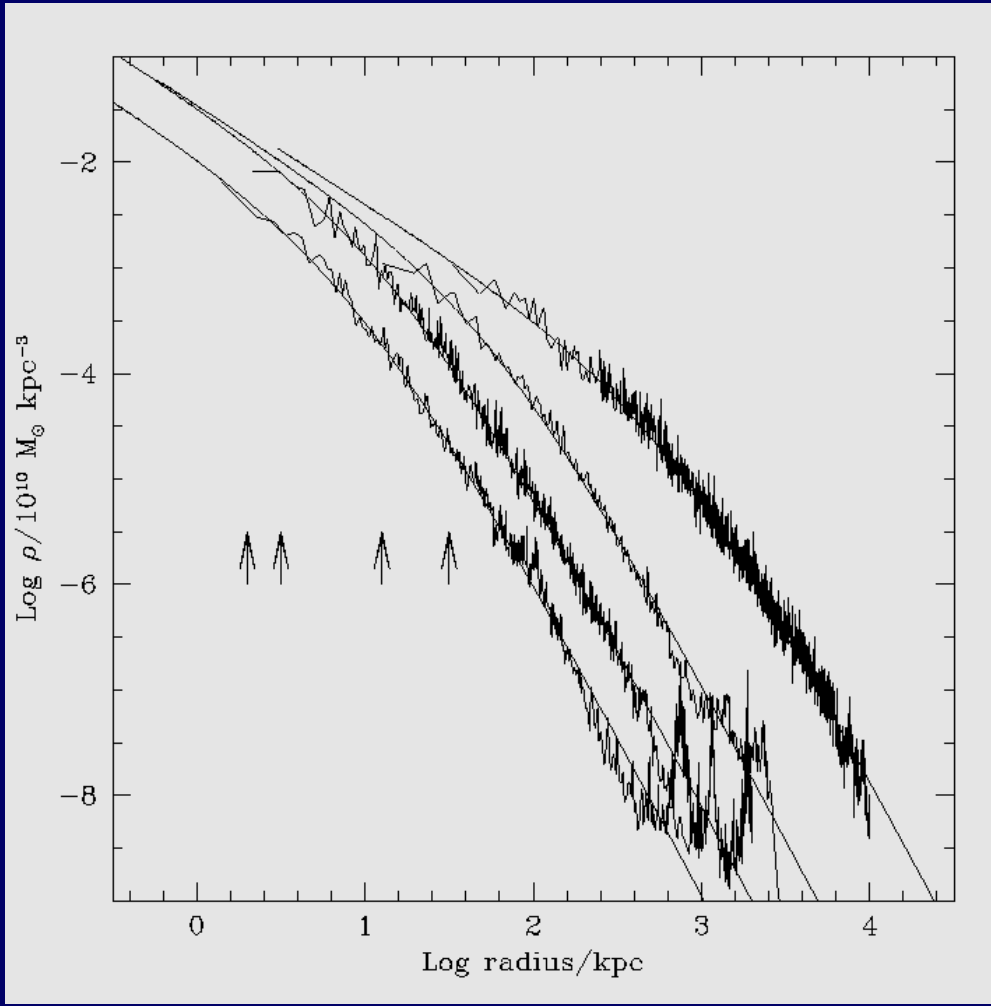
$$P = nkT = \rho \frac{kT}{m} = \rho(r)\sigma^2 \rightarrow \frac{dP}{dr} = -\frac{\alpha\rho(r)\sigma^2}{r}$$

isothermal

$$\rightarrow M(r) = \frac{2\sigma^2}{G} r \rightarrow \rho(r) = \frac{\sigma^2}{2\pi G} r^{-2}$$

$$V^2(r) = \frac{GM(r)}{r} = 2\sigma^2$$

Universal Mass Profile of CDM Halos



Density

Radius

Mass profile general shapes are independent of halo mass & cosmological parameters

Density profiles differ from power law

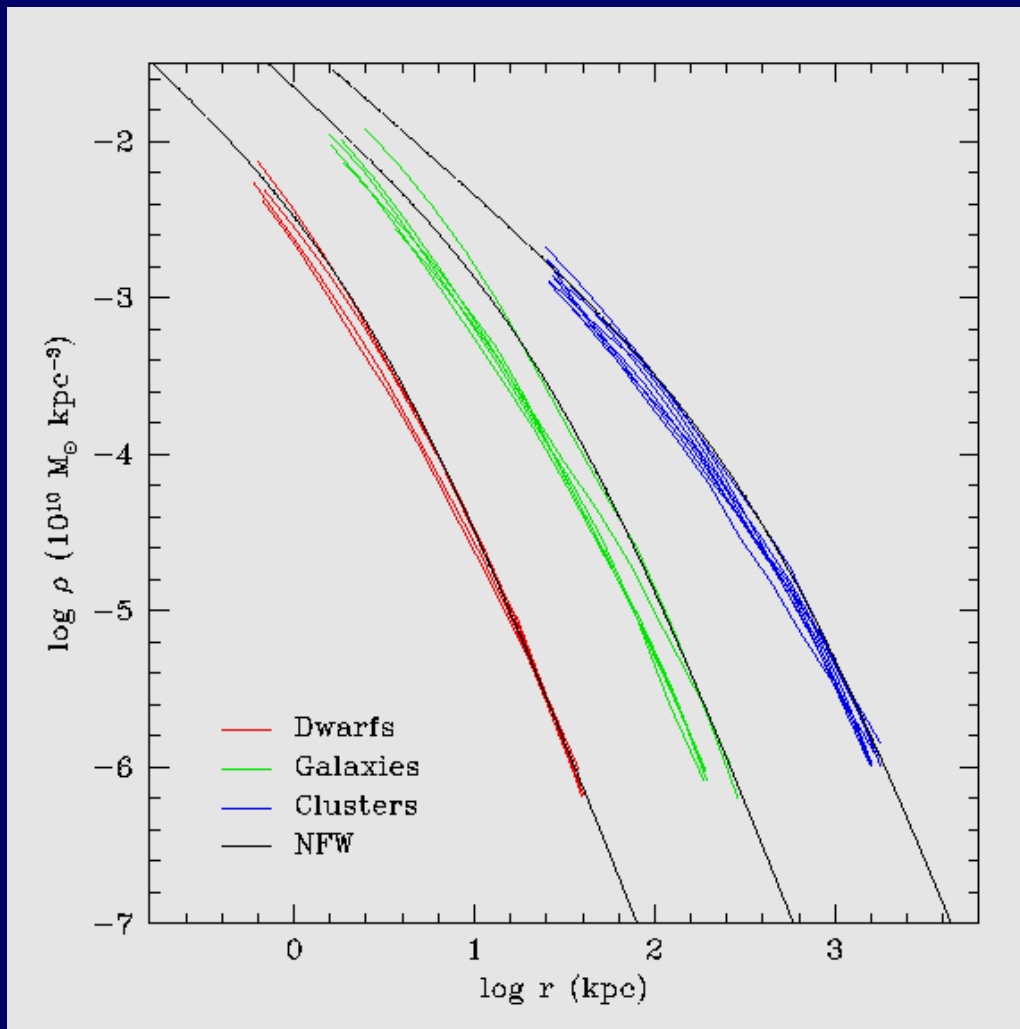
The profile is shallower than isothermal near the center

But no obvious flat-density core near the center

A cusp; some controversy about inner slope

New results for Λ CDM halos

Density



Radius

Simulations span ~ 6 decades in M_{vir} , from dwarf galaxies ($V_c \sim 50$ km/s) to galaxy clusters ($V_c \sim 1000$ km/s)

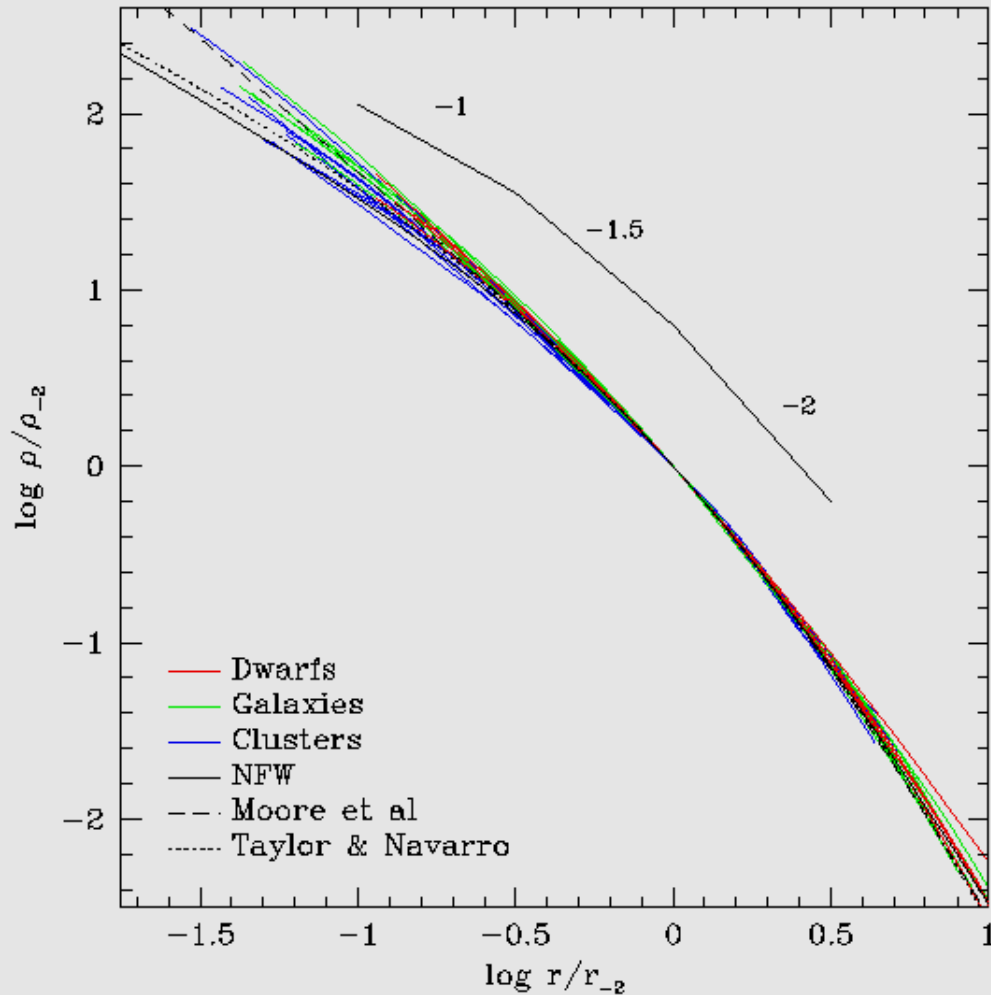
\sim million particles within R_{vir}

Controlled numerical effects via convergence studies

Navarro, Frenk, White, Hayashi, Jenkins, Power, Springel, Quinn, Stadel

Recent results for Λ CDM halos

Scaled Density



Properly scaled, all halos look alike: CDM halo structure appears to be "universal"

Scaled Radius

Navarro, Frenk, White, Hayashi, Jenkins, Power, Springel, Quinn, Stadel

Universal Profile: NFW

$$\rho(r) = \frac{\rho_s}{x(1+x)^2} \quad x \equiv \frac{r}{r_s} \quad \text{for } 0.01R_{vir} < r < R_{vir}$$

generalized
cusp:

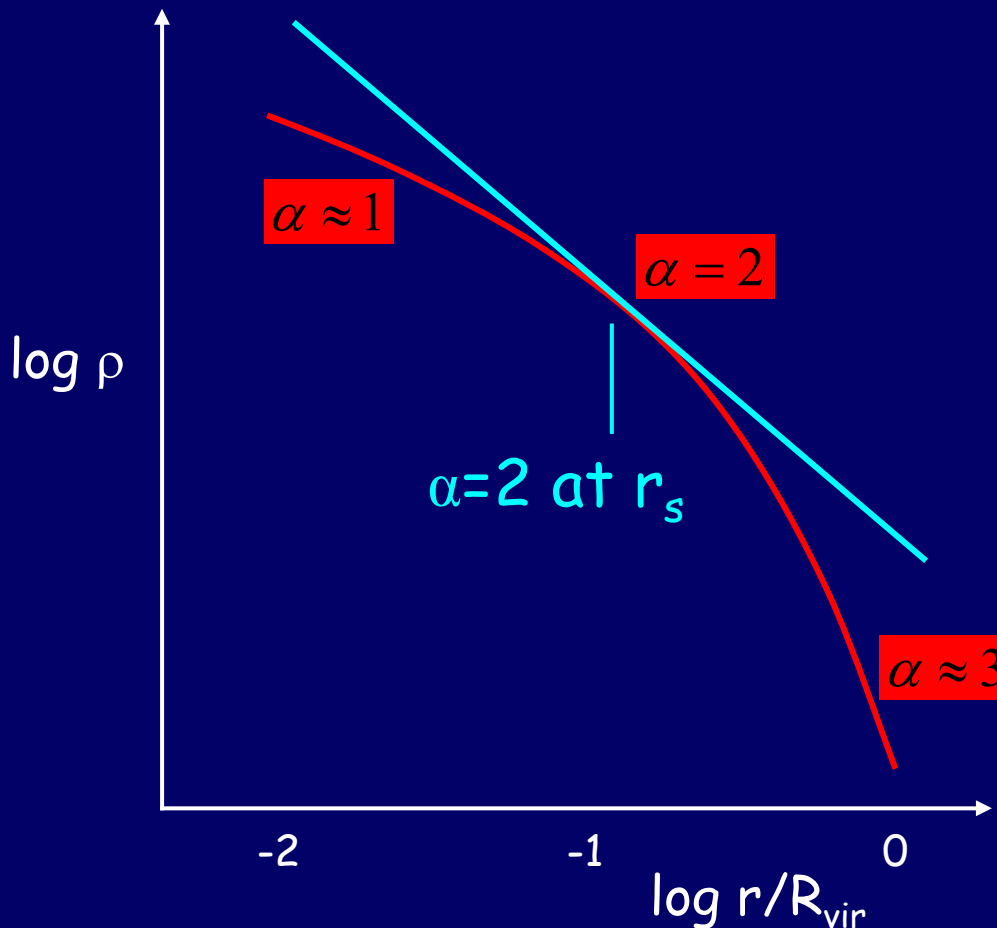
$$\rho(r) = \frac{\rho_s}{x^{\alpha_0} (1+x)^{3-\alpha_0}}$$

slope: $\alpha(r) = -\frac{d \ln \rho}{d \ln r}$

two parameters:

$$M_{vir} \quad C \equiv \frac{R_{vir}}{r_s} \sim 10$$

Ellipsoidal shape: $a_3/a_1 \sim 0.5$



- Navarro, Frenk & White 95, 96, 97
- Cole & Lacey 96
- Moore et al. 98
- Ghinga et al. 00
- Klypin et al. 01
- Power et al. 02
- Navarro, Hayashi et al. 03, 04
- Stoehr et al. 04, 05

Halo Concentration vs Mass and History

Self-similar Toy model (Bullock et al. 2001):

Define a_c as the time when typically a constant fraction f of M is collapsing:

$$M_*(a_c) \equiv f M \quad (1)$$

Define a characteristic halo density:

$$\tilde{\rho}_s \equiv \frac{M}{(4\pi/3)r_s^3} = 3\rho_s \left(\ln(1+C) - \frac{C}{1+C} \right) \quad \text{for NFW}$$

Assume additional contraction of inner halo by a constant factor k :

$$\tilde{\rho}_s = k^3 \Delta(a) \rho_u(a_c) = k^3 \Delta(a) \rho_u(a) \frac{a^3}{a_c^3}$$

$$C \equiv \frac{R_{vir}}{r_s} \longrightarrow C(\mu, a) = k \frac{a}{a_c} \quad (2)$$

EdS
 $P_k \propto k^n$

$$\sigma \propto M^{-\alpha} \rightarrow M_* \propto a^{1/\alpha} \rightarrow \frac{a_c}{a_0} = (\mu f)^\alpha$$

$$\mu \equiv M(a) / M_*(a)$$

$$C(\mu, a) = k (f\mu)^{-\alpha}$$

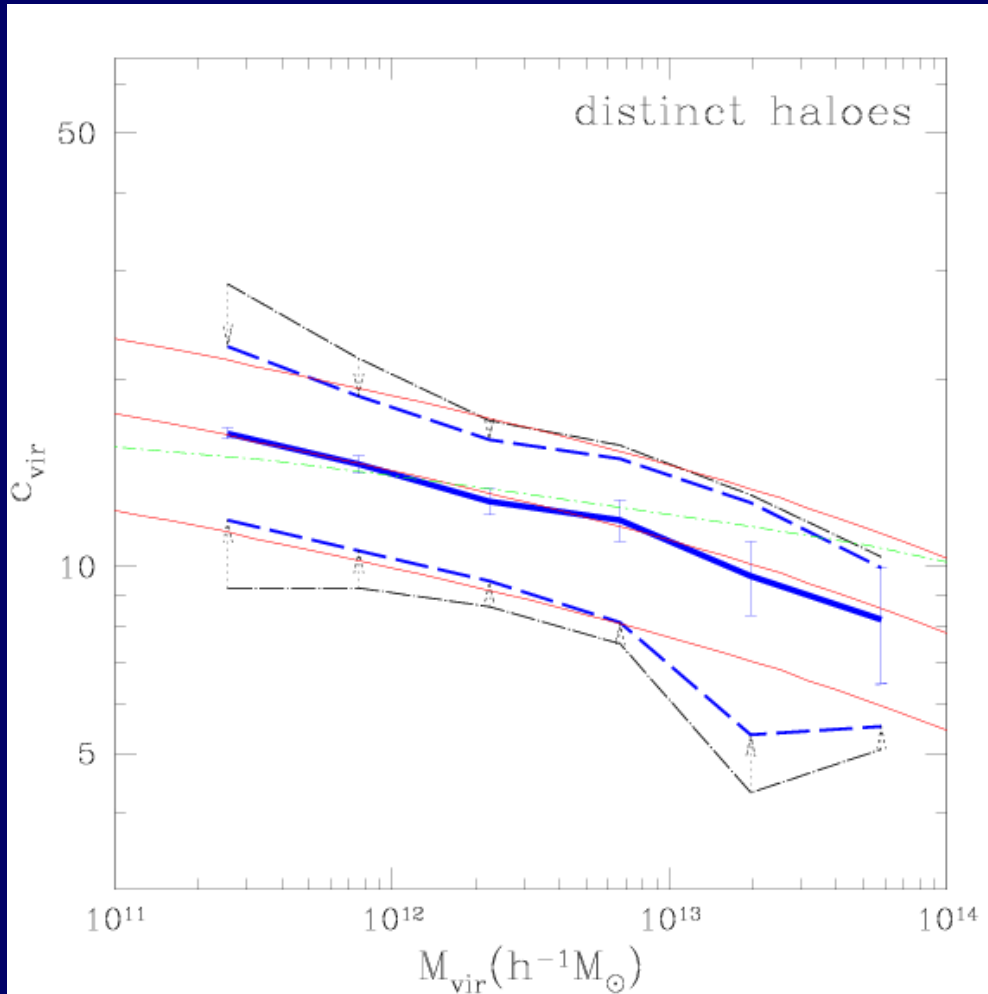
Determine parameters from simulations:

$$f \sim 0.01 \quad k \approx 4 \quad \alpha \approx 0.13$$

Excellent fit!

$$C(\mu, a) \approx 4 (0.01\mu)^{-0.13} \approx 4 \frac{a}{a_c}$$

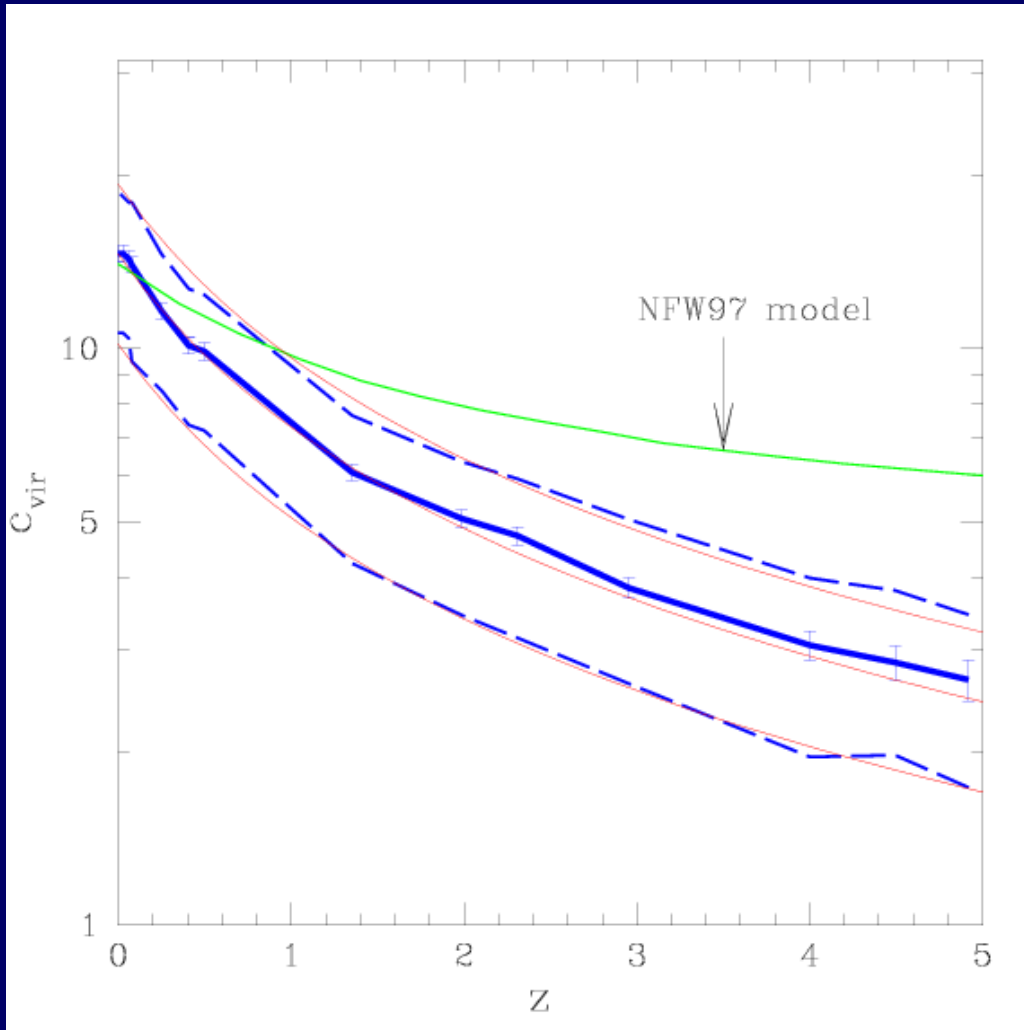
Concentration vs Mass



$$C(\mu, a) \approx 4 (0.01\mu)^{-0.13} \approx 4 \frac{a}{a_c}$$

Bullock et al. 2001

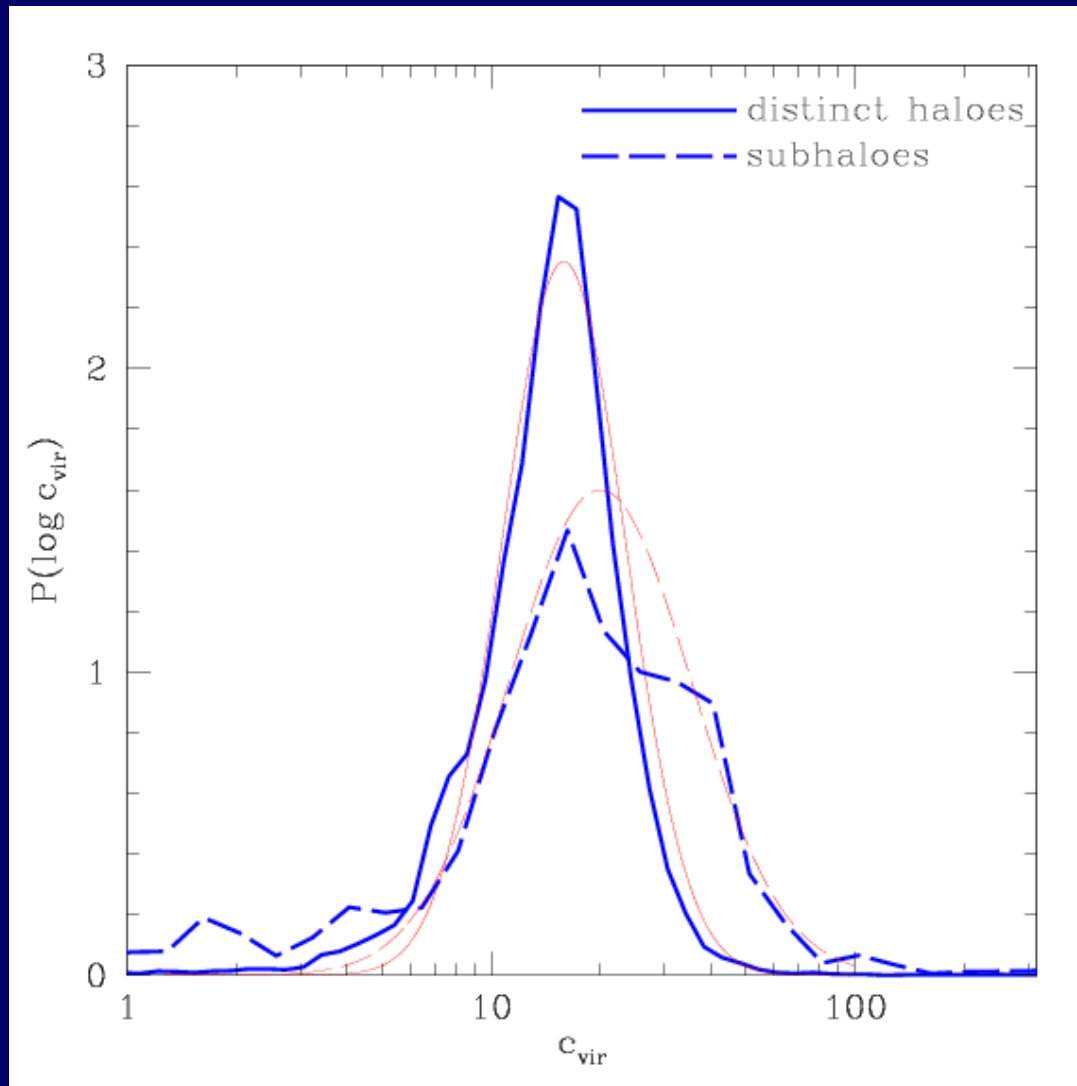
Concentration vs time, given mass



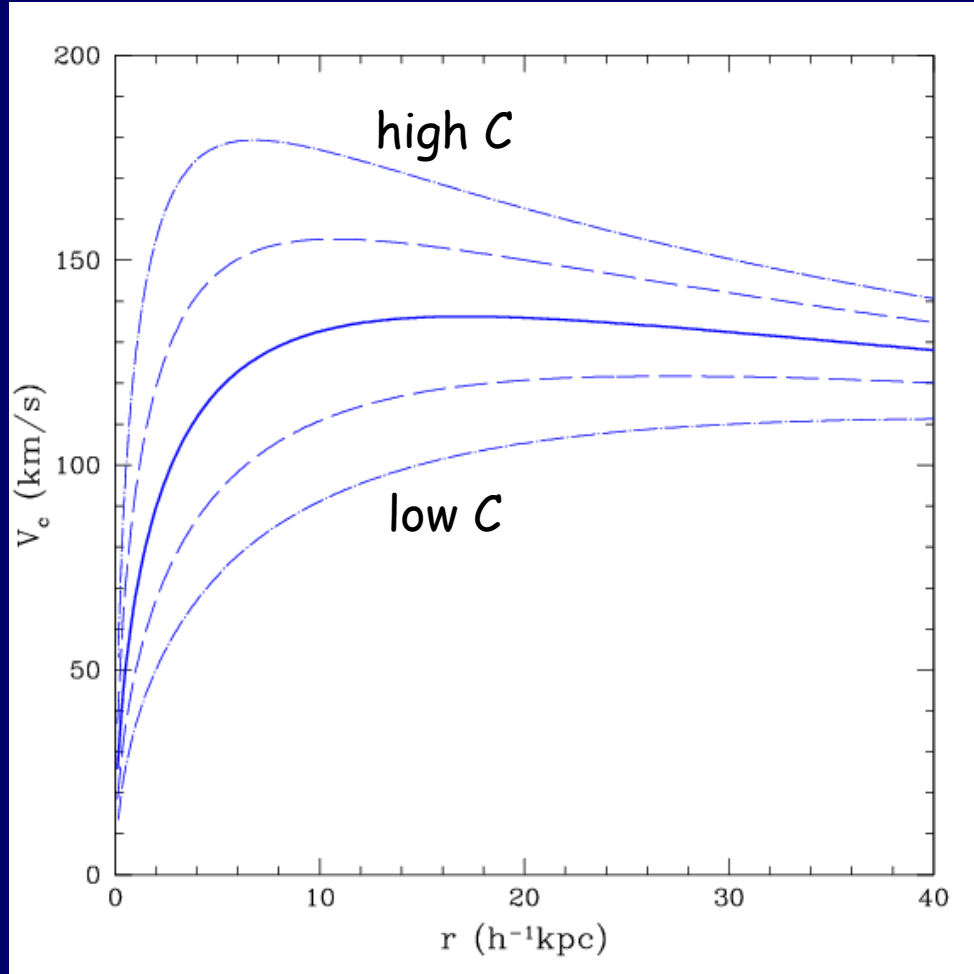
$$C(\mu, a) \approx 4 (0.01\mu)^{-0.13} \approx 4 \frac{a}{a_c}$$

Bullock et al. 2001

Distribution of C : log-normal



NFW Rotation Curve



$$M = 4\pi\rho_s A(C) \quad A(C) \equiv \ln(1+C) - \frac{C}{1+C}$$

$$V^2(x) = V_{vir}^2 \frac{C}{A(C)} \frac{A(x)}{x}$$

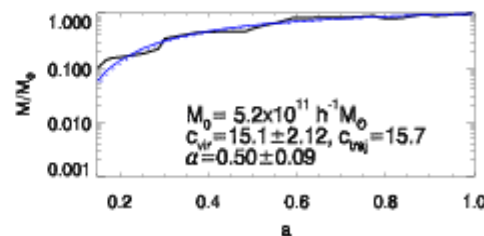
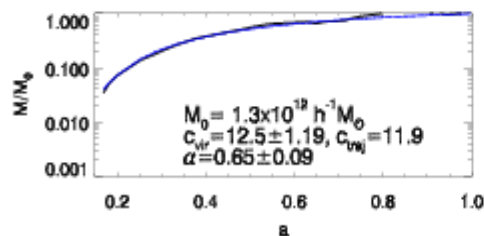
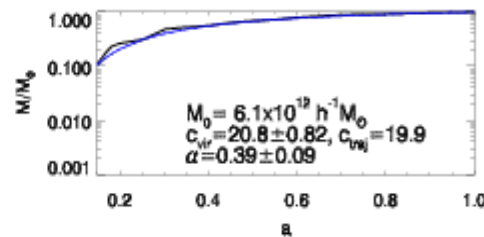
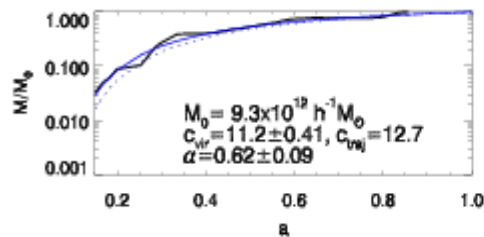
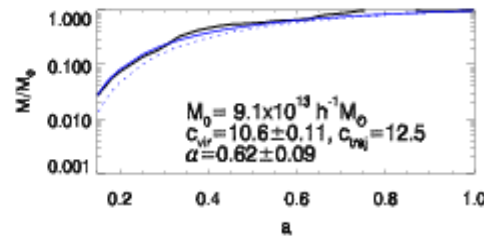
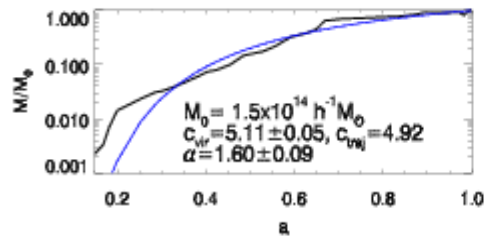
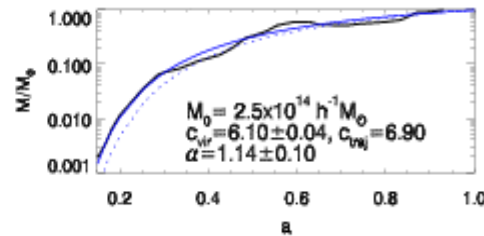
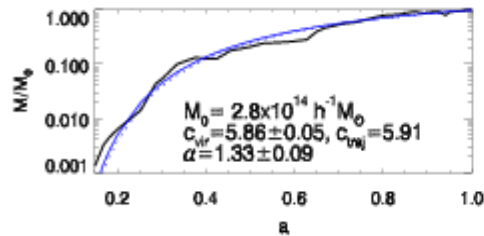
$$r_{max} \approx 2.16r_s \quad \frac{V_{max}^2}{V_{vir}^2} \approx 0.216 \frac{C}{A(C)}$$

Mass Assembly History

Wechsler et al. 2002

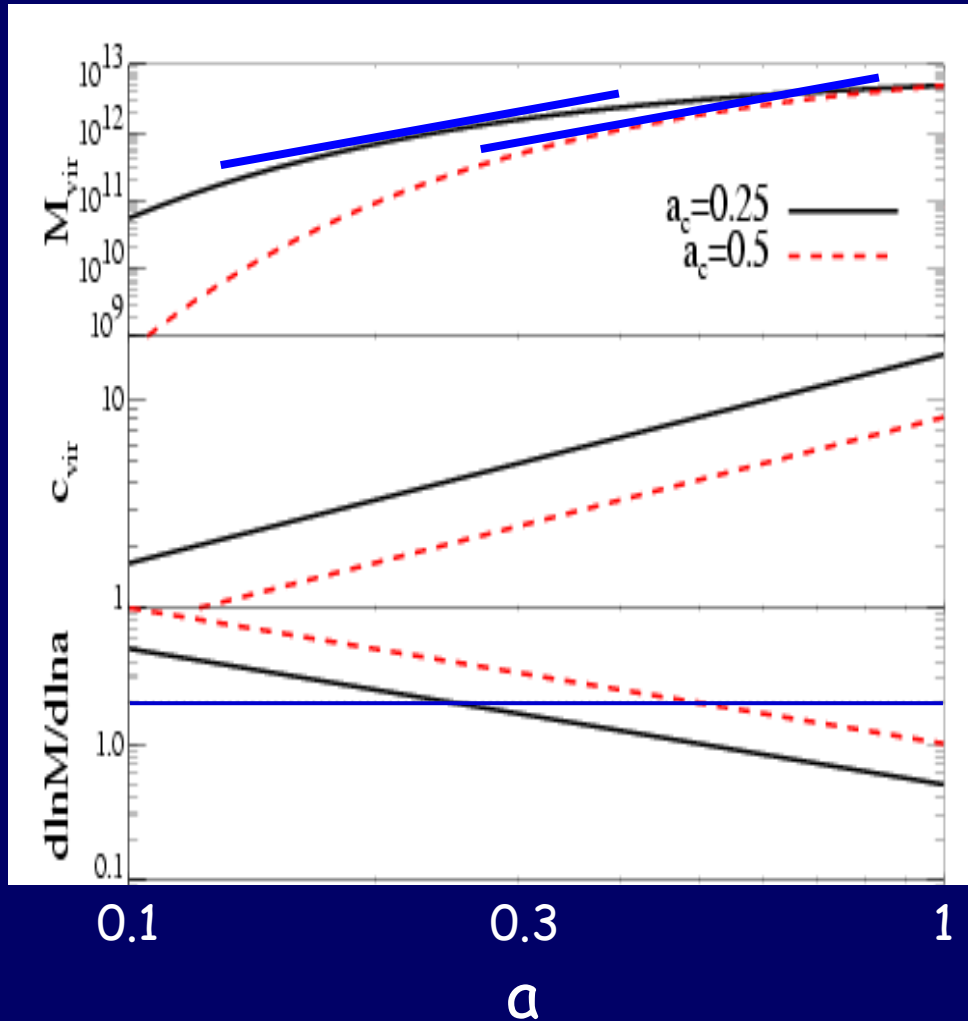
$$M(a) \propto e^{-2a_c z}$$

$$\frac{d \log M}{d \log a} = 2 \quad \text{defines } a_c$$



Mass Assembly History

Wechsler et al. 2002

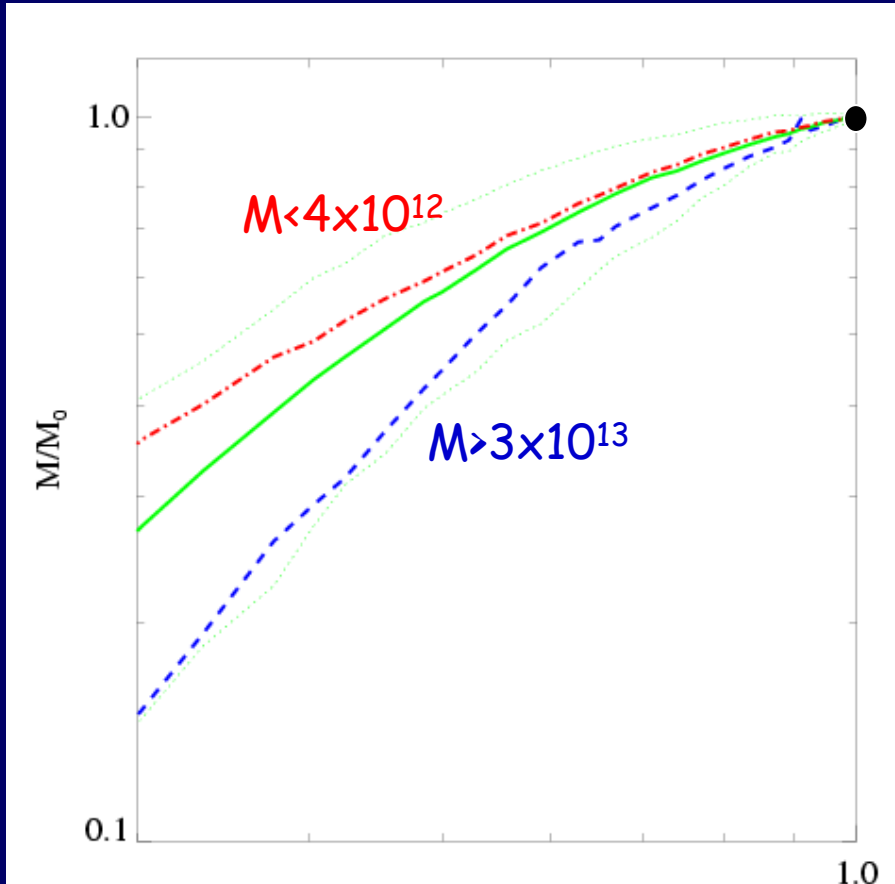


$$M(a) \propto e^{-2a_c z}$$

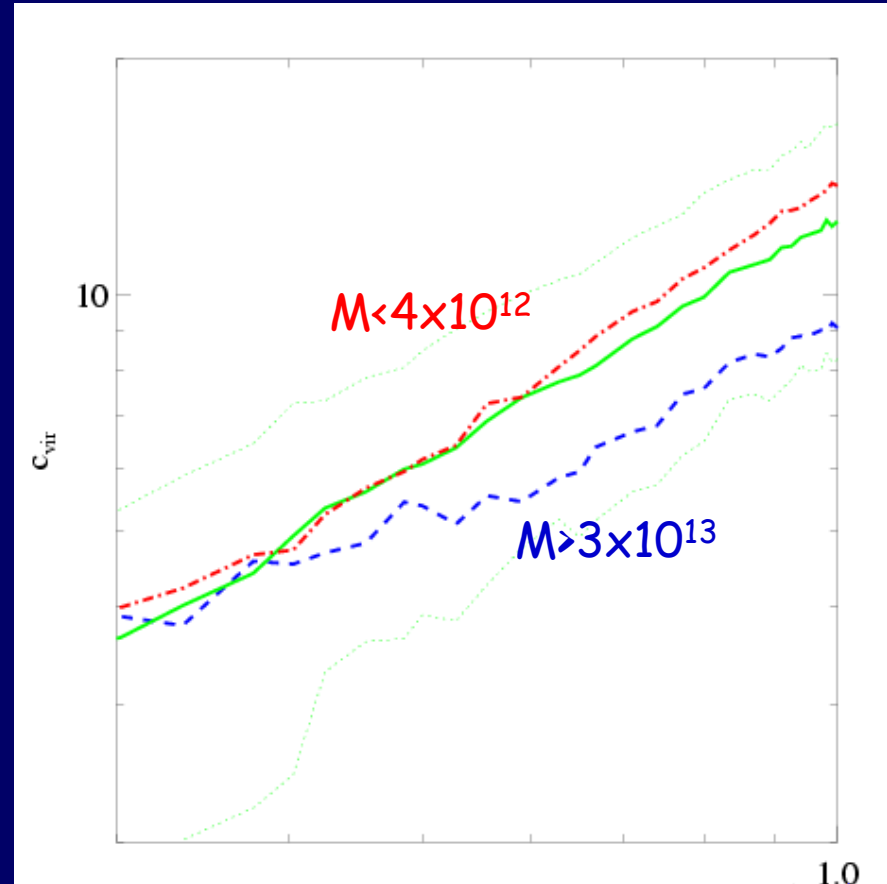
$$\frac{d \log M}{d \log a} = 2 \quad \text{defines } a_c$$

Mass dependence of History and Concentration

Wechsler et al. 2002



a

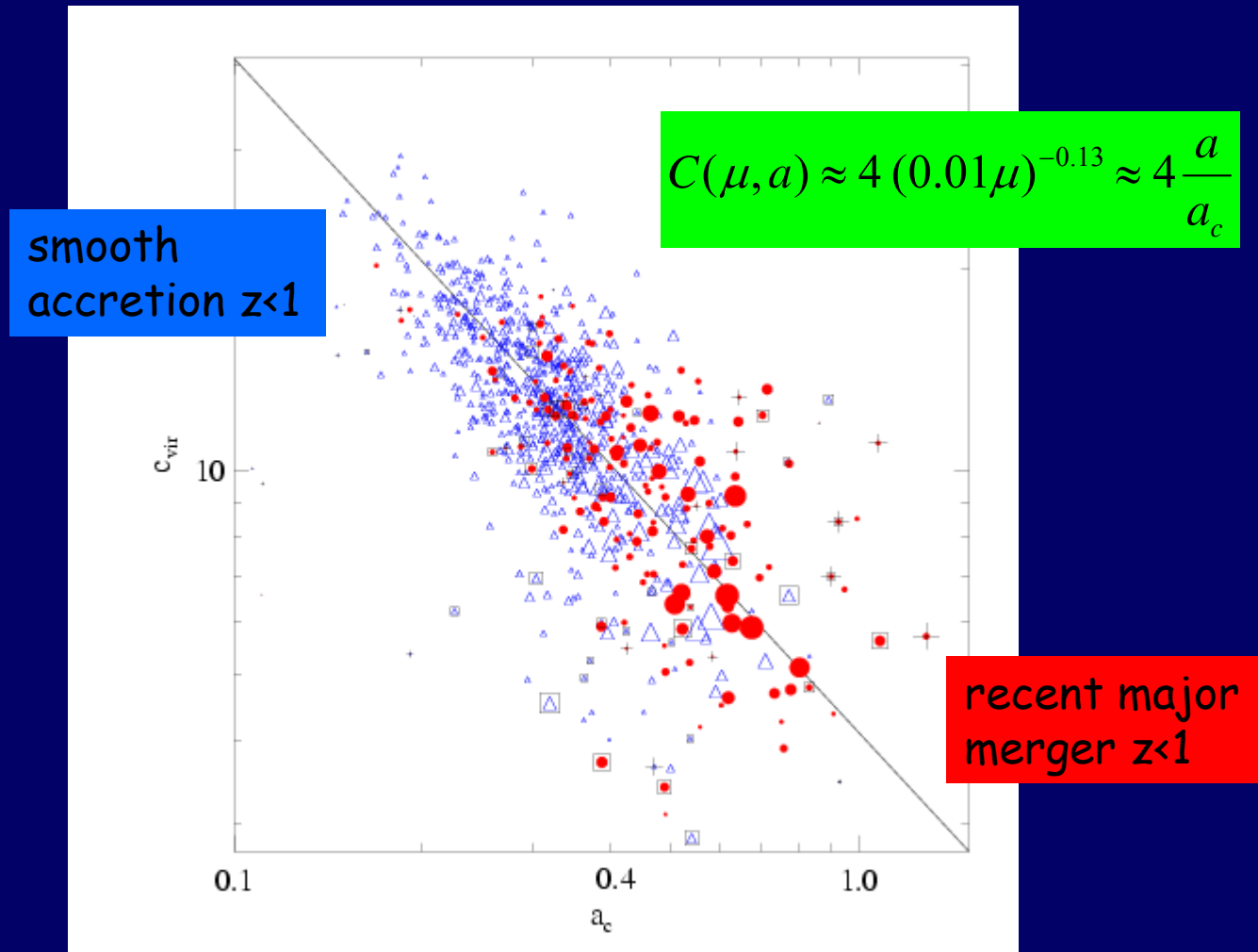


a

$$C(\mu, a) \approx 4 (0.01\mu)^{-0.13} \approx 4 \frac{a}{a_c}$$

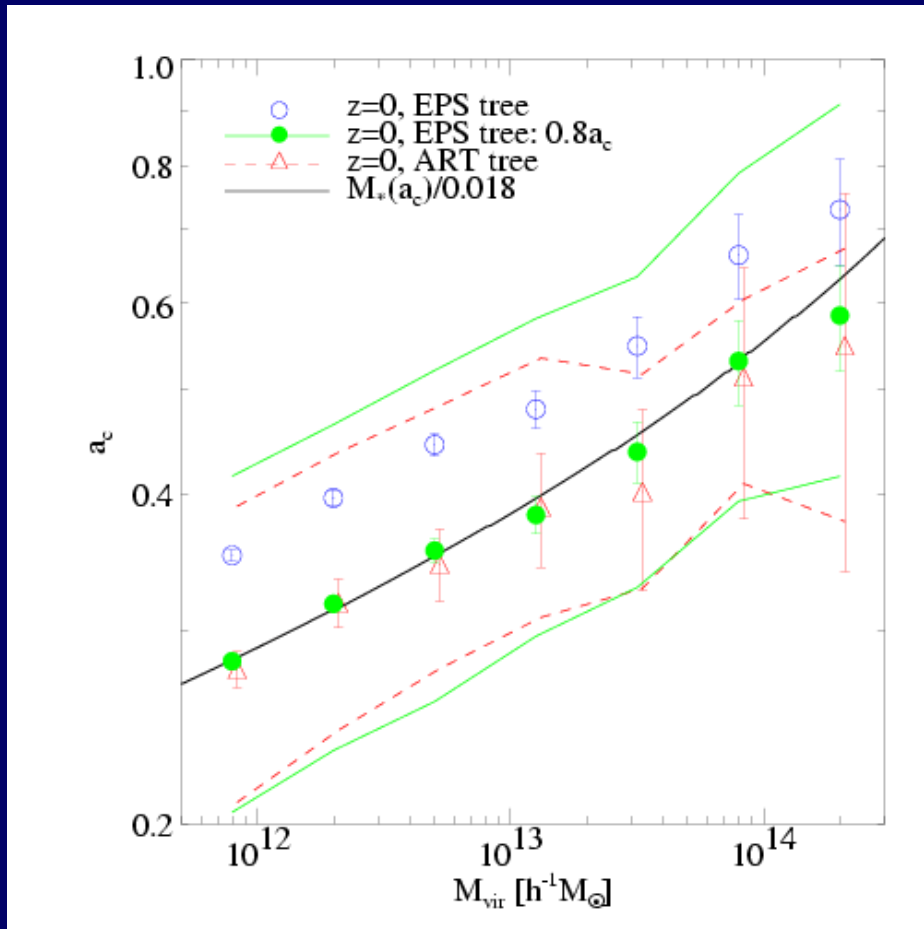
Concentration vs History

Wechsler et al. 2002



History vs Mass

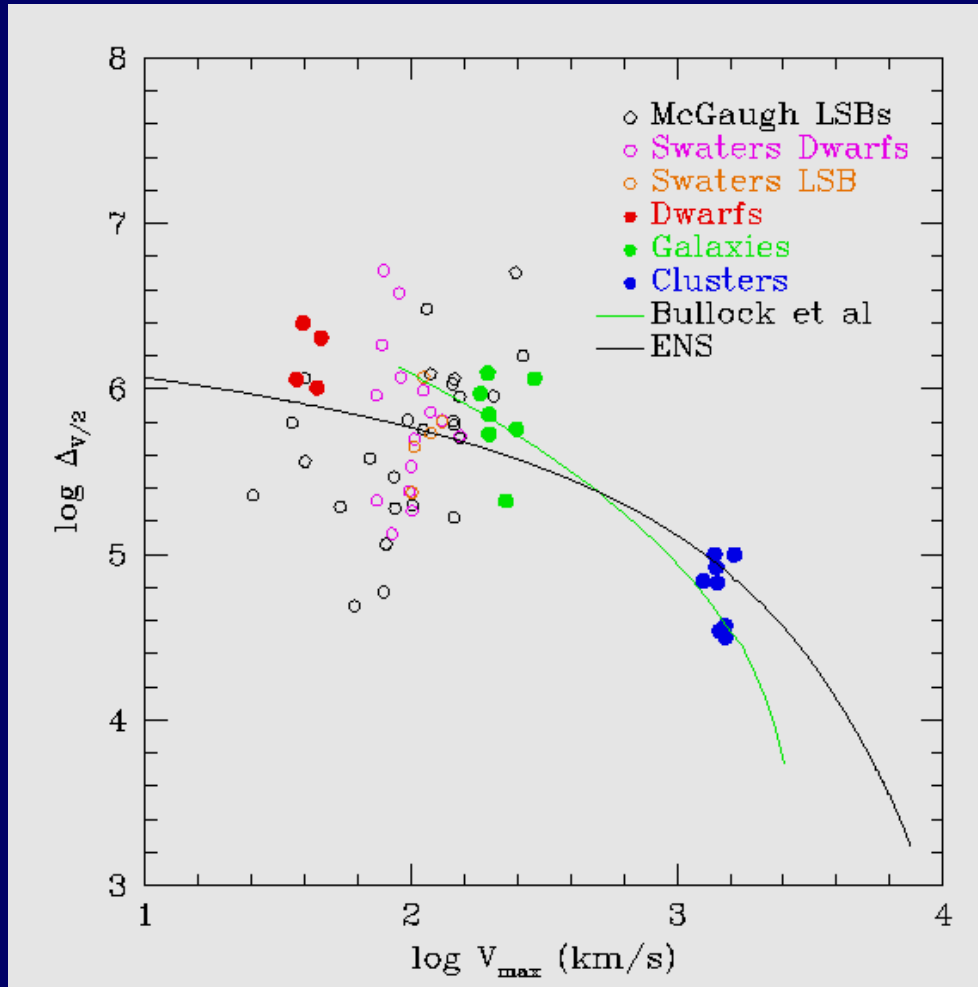
Wechsler et al. 2002



$$C(\mu, a) \approx 4 (0.01\mu)^{-0.13} \approx 4 \frac{a}{a_c}$$

Concentration of LSB galaxies and Λ CDM halos

Mean density contrast within $r(V_{\max}/2)$



Maximum Rotation Speed

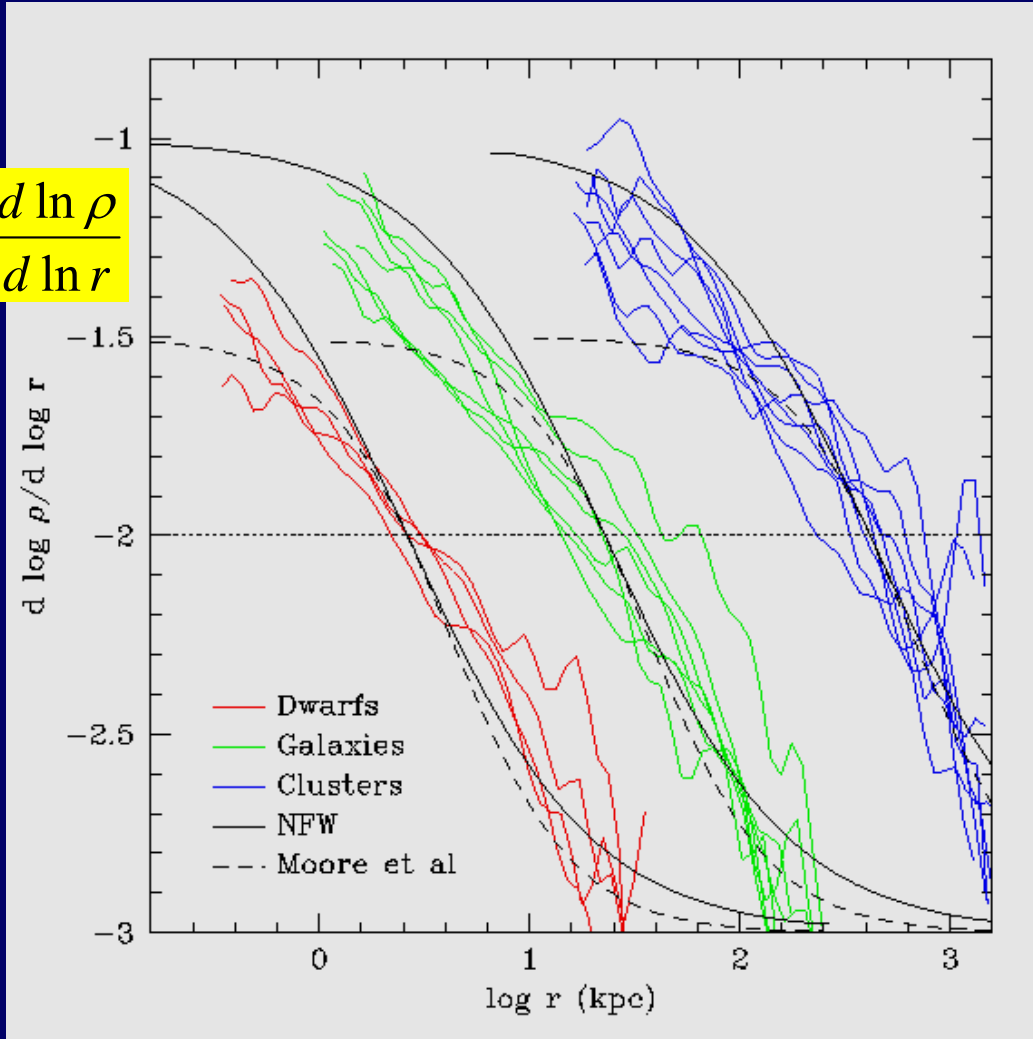
The average intermediate-scale concentration and scatter of Λ CDM halos is roughly consistent with observations of LSB and dwarf galaxies

Simulated Cusp

Recent results for Λ CDM halos

$$-\alpha(r) = \frac{d \ln \rho}{d \ln r}$$

Logarithmic Slope



Radius

No obvious convergence to a power law: profiles get shallower all the way in.

Innermost slopes are shallower than -1.5

Improved profile:

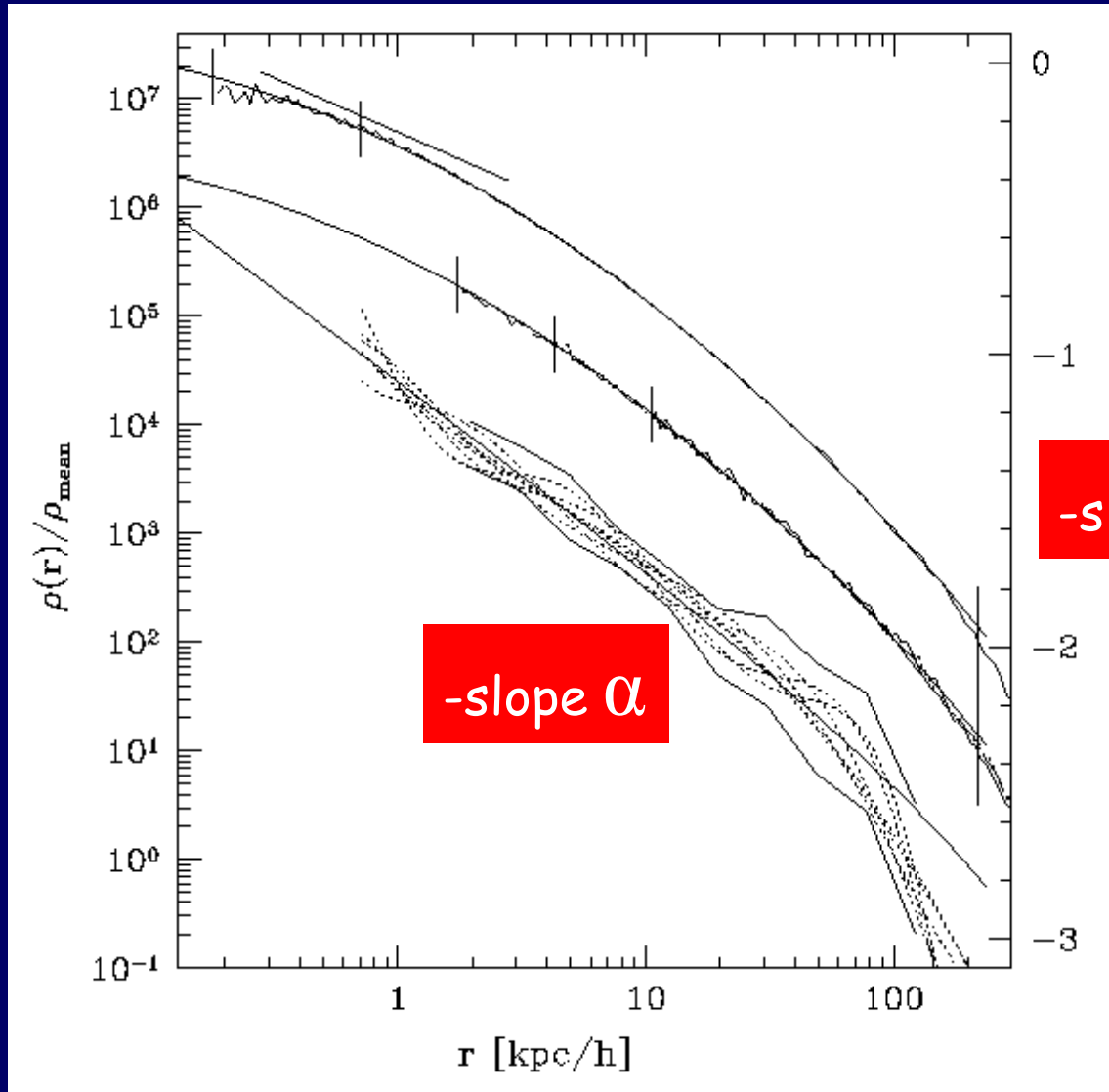
$$\alpha_{\beta}(r) \equiv -\frac{d \ln \rho}{d \ln r} = 2 \left(\frac{r}{r_s} \right)^{\beta}$$

$$\ln \left(\frac{\rho_{\beta}}{\rho_s} \right) = -\frac{2}{\beta} \left[\left(\frac{r}{r_s} \right)^{\beta} - 1 \right]$$

$$\beta \sim 0.1 - 0.2$$

Navarro, Frenk, White, Hayashi, Jenkins, Power, Springel, Quinn, Stadel

Improved Cusp Profiles



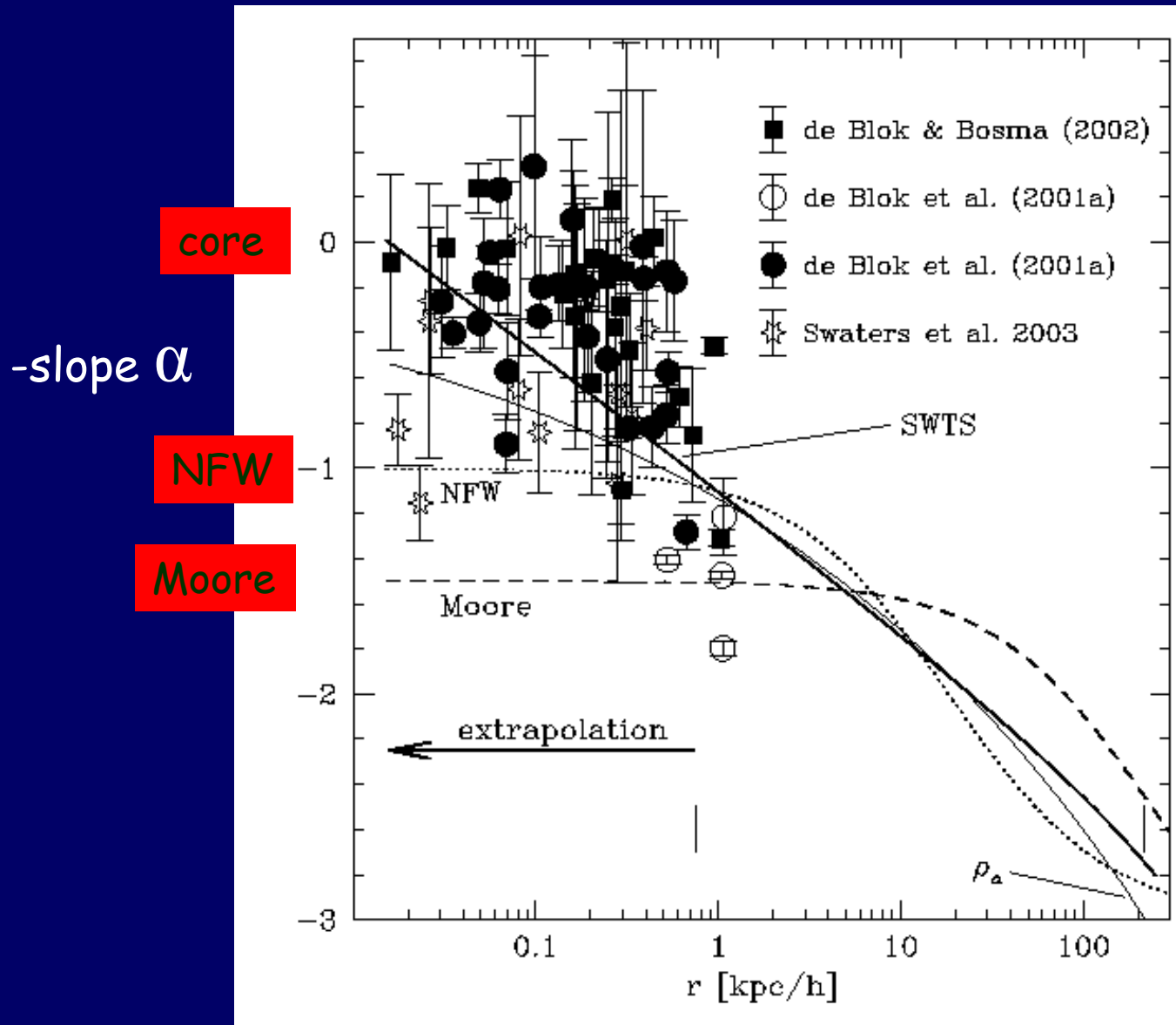
Stoehr et al. 2004

$$\log\left(\frac{V}{V_{\text{max}}}\right) = -a \left[\log\left(\frac{r}{r_{\text{max}}}\right) \right]^2$$

-slope α

-slope α

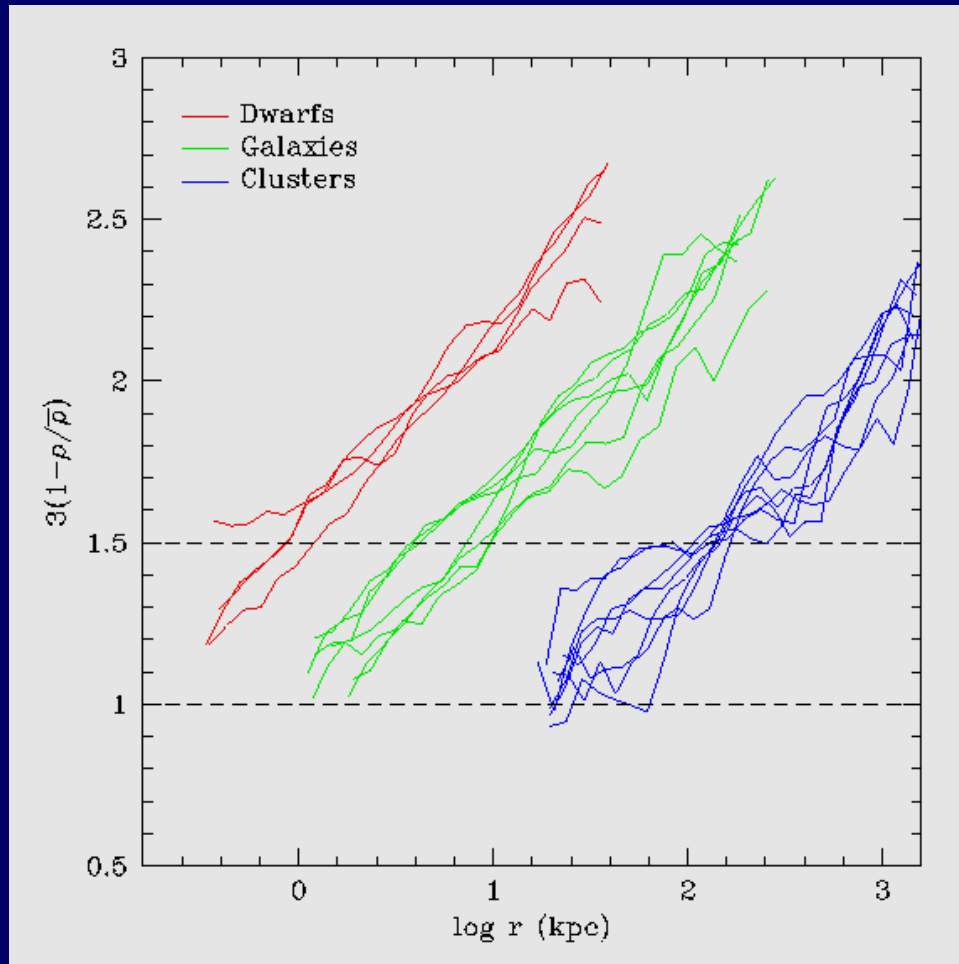
Improved Cusp Profiles: extrapolated to the inner cusp



Maximum Asymptotic Inner Slope

$$\rho = r^{-\alpha} \quad r < r_p \quad \rightarrow \quad \bar{\rho}(r) = \frac{1}{(4\pi/3)r^3} \int_0^r 4\pi r'^2 dr' \rho(r') = \frac{3}{3-\alpha} r^{-\alpha}$$

$$\rightarrow \alpha = 3[1 - \rho(r) / \bar{\rho}(r)] \quad \text{upper limit for slope in } r < r_p$$



Radius

$M(r)$ is robustly measured in the simulations.

With the local density, it provides an upper limit to the inner asymptotic log slope

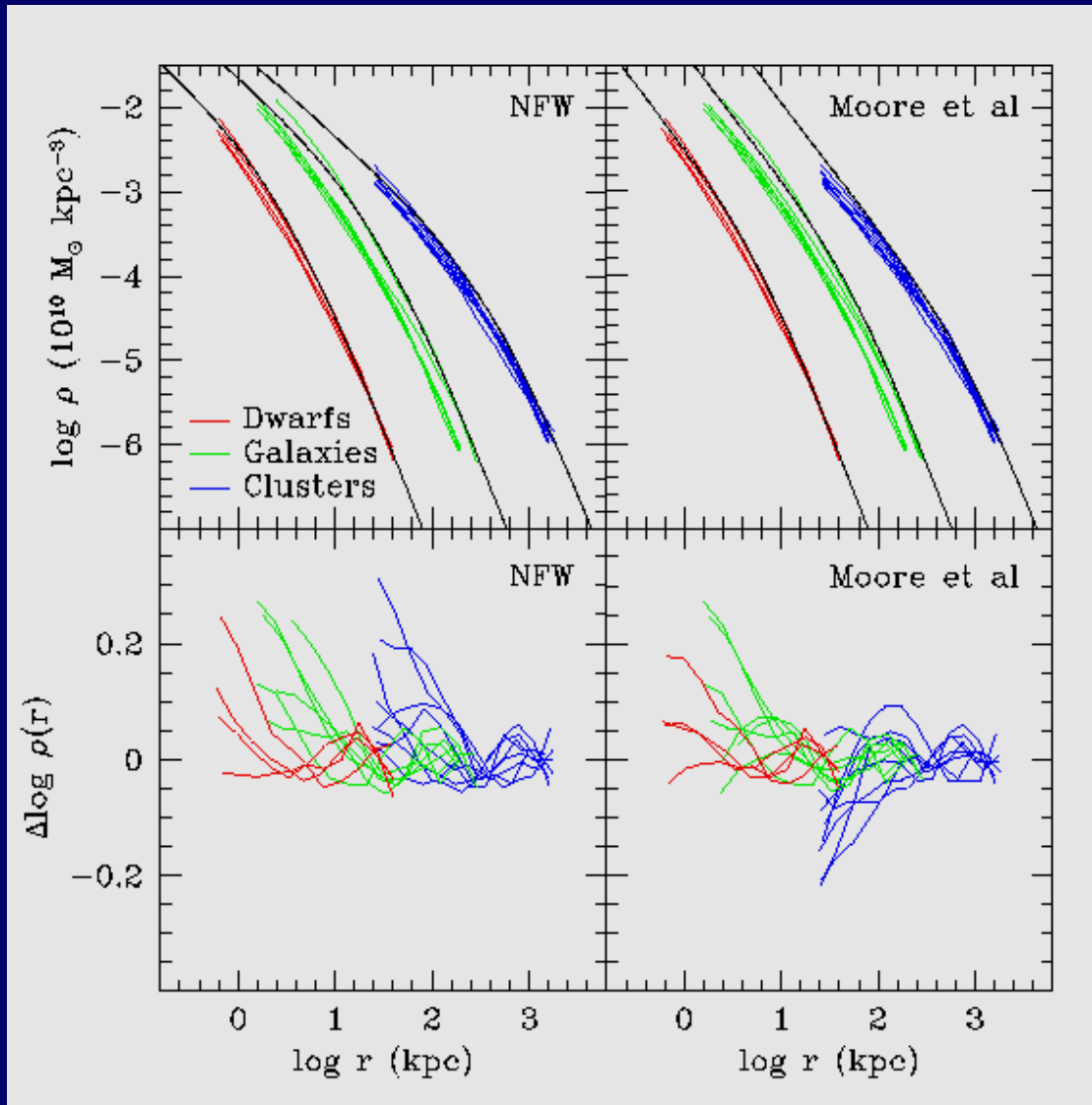
→ There is not enough mass in cusp to sustain a power-law as steep as $\rho \sim r^{-1.5}$

Navarro, Hayashi, Frenk, Jenkins, White, Power, Springel, Quinn, Stadel

How good or bad are simple fits?

Density

residuals



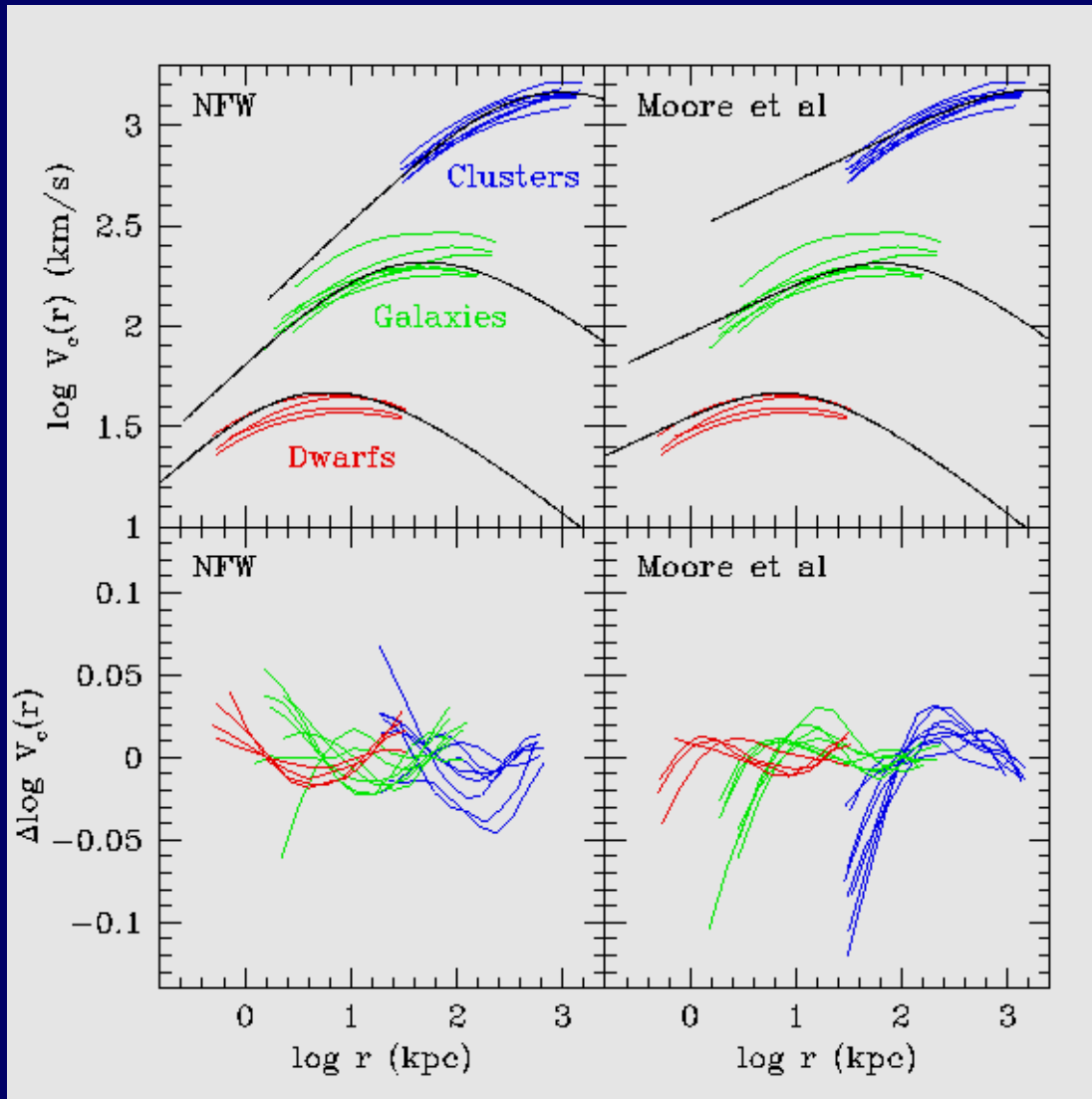
Radius

Over the well resolved regions, both NFW and Moore functions exhibit comparable systematic deviations when fitted to simulated CDM halos.

Navarro, Frenk, White,
Hayashi, Jenkins, Power,
Springel, Quinn, Stadel

How good or bad are simple fits?

Circular Velocity



residuals

Radius

Over the well resolved regions, both NFW and Moore functions exhibit comparable systematic deviations when fitted to simulated CDM halos.

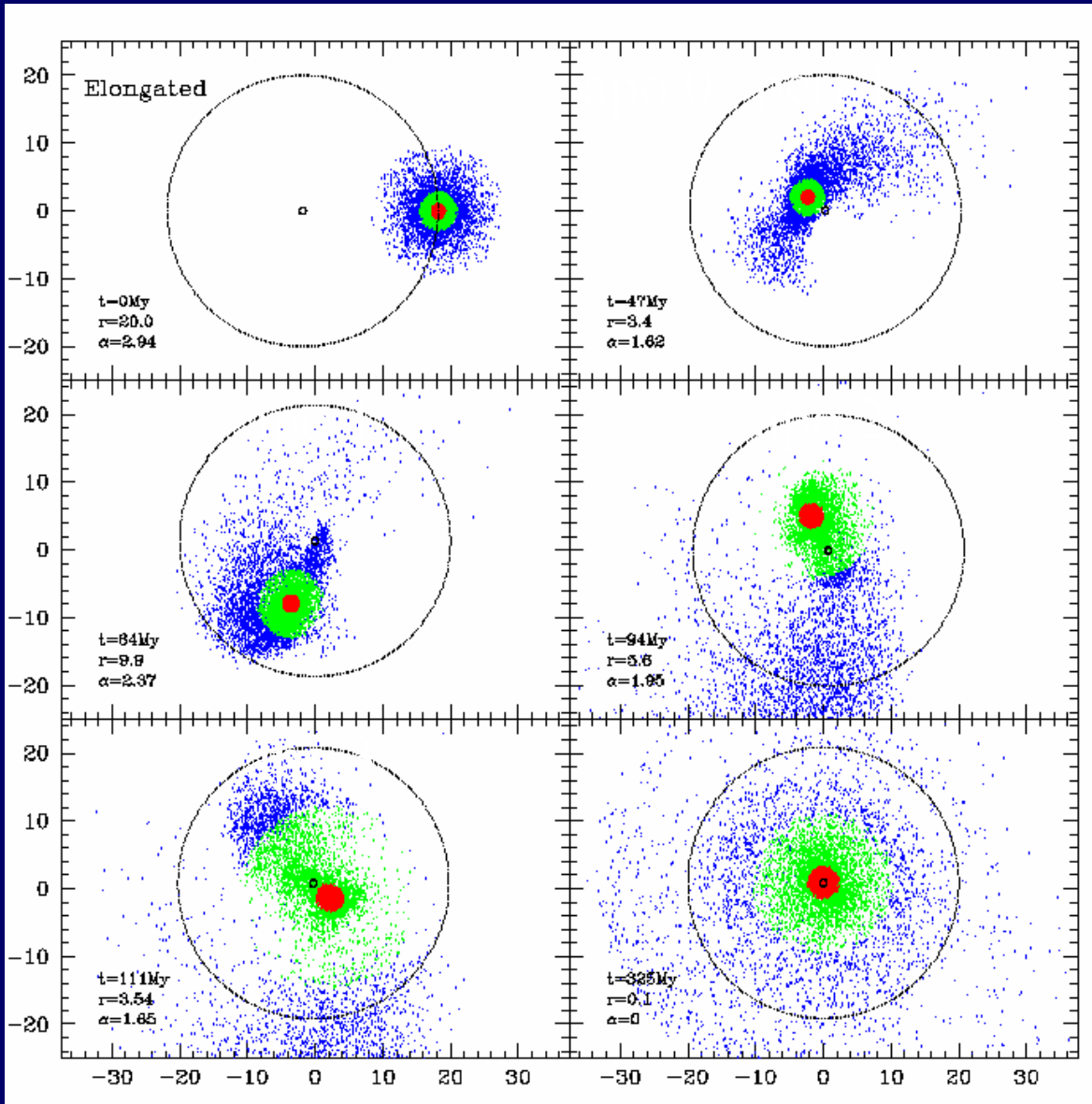
Navarro, Frenk, White,
Hayashi, Jenkins, Power,
Springel, Quinn, Stadel

Origin of the Halo inner Cusp?

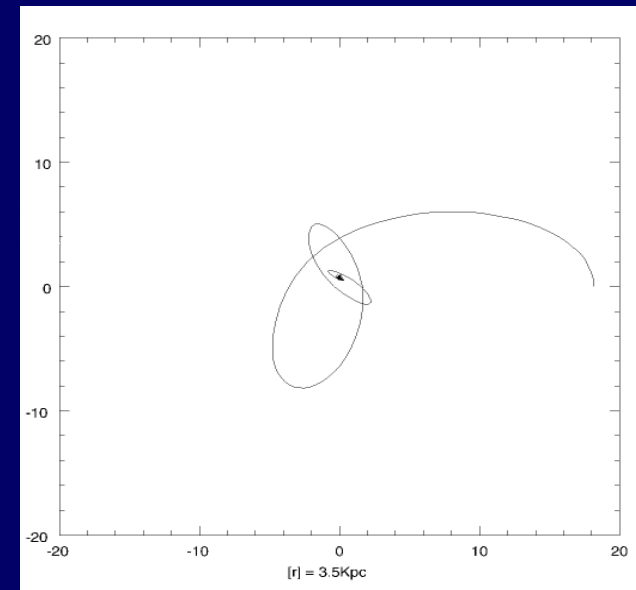
Dynamical Friction and Tidal Effects

Dekel, Arad, Devor, et al. 2003

Halo Bulidup by Mergers

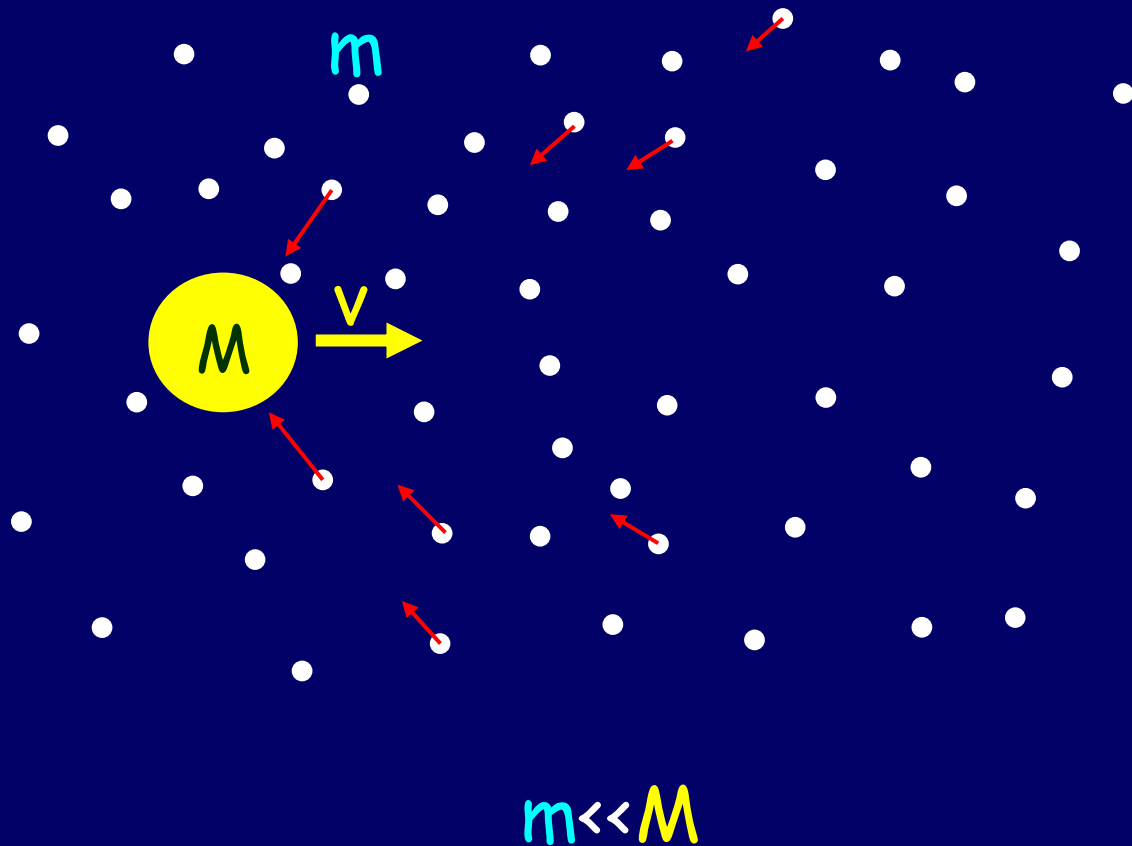


tidal stripping & dynamical friction

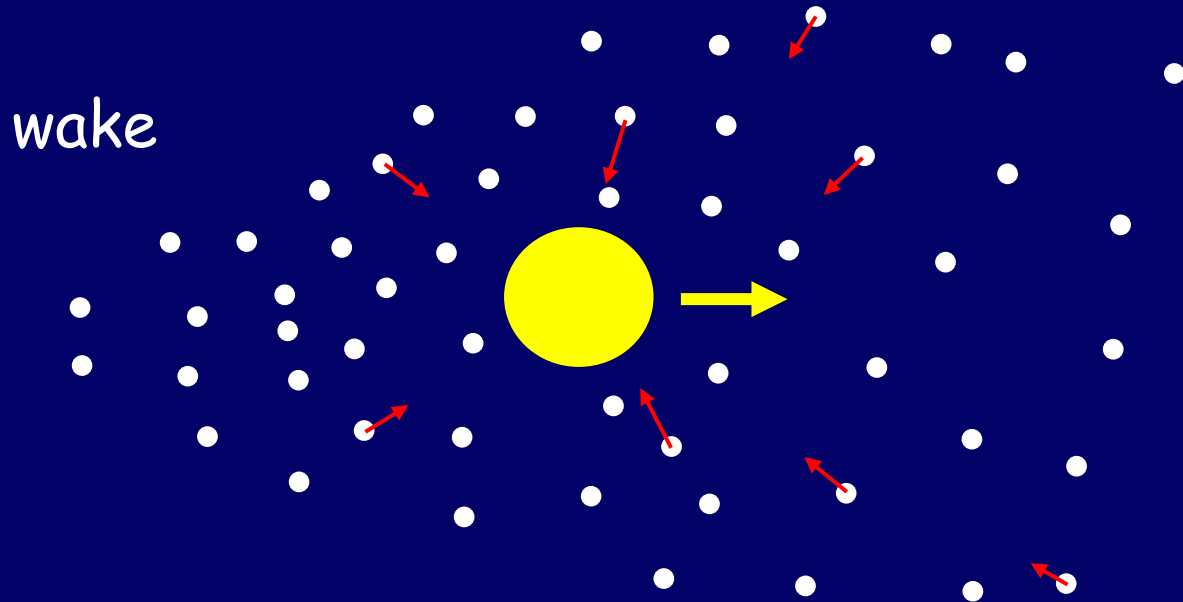


Dynamical Friction

Dynamical Friction



Dynamical Friction



Dynamical Friction

Chandrasekhar formula:

$$\frac{d\vec{v}}{dt} = -4\pi G^2 \ln \Lambda \rho(< v) M_{sat} \frac{\vec{v}}{v^3} \left[\operatorname{erf}(X) - \frac{2X}{\pi^{1/2}} e^{-X^2} \right] \quad m \ll M_{sat}$$

$$X \equiv \frac{v}{\sqrt{2}\sigma}$$

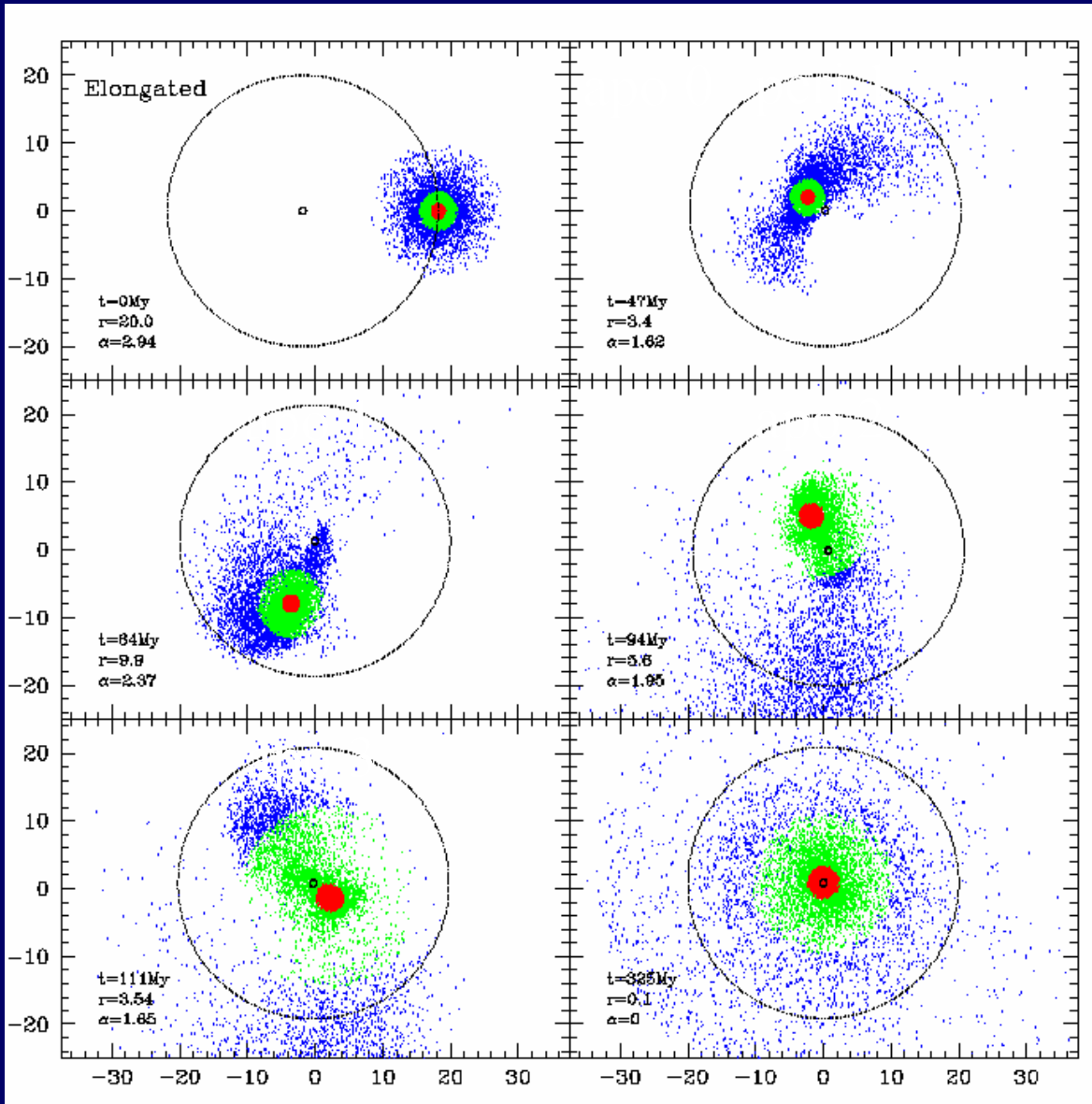
Coulomb logarithm:

$$\Lambda = \frac{b_{max} v_0^2}{GM_{sat}} \approx \frac{M_{halo}}{M_{sat}}$$

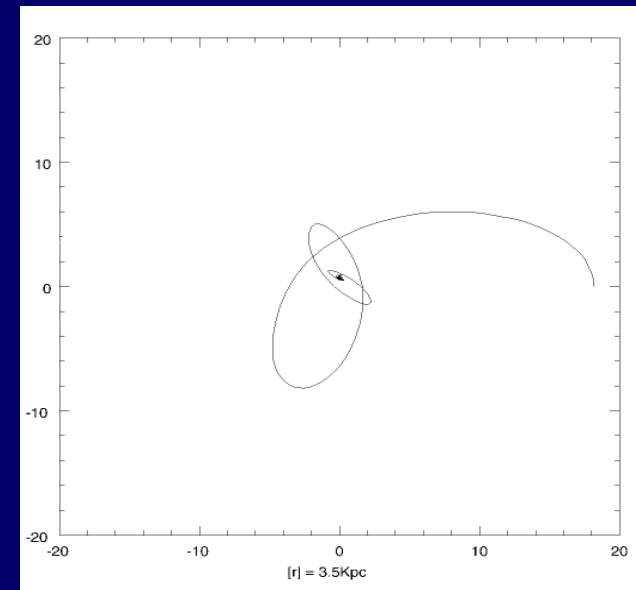
drag proportional to ρ but independent of m

acceleration propto M (because wake density propto M)

Halo Bulidup by Mergers



tidal stripping & dynamical friction

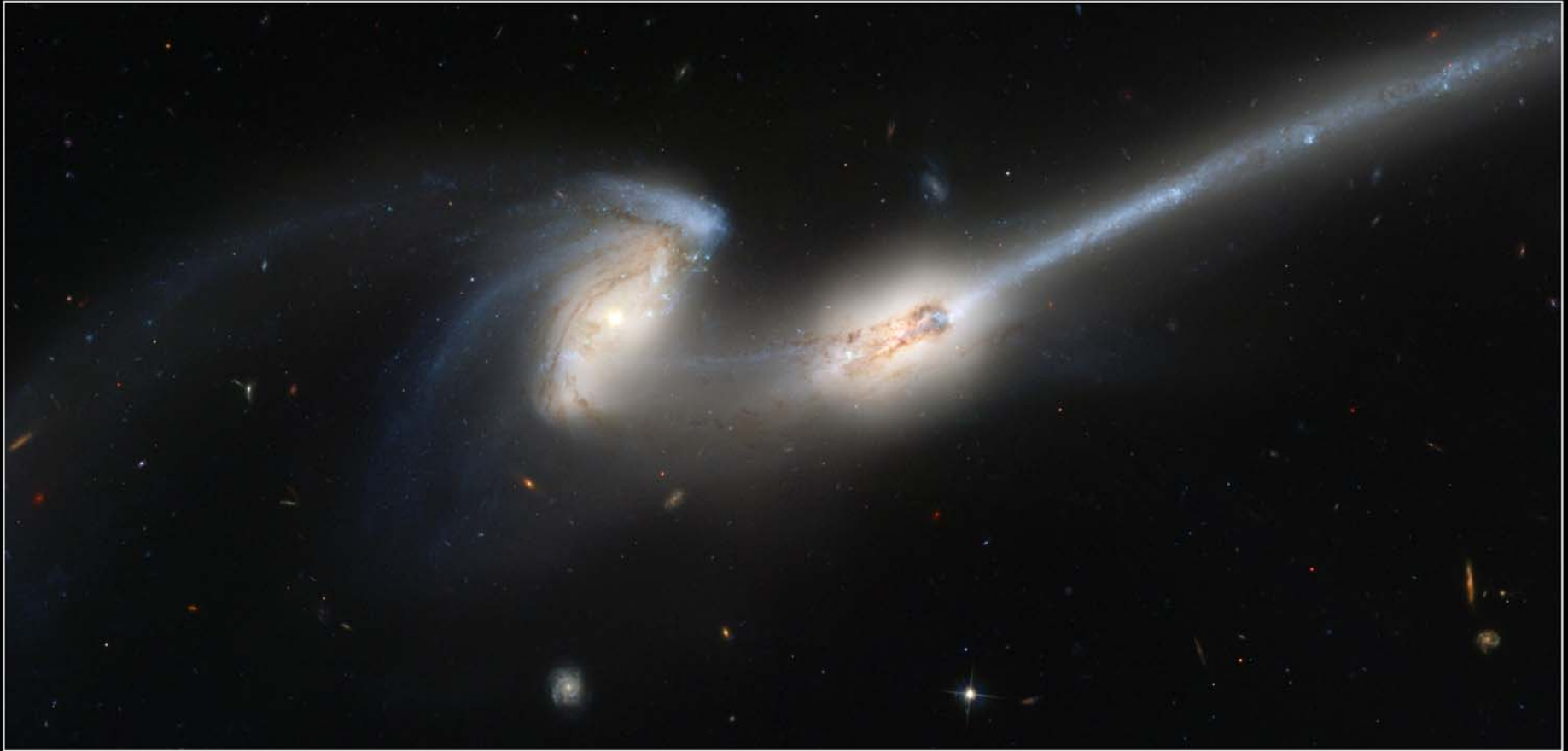


Tidal Effects



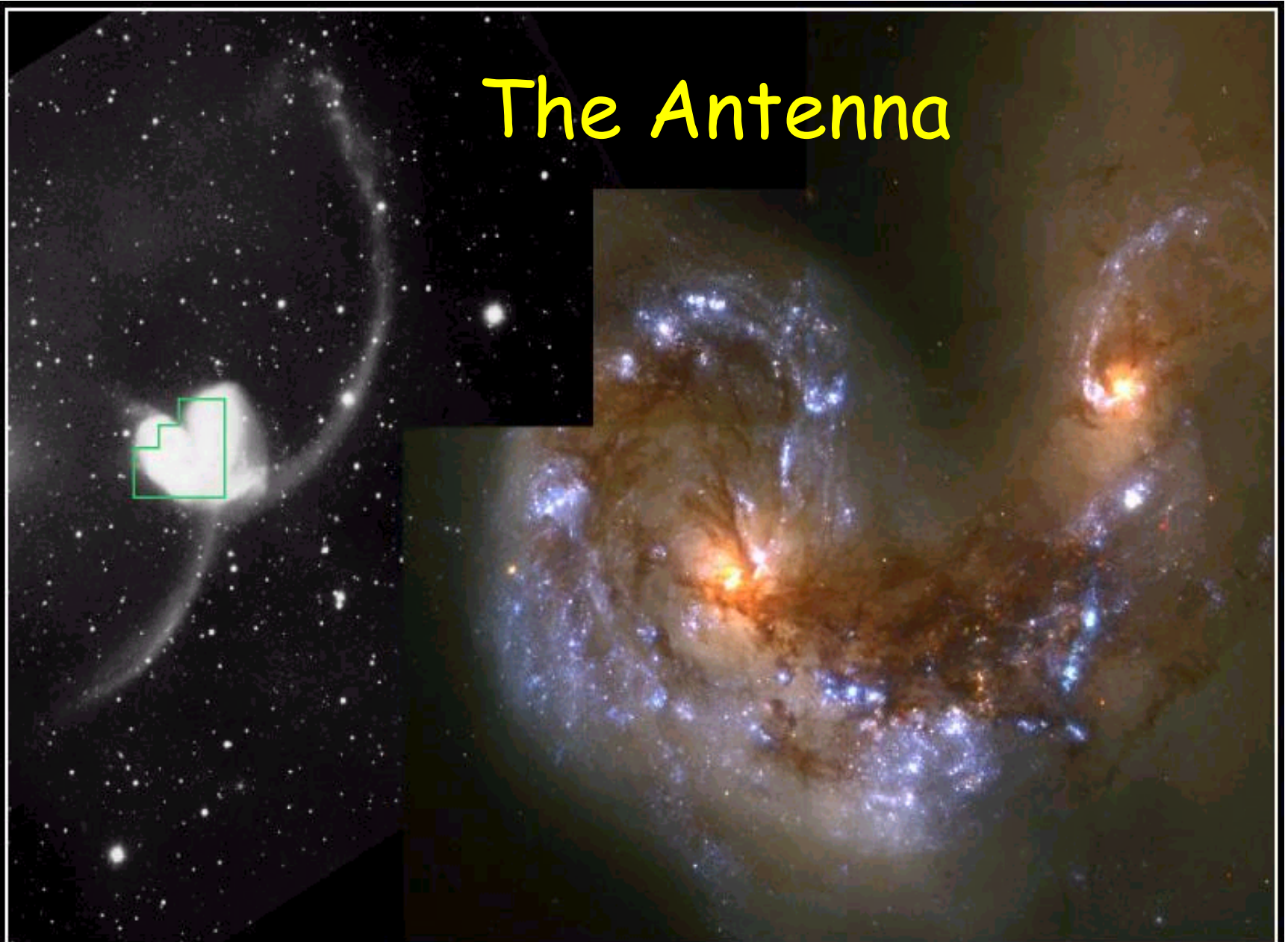
12-hour period

Tidal interaction & Merger



The Mice • Interacting Galaxies NGC 4676
Hubble Space Telescope • Advanced Camera for Surveys

The Antenna

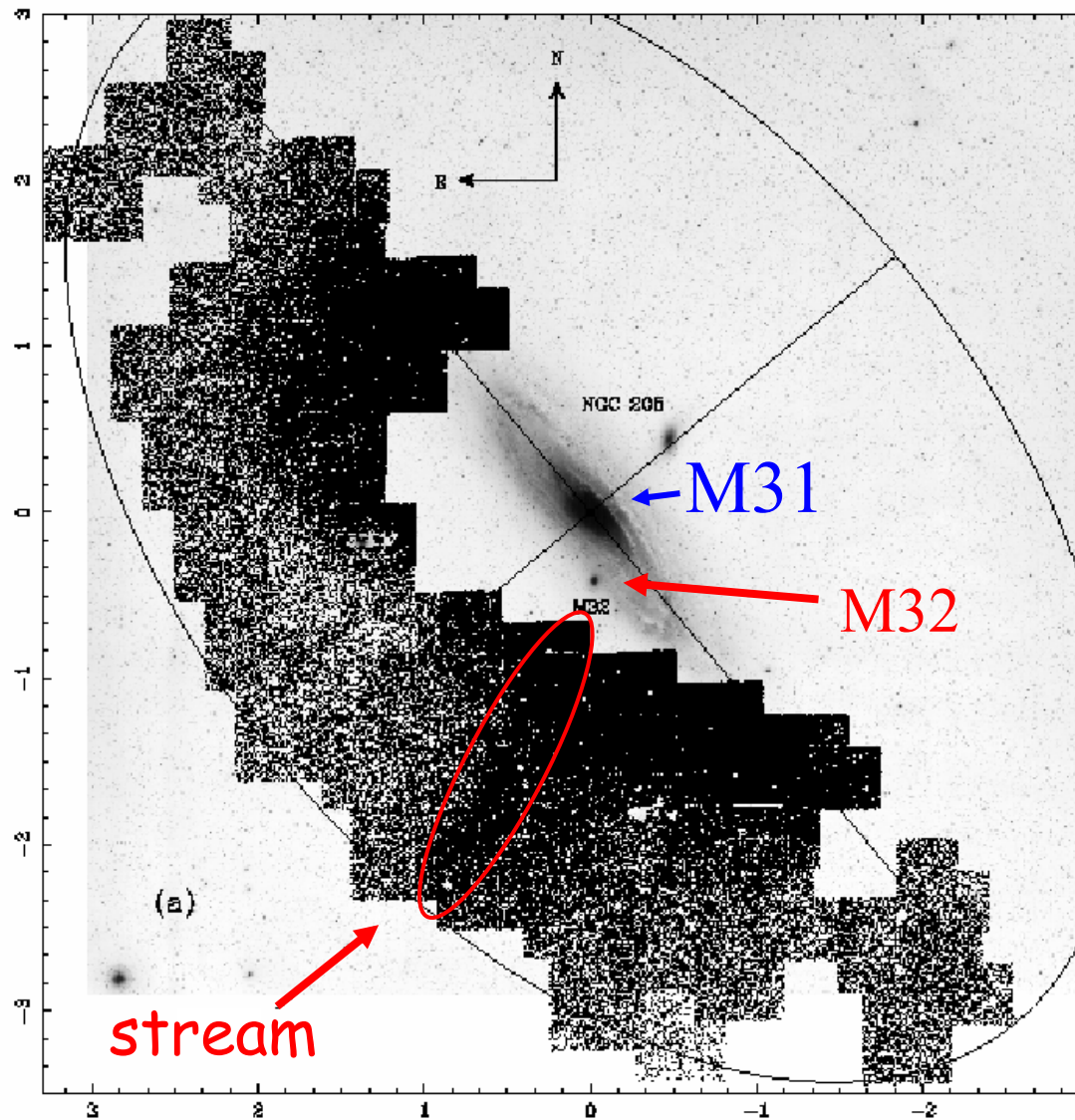


Colliding Galaxies NGC 4038 and NGC 4039

HST • WFPC2

PRC97-34a • ST ScI OPO • October 21, 1997 • B, Whitmore (ST ScI) and NASA

Tidal stripping of a satellite?

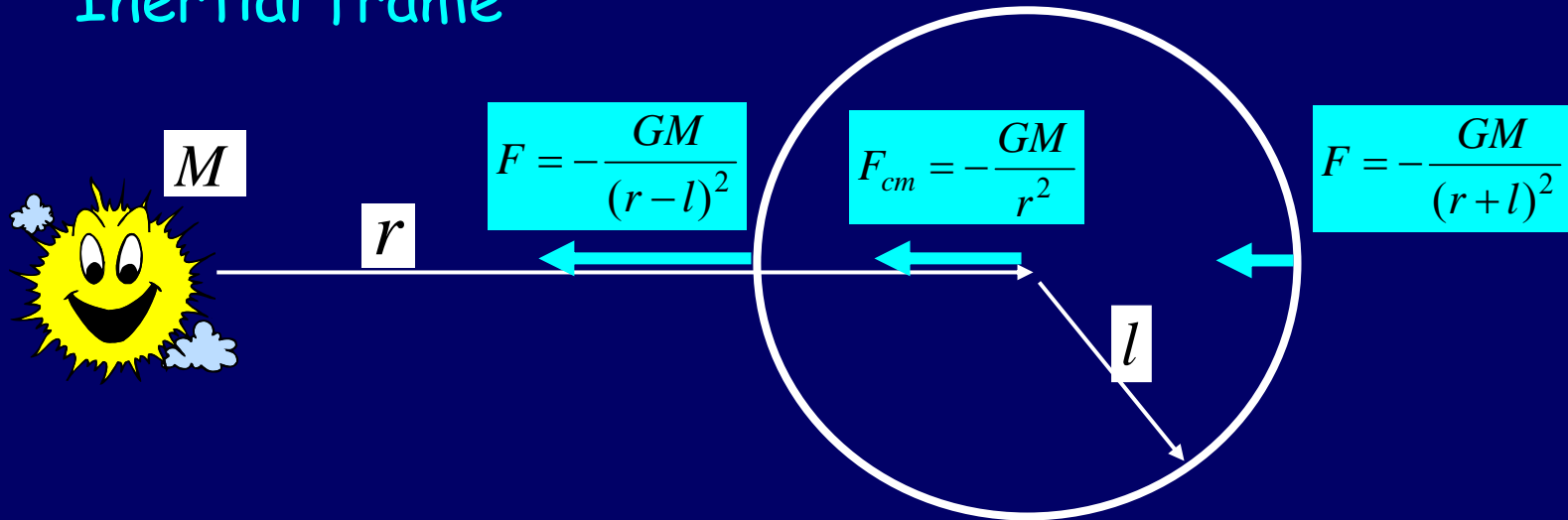


Ibata et al. 2001

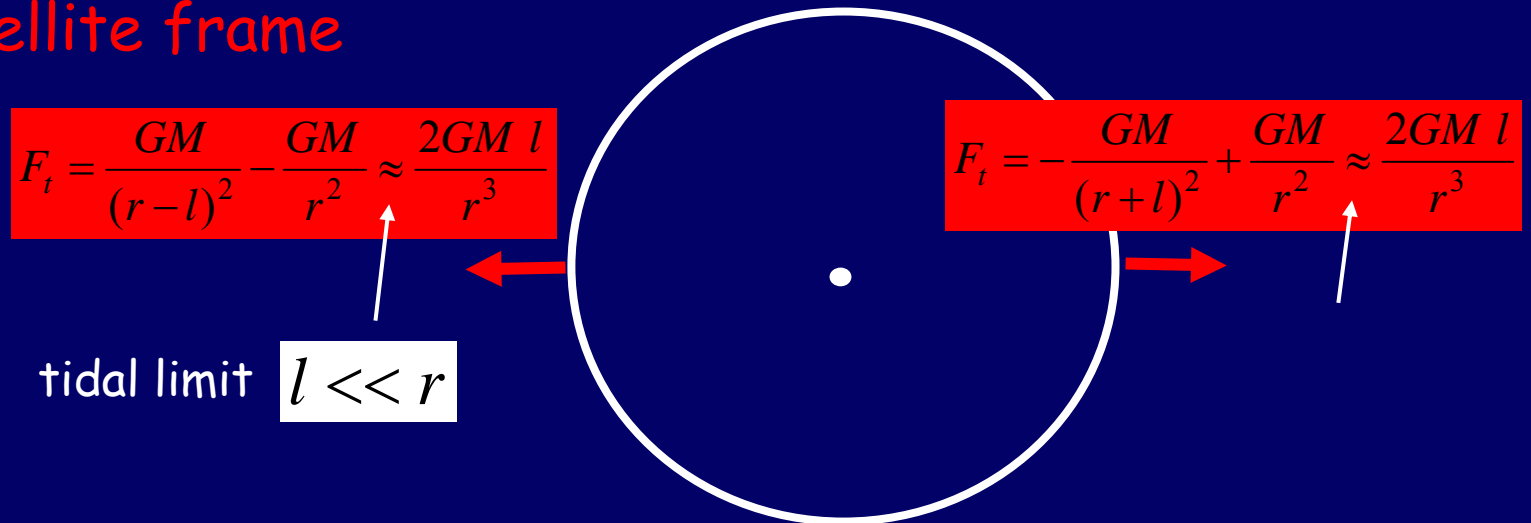
(degrees)

Tidal Force by a Point Mass

Inertial frame



Satellite frame



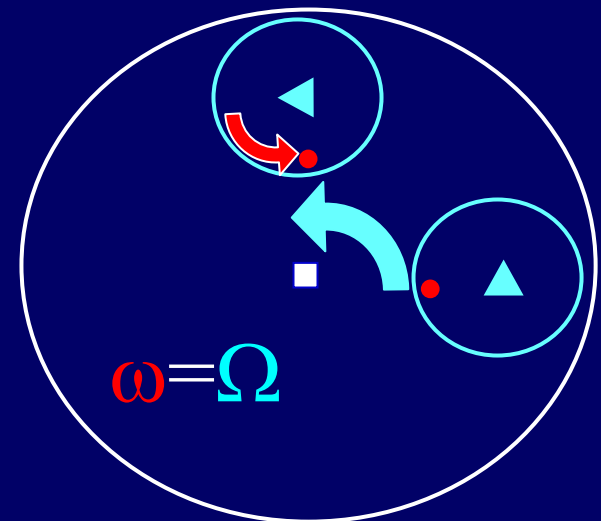
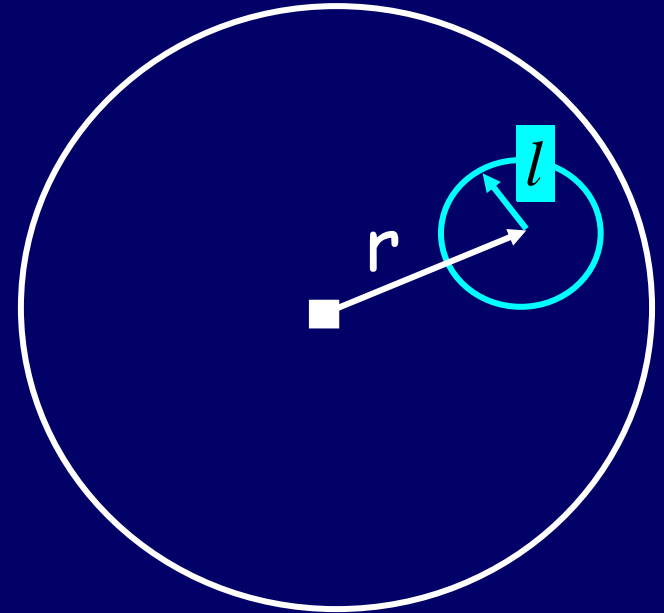
Tidal Radius of a Satellite

self-gravity force $\frac{Gm(l_t)}{l_t^2} = \frac{2GM(r)l_t}{r^3}$ tidal force

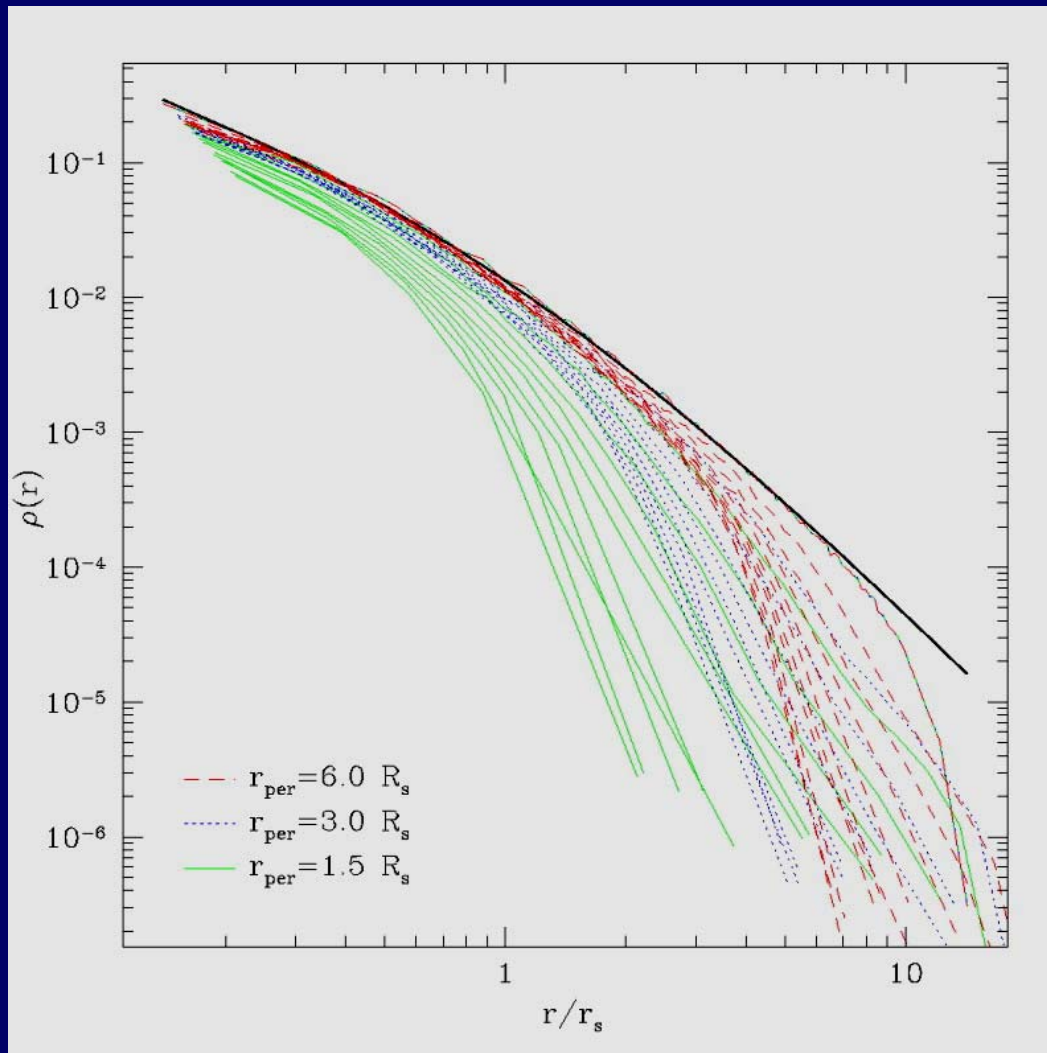
$\rightarrow \bar{\rho}_{sat}(l_t) \sim \frac{m(l_t)}{l_t^3} \sim \frac{M(r)}{r^3} \sim \bar{\rho}_{halo}(r)$

$t \propto \frac{R}{V} \propto \frac{R}{(GM/R)^{1/2}} \propto \left(\frac{R^3}{M}\right)^{1/2} \propto \rho^{-1/2}$

$\rightarrow t_{sat}(l_t) \sim t_{halo}(r)$
resonance



Density Profiles of stripped NFW halos



Profiles of sub-halos
Stoehr et al 2004:

$$\log\left(\frac{V}{V_{\text{max}}}\right) = -a \left[\log\left(\frac{r}{r_{\text{max}}}\right) \right]^2$$

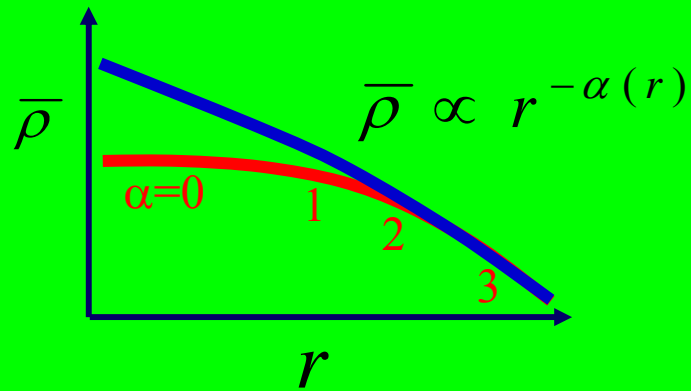
$$a \approx 0.45 \leftrightarrow \beta \approx 0.7$$

Origin of a cusp: tidal effects in mergers

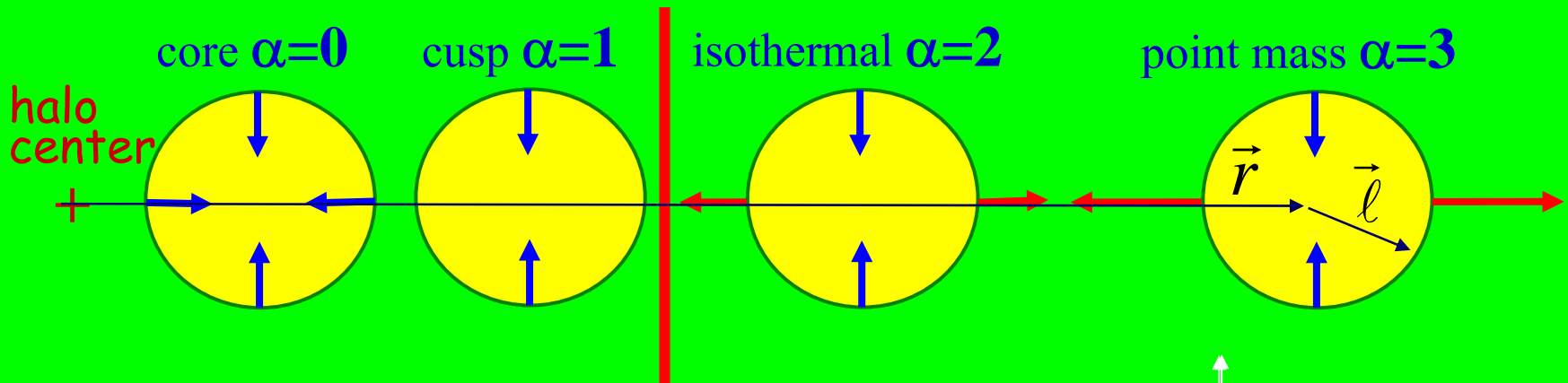
Dekel, Devor, Arad et al.

- a. If satellites settle in halo core →
steepening to a cusp $\alpha \geq 1$
- b. Mass-transfer recipe →
convergence to a universal slope $\alpha > 1$
- c. Flat-density core? Only if satellites are
puffed up, e.g. by gas blowout

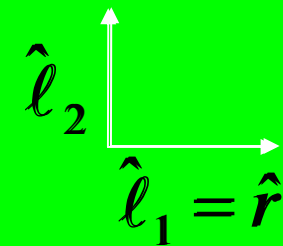
Tidal force on a satellite



$$\alpha(r) \equiv -\frac{d \ln \bar{\rho}(r)}{d \ln r}$$

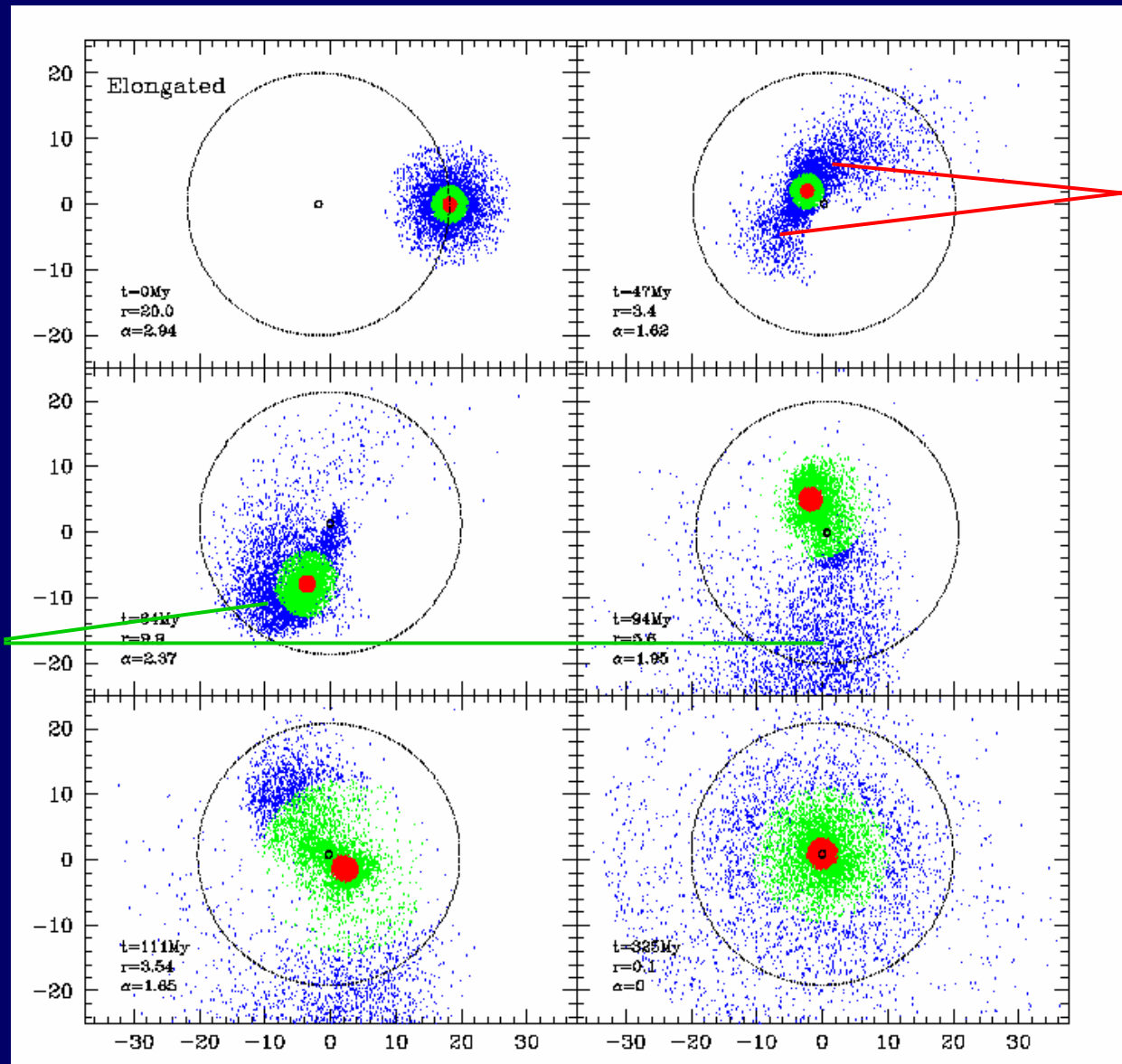


$$\vec{F}_{\text{tidal}} \propto \frac{GM(r)}{r^3} \left(\underline{[\alpha(r)-1]} \vec{l}_1 - \vec{l}_2 - \vec{l}_3 \right)$$



→ no mass transfer where $\alpha < 1$

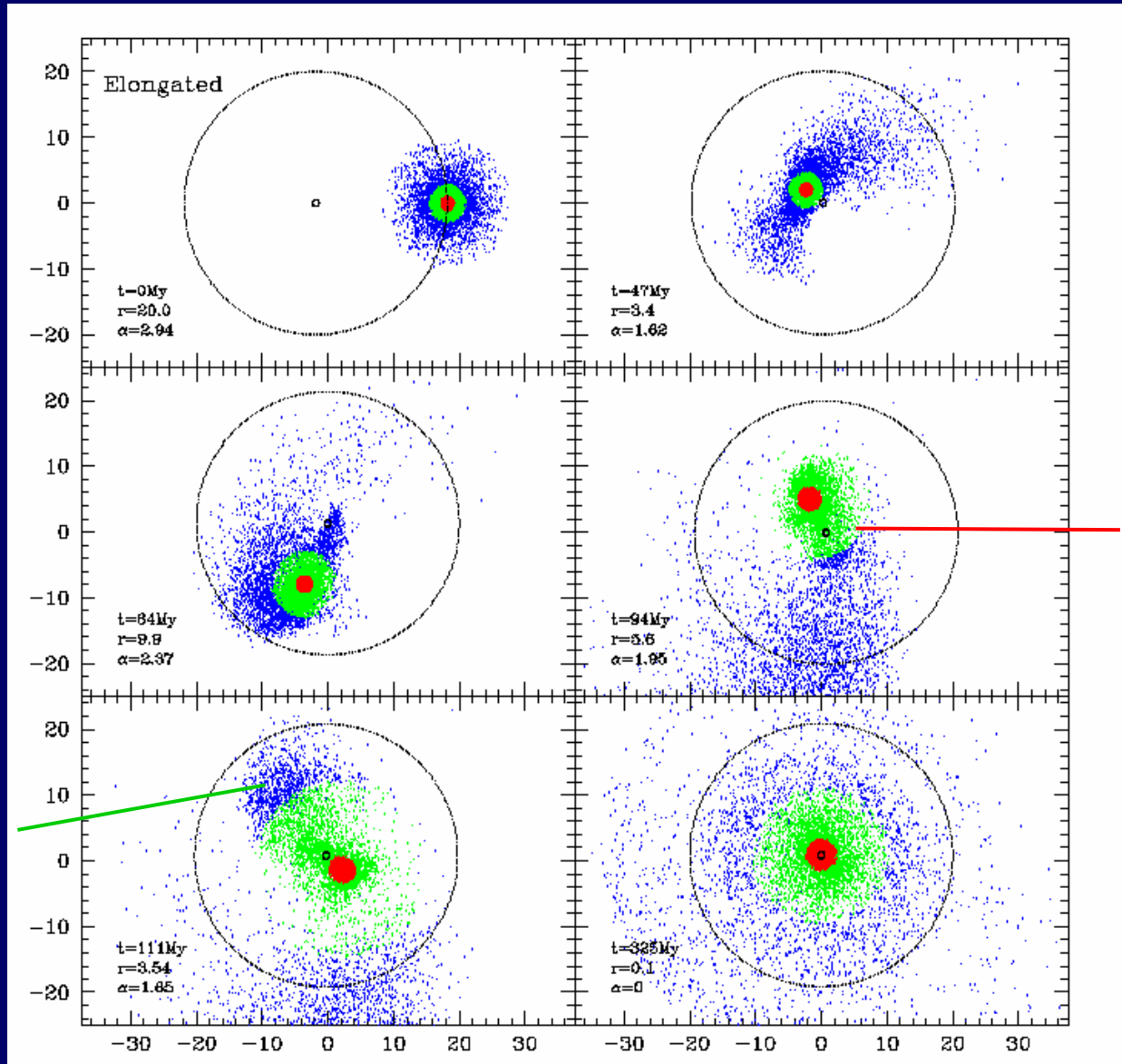
Impulsive stripping and deposit



pericenter stripping

apocenter deposit

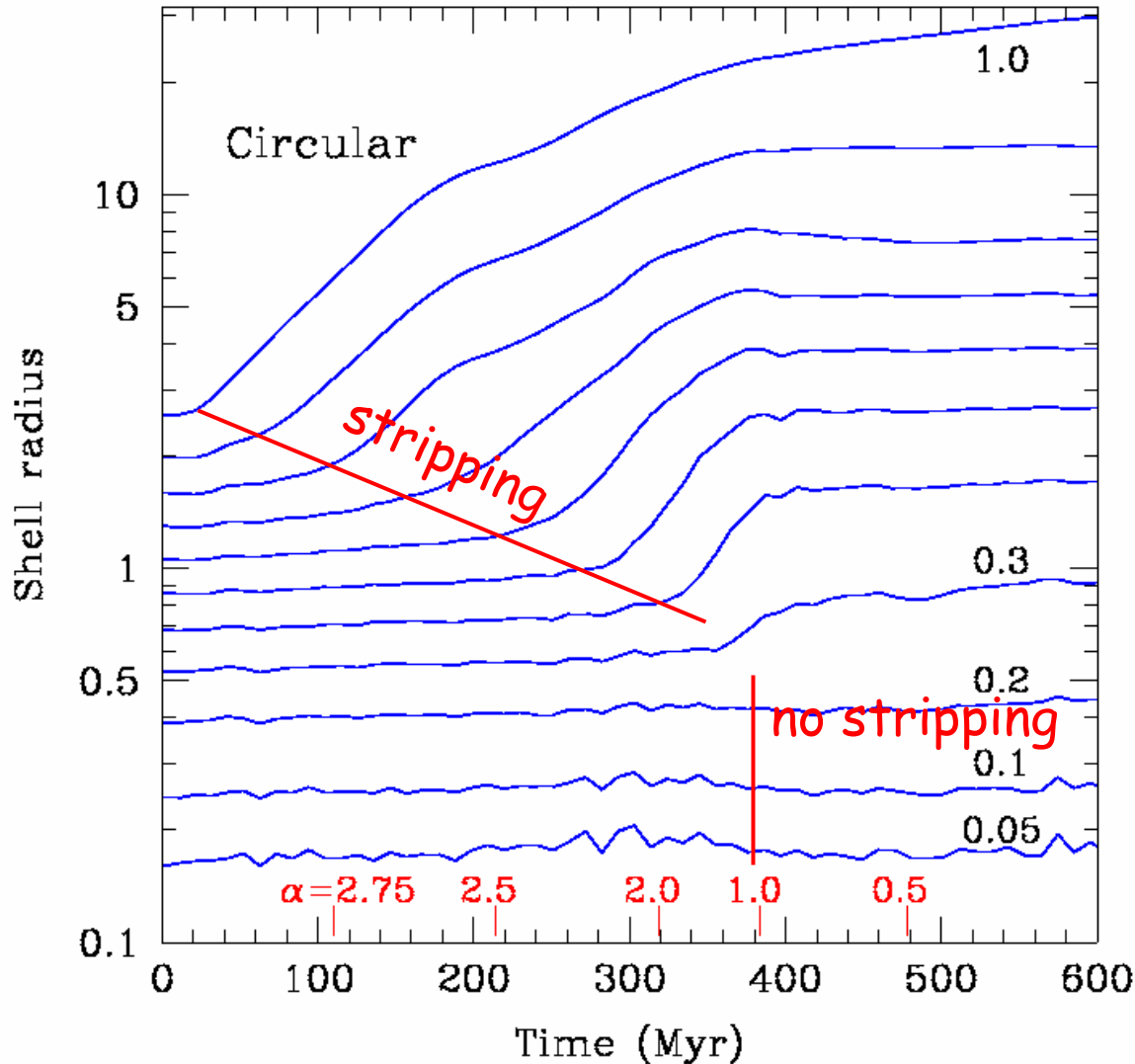
Impulsive stripping and deposit



pericenter stripping

apocenter deposit

Adiabatic evolution of satellite profile



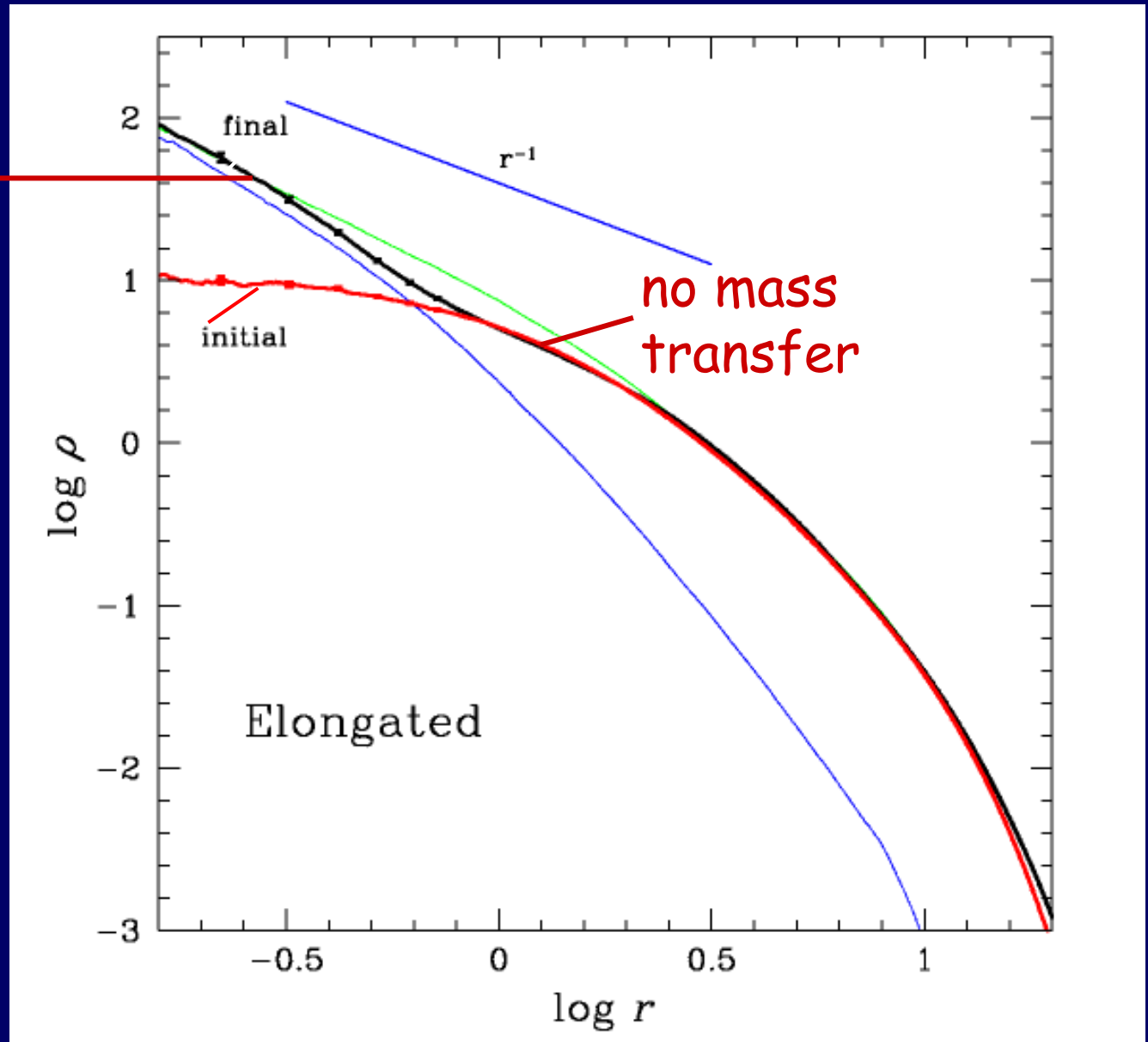
tidal
compression in
halo core

Merger of a compact satellite

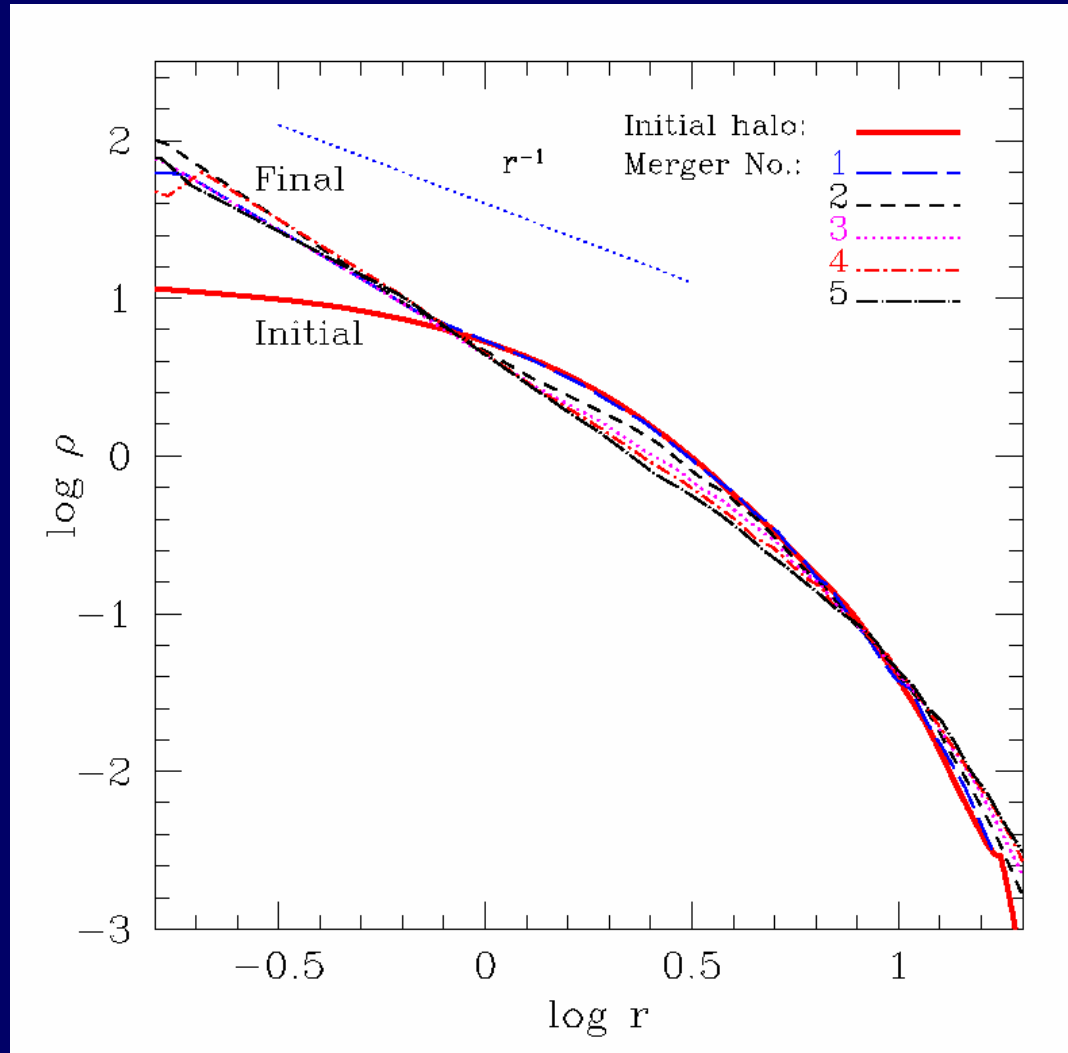
satellite
decays intact
to halo center

N-body
simulation

Dekel, Devor &
Hetzroni 03



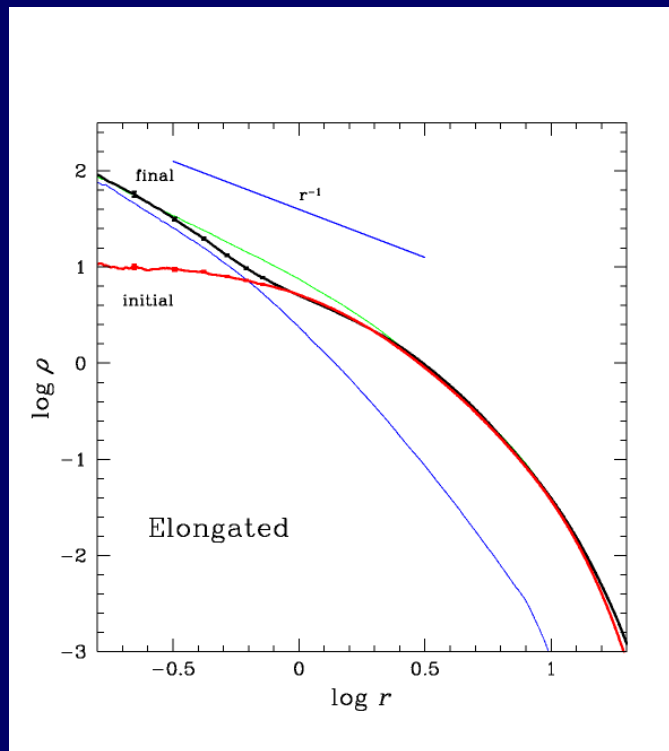
Tandem mergers with compact satellites



→ The cusp is stable!

Result:

No mass transfer in core \rightarrow
rapid **steepening to a cusp** of $\alpha \geq 1$

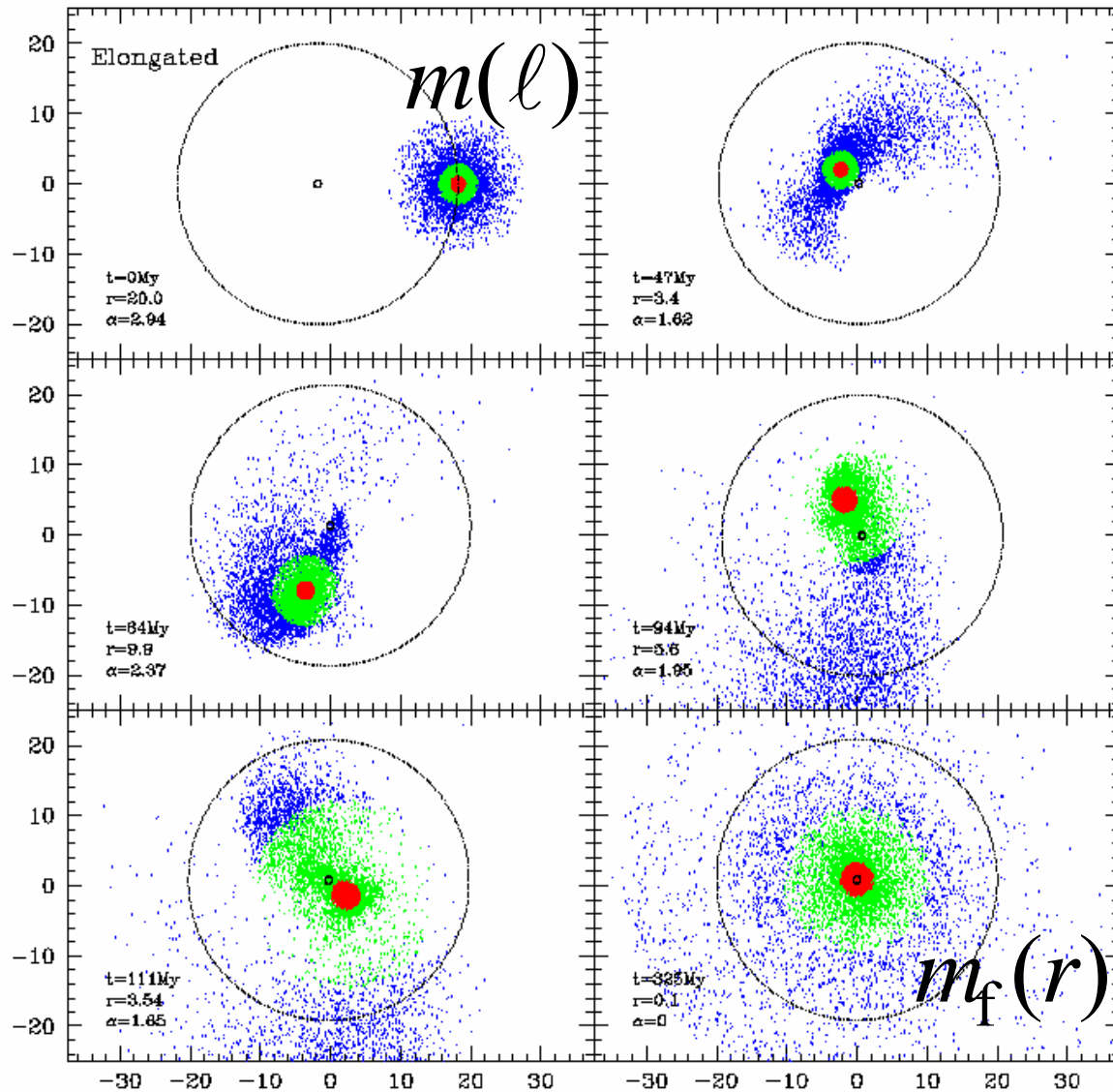


Tidal mass-transfer recipe at $\alpha > 1$

final initial satellite profile

$$m_f(r) = m(\ell) \rightarrow \ell(r)$$

Deposit radius



Tidal mass-transfer recipe at $\alpha > 1$

final initial satellite profile

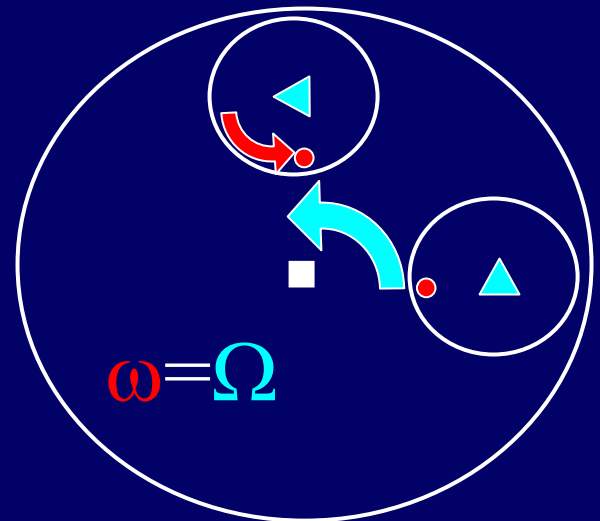
$$m_f(r) = m(\ell) \rightarrow \ell(r)$$

halo

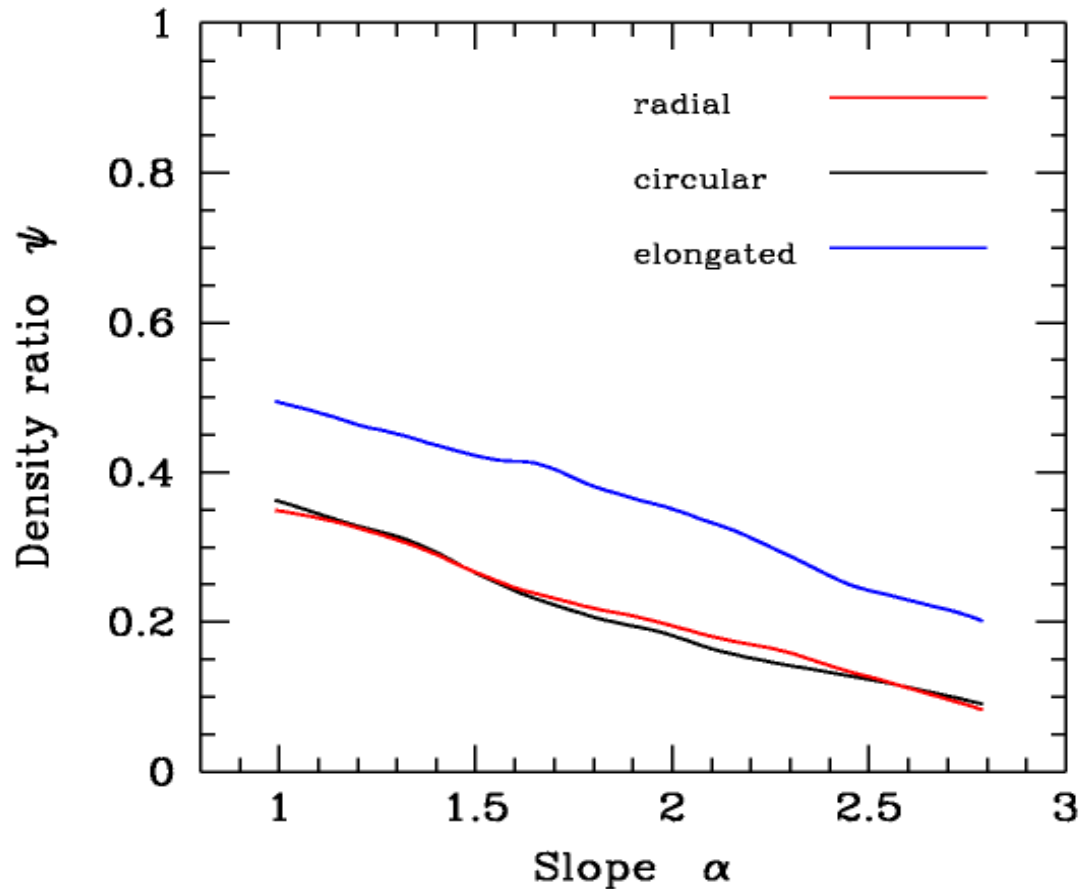
$$\frac{\bar{\rho}(r)}{\bar{\sigma}[\ell(r)]} = \psi[\alpha(r)]$$

initial satellite

=1? resonance



Tidal mass-transfer recipe at $\alpha > 1$



final initial sat. profile

$$m_f(r) = m(\ell) \rightarrow \ell(r)$$

halo

$$\frac{\bar{\rho}(r)}{\bar{\sigma}[\ell(r)]} = \psi[\alpha(r)] \approx \frac{1}{2\alpha}$$

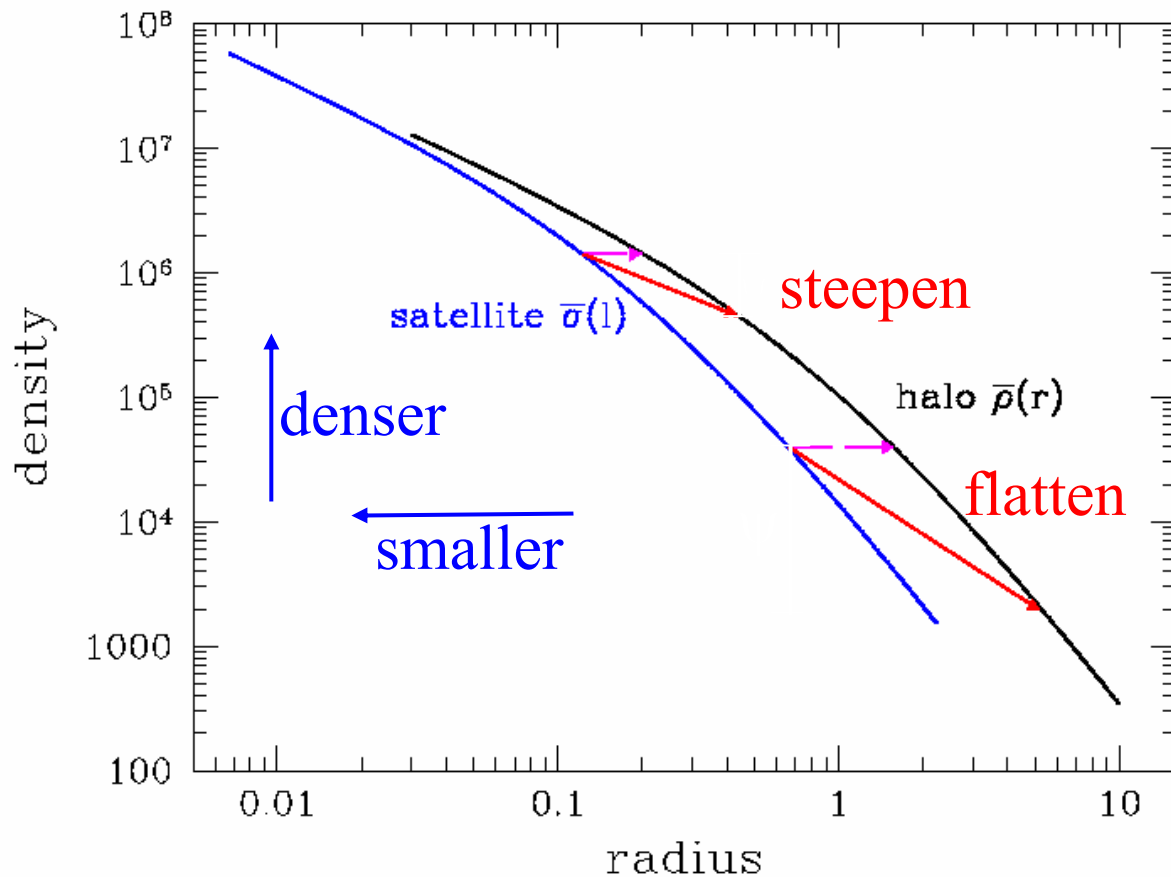
satellite

→ stripping efficiency grows with α

Steepening / flattening

homologous halo and satellite

scaling: $\rho_s \propto m^{-(3+n)/2}$ $r_s \propto m^{(5+n)/6}$



$$\frac{\bar{\rho}(r)}{\bar{\sigma}[\ell(r)]} = \psi[\alpha(r)]$$
$$\psi \approx \frac{1}{2\alpha}$$

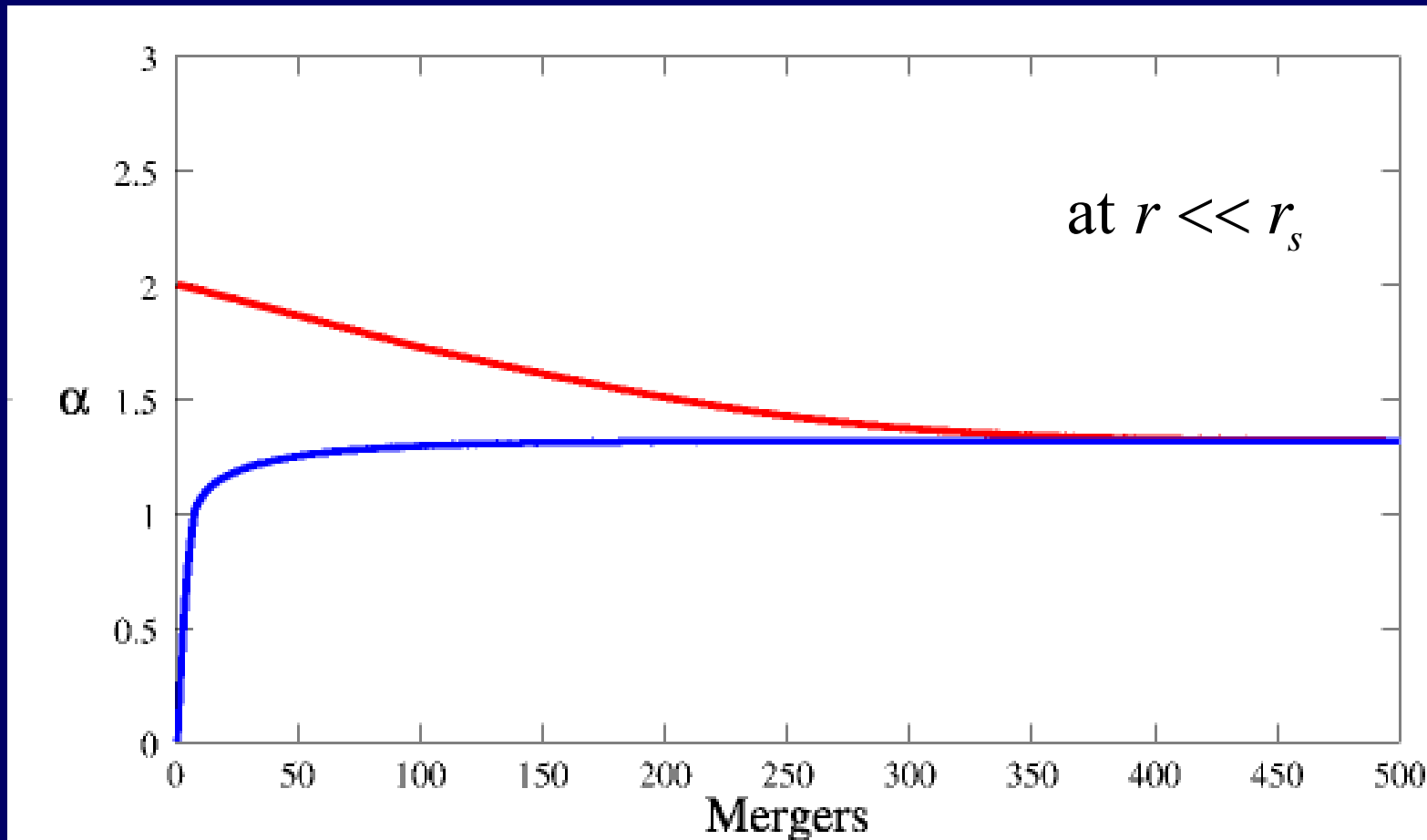
Adding satellite to halo profile

$$\bar{\rho}_{\text{new}}(r) = \bar{\rho}_{\text{old}}(r) + \bar{\sigma}(\ell) \frac{\ell^3}{r^3}$$

$$\Rightarrow \Delta\alpha(r) \propto -\frac{d}{dr} \left[\frac{\bar{\sigma}(\ell) \ell^3}{\bar{\rho}(r) r^3} \right]$$

linear perturbation analysis $\Rightarrow \alpha \rightarrow \alpha_{\text{asymptotic}}$

Convergence to an asymptotic slope



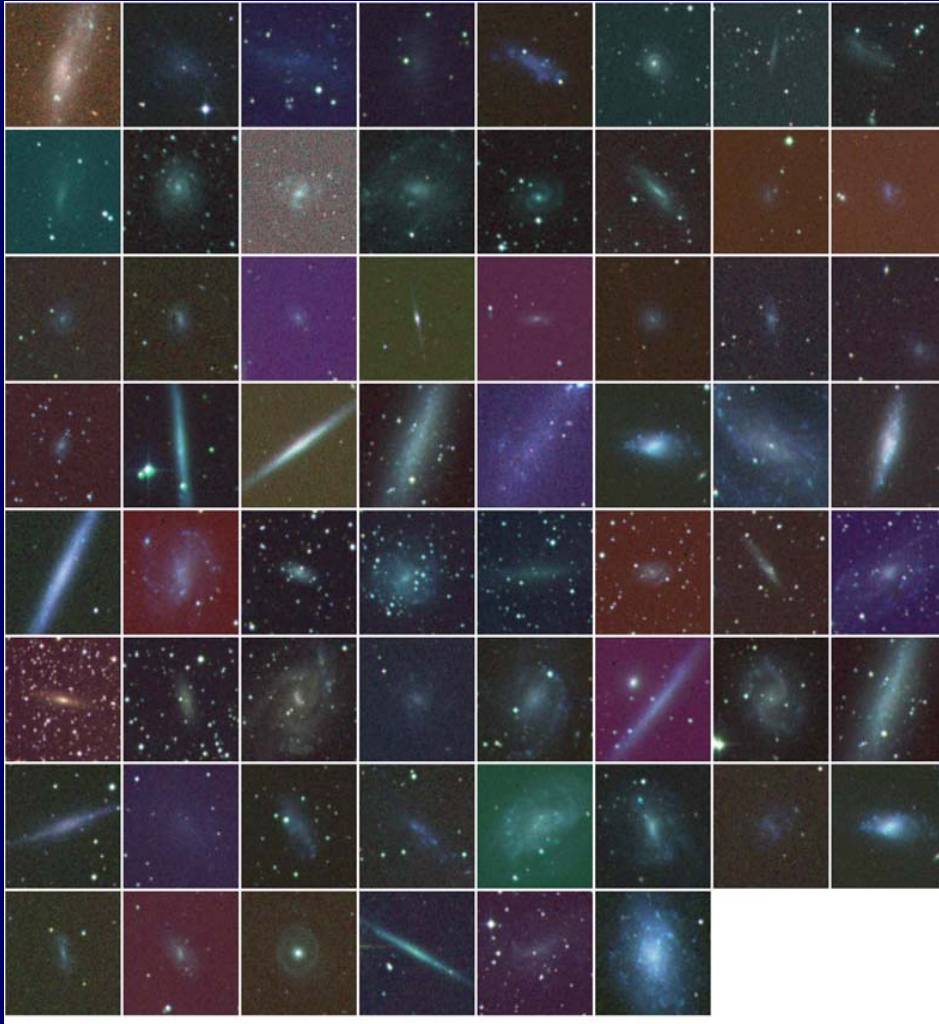
Dekel, Arad, Devor, Birnboim 03

Summary: Cusp

Dark-matter halos in CDM naturally form **cusps** due to merging compact satellites

Observed Core

Low Surface Brightness Galaxies



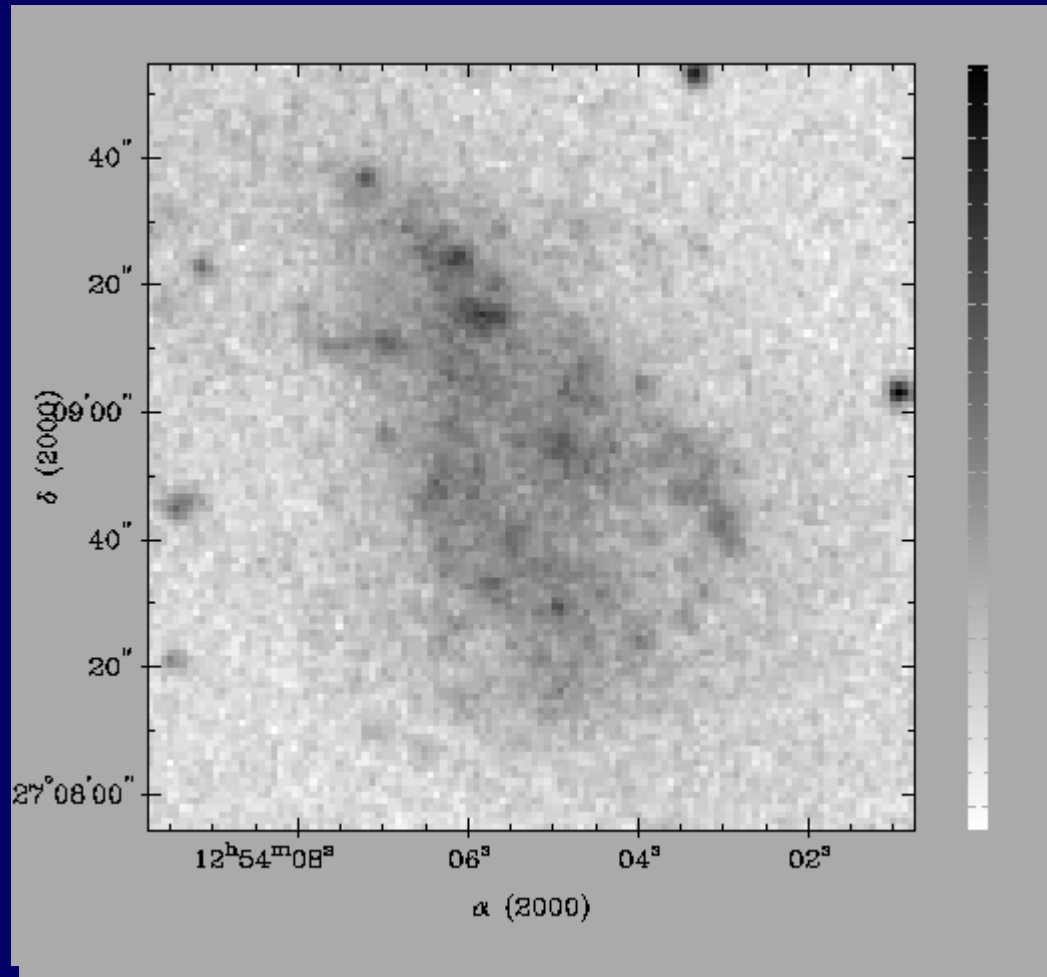
Compare simulated $V_c(r)$ with rotation curves of dark-matter dominated LSB galaxies

Observations:

de Blok et al (2001) (B01),
de Blok & Bosma (2002) (B02),
and Swaters et al (2003) (S03)

Peak velocities range from 25
km/s to 270 km/s

These measurements are hard!



DDO154 (a dwarf LSB)

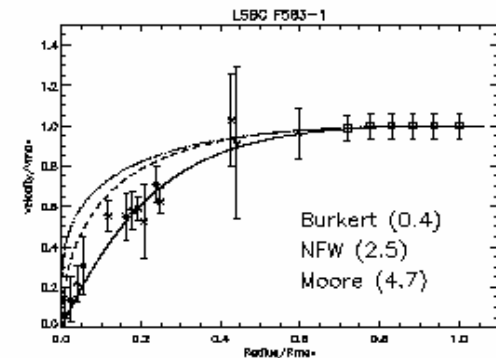
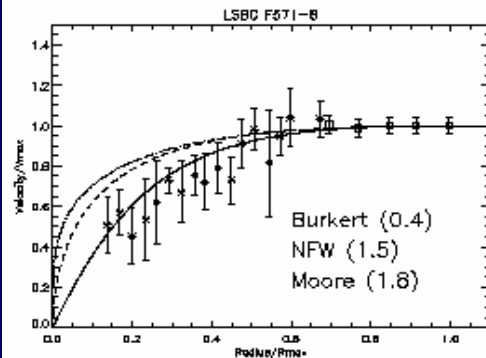
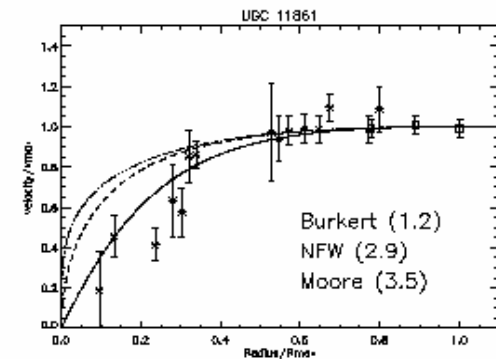
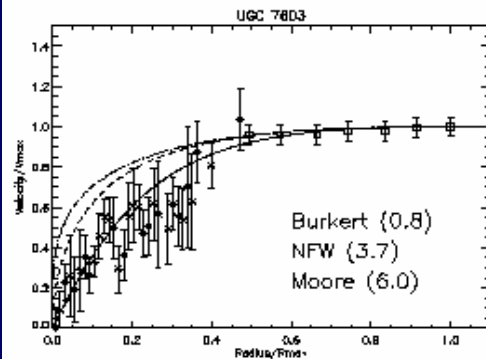
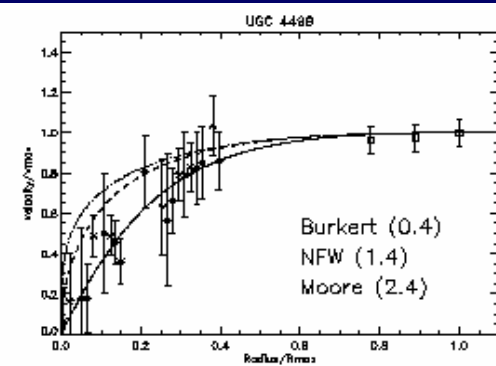
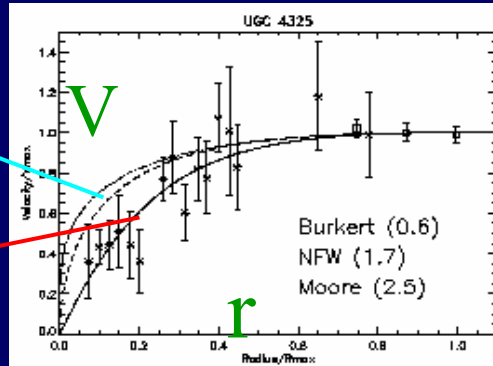
Observed cores vs. simulated cusps

core $\alpha=0$

cusp $\alpha \geq 1$

$$V^2 = \frac{GM(r)}{r}$$

$$\rightarrow V \propto r^{1-\alpha/2}$$

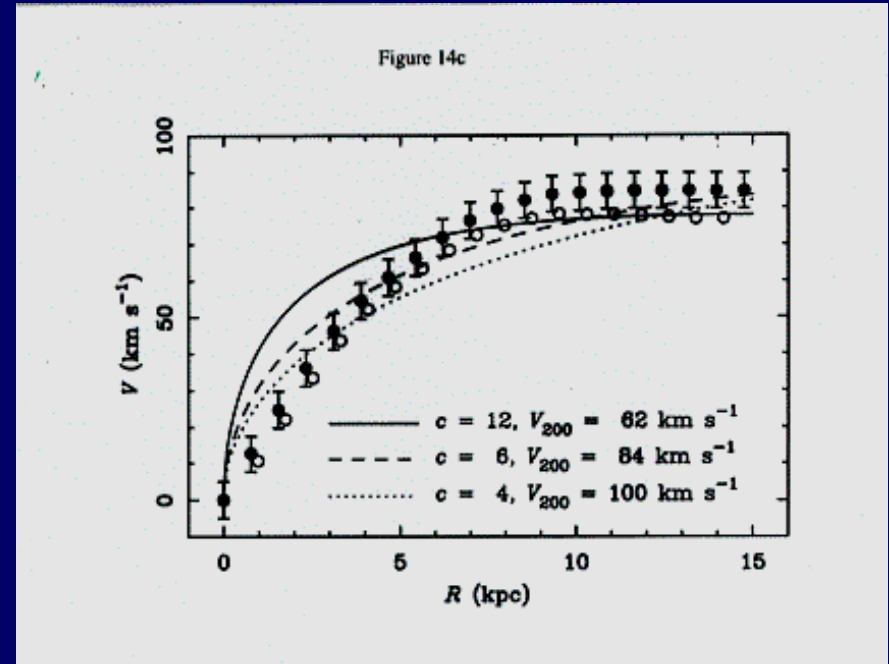


LSB rotation curves and CDM halos

Two problems:

The **shape** of LSB galaxy rotation curves is inconsistent with the circular velocity curves of CDM halos.

The **concentration** of dark matter halos is inconsistent with rotation curve data: there is too much dark matter in the inner regions of LSB galaxies.



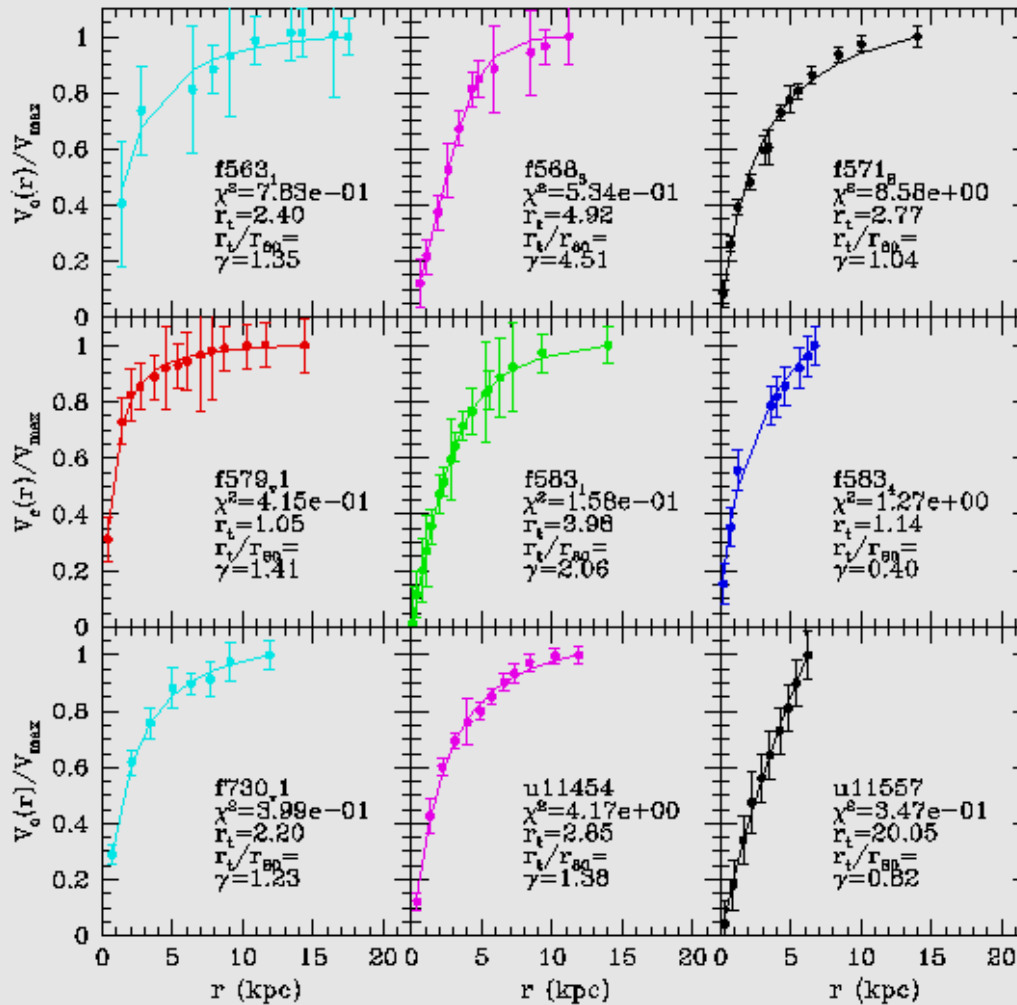
McGaugh & de Block 1998

see also Moore 1994

Flores & Primack 1994

LSB rotation curves (McGaugh et al sample)

Rotation Speed



Radius

The shape of $V(r)$ varies from galaxy to galaxy

A fitting function:

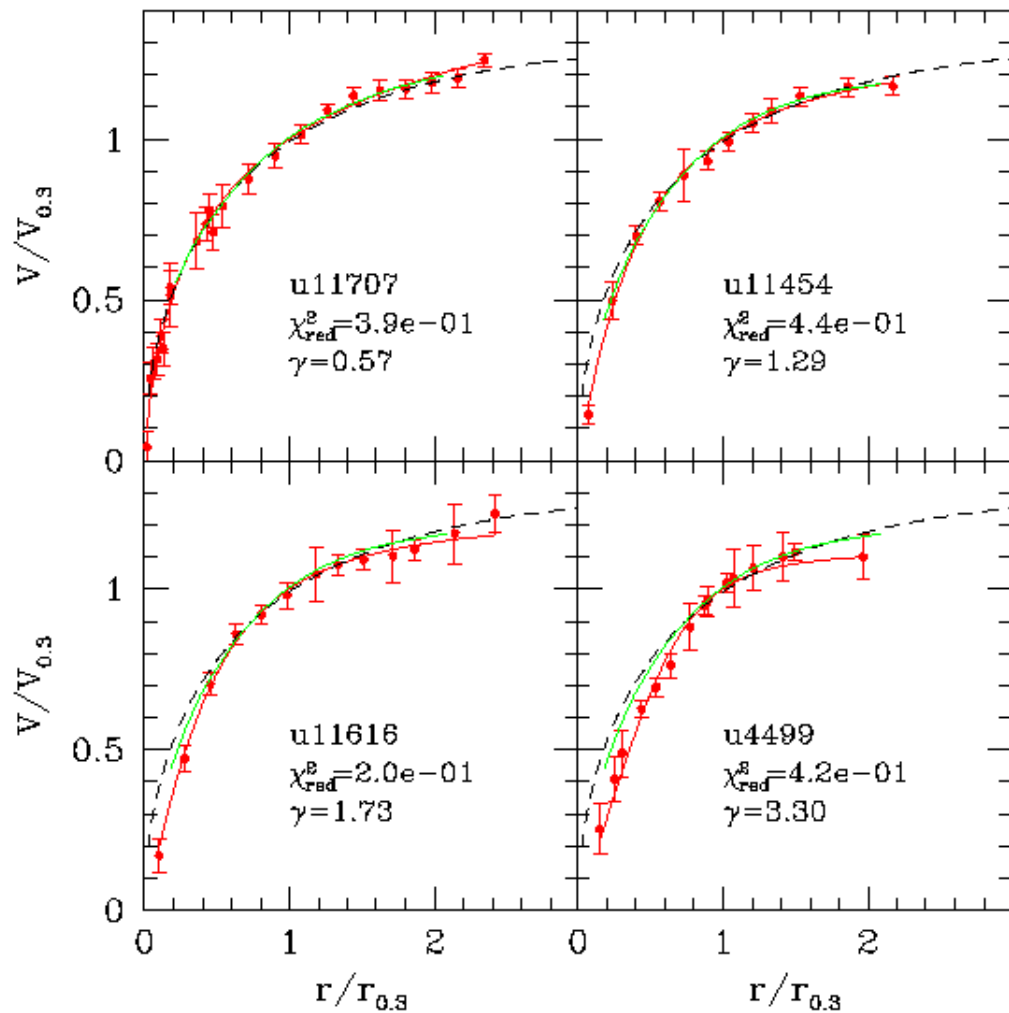
$$V_{\gamma}(r) = V_0 (1 + (r/r_t)^{-\gamma})^{-1/\gamma}$$

The parameter γ is a good indicator of the shape of the rotation curve, the rate of change from rising to flat.

Hayashi et al 2003

Scaled LSB rotation curves: a representative sample

Rotation Speed



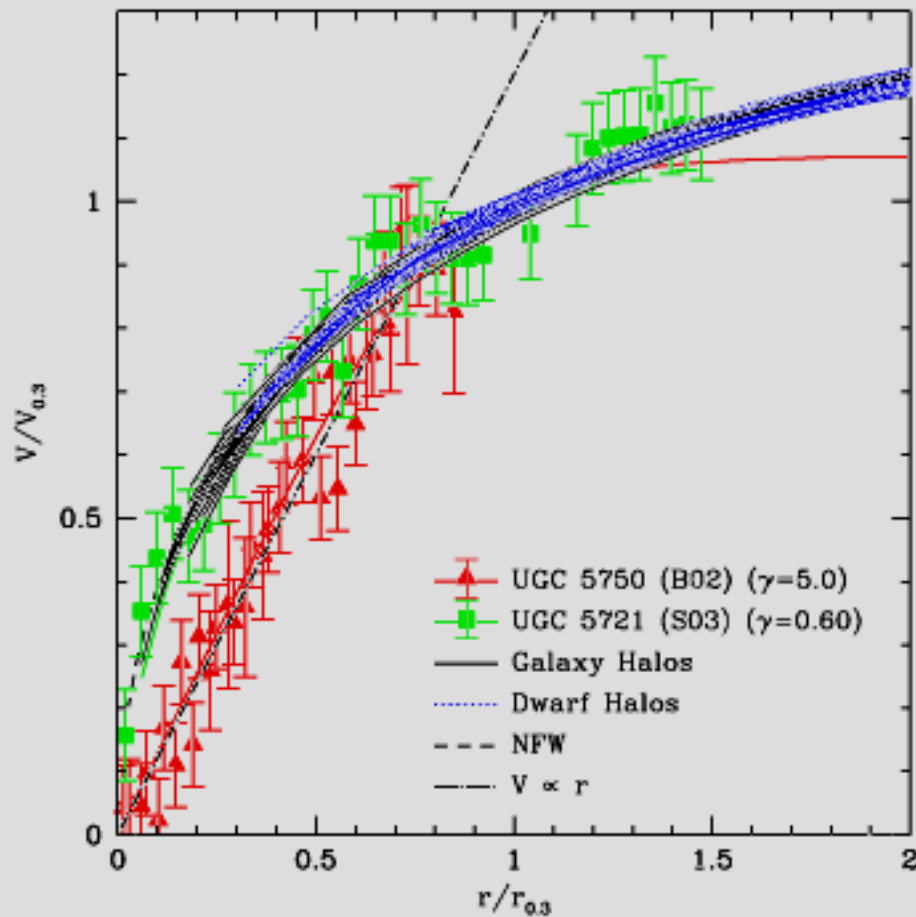
Radius

75% of LSB have $0.5 < \gamma < 2$
(~CDM halos)

25% have $\gamma \gg 2$
(in conflict with CDM halos)

Scaled LSB rotation curves

Rotation Speed

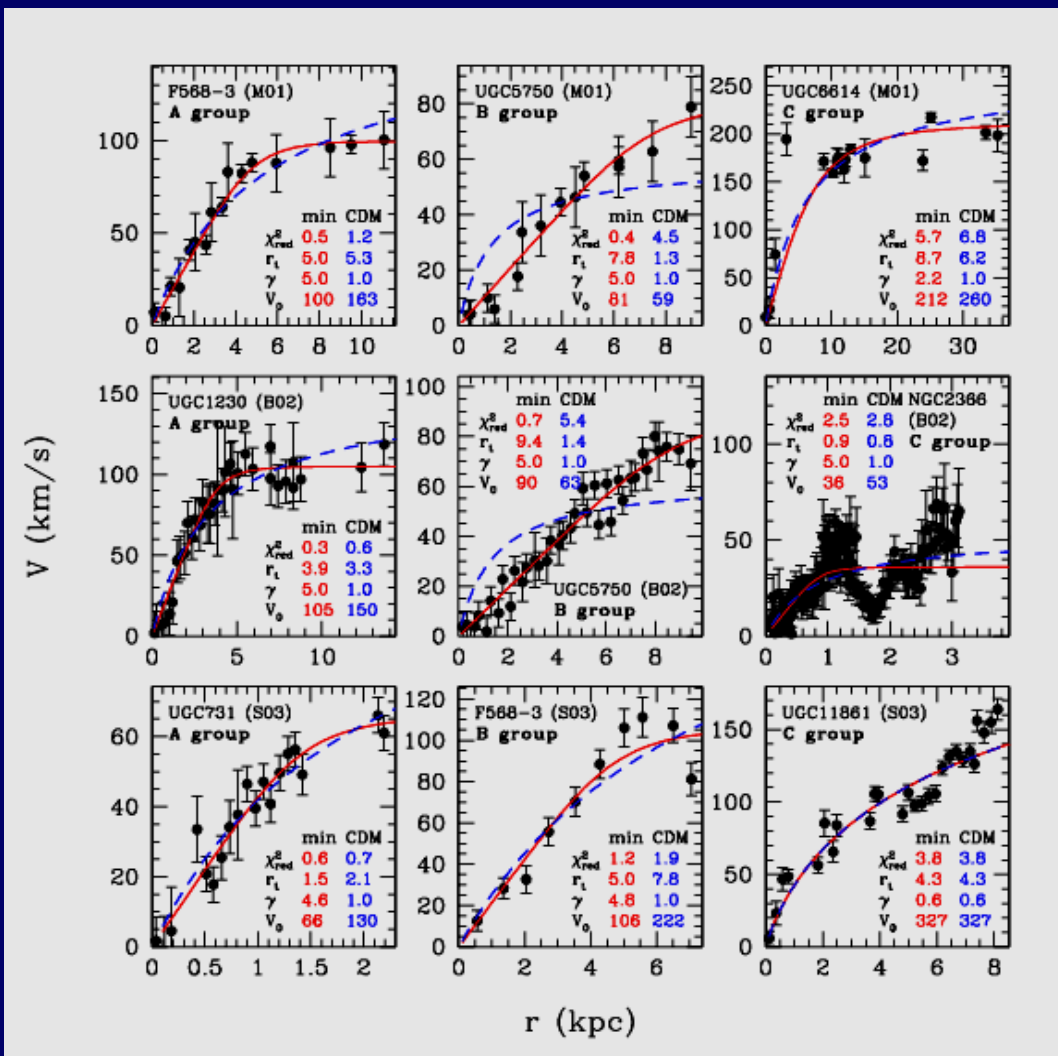


Radius

75% of LSB have $0.5 < \gamma < 2$
(~CDM halos)

25% have $\gamma \gg 2$
(in conflict with CDM halos)

Rotation Curves Inconsistent with CDM Halos



Three categories of rotation curves:

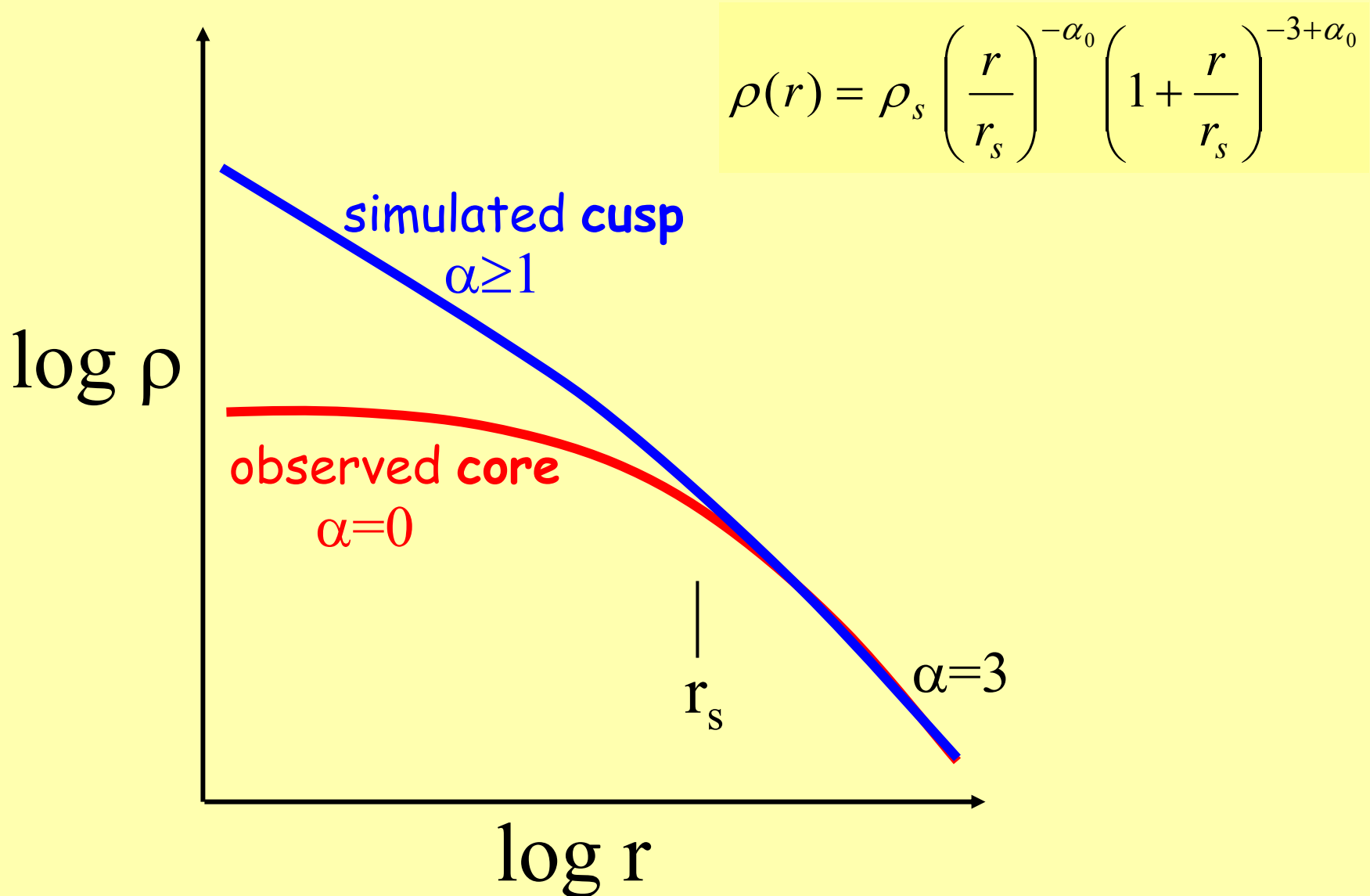
A) Well fit by V_g with LCDM compatible parameters (70%)

B) Poorly fit by V_g with LCDM-compatible parameters (10%)

C) Poorly fit by V_g with any parameters (20%)

Only 10% of LSB rotation curves are robustly inconsistent with LCDM halo structure

The dark-halo cusp/core problem

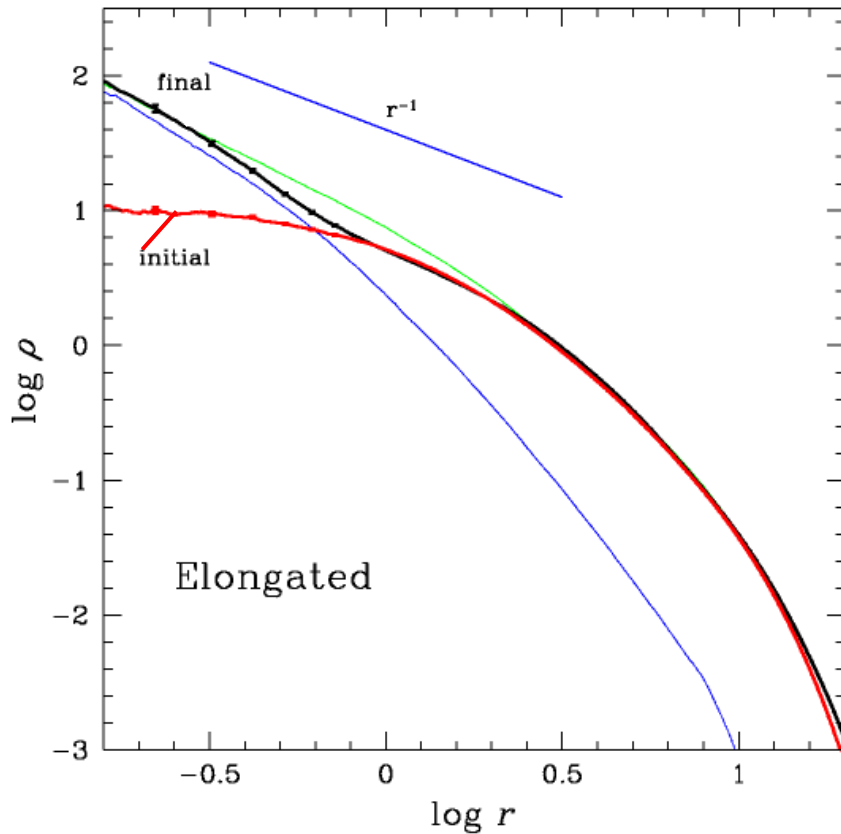


How to make and maintain a core?

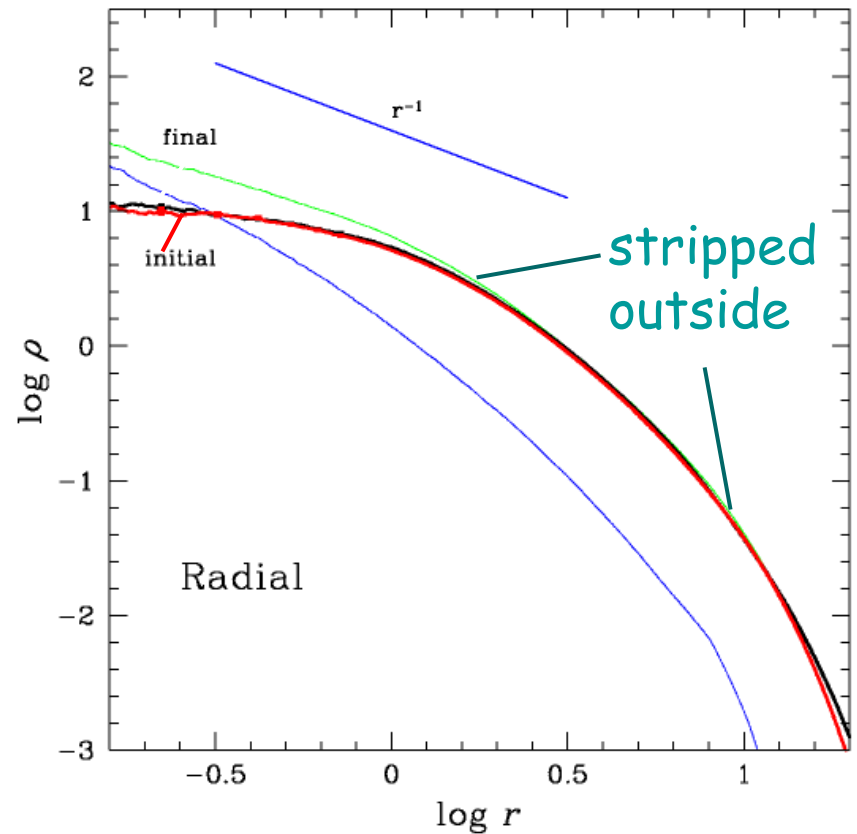
must suppress satellite
mergers with the halo core!

Compact vs. puffy satellite

compact



puffy 1/3 density



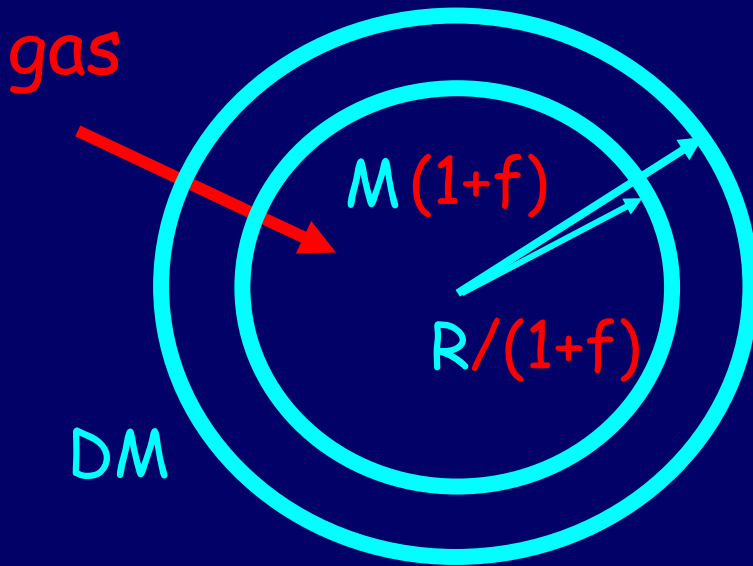
Adiabatic Contraction

Periodic motion under a slowly varying potential

Adiabatic invariant:

$$I \approx \int_0^T v^2 dt \approx v^2 T$$

$$t_{\text{dyn}} \sim \frac{R}{V} \sim \frac{R}{(GM/R)^{1/2}} \sim (GM/R^3)^{-1/2} \sim (G\rho)^{-1/2}$$



$$I \approx \frac{GM}{R} \left(\frac{M}{R^3} \right)^{-1/2} \propto (MR)^{1/2}$$

$$R \propto M^{-1}$$

Instant Blowout

$$E_{\text{before}} = -\frac{GM^2}{R} + \frac{1}{2}MV^2$$

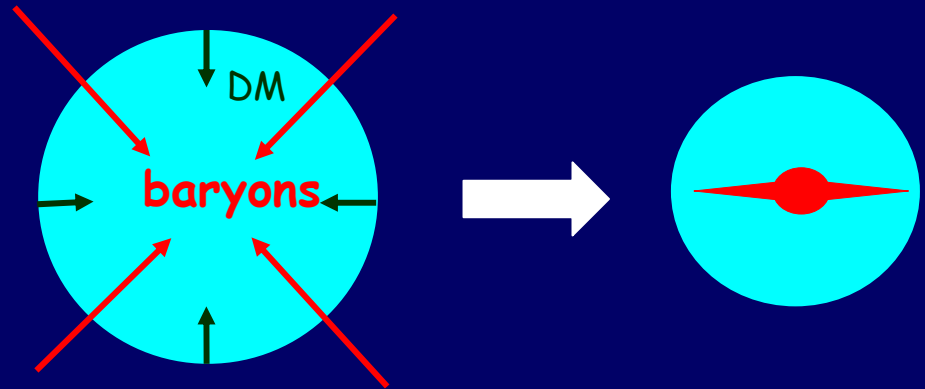
Lose $M/2$ while V^2 is unchanged:

$$E_{\text{after}} = -\frac{G(M/2)^2}{R} + \frac{1}{2}(M/2)V^2 = 0$$

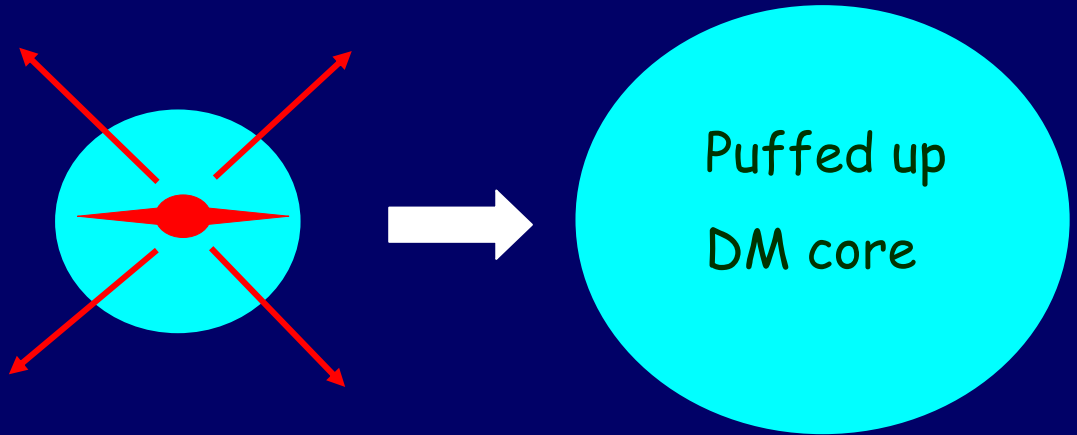
unbound!

DM-halo reaction to blowout

Adiabatic contraction:



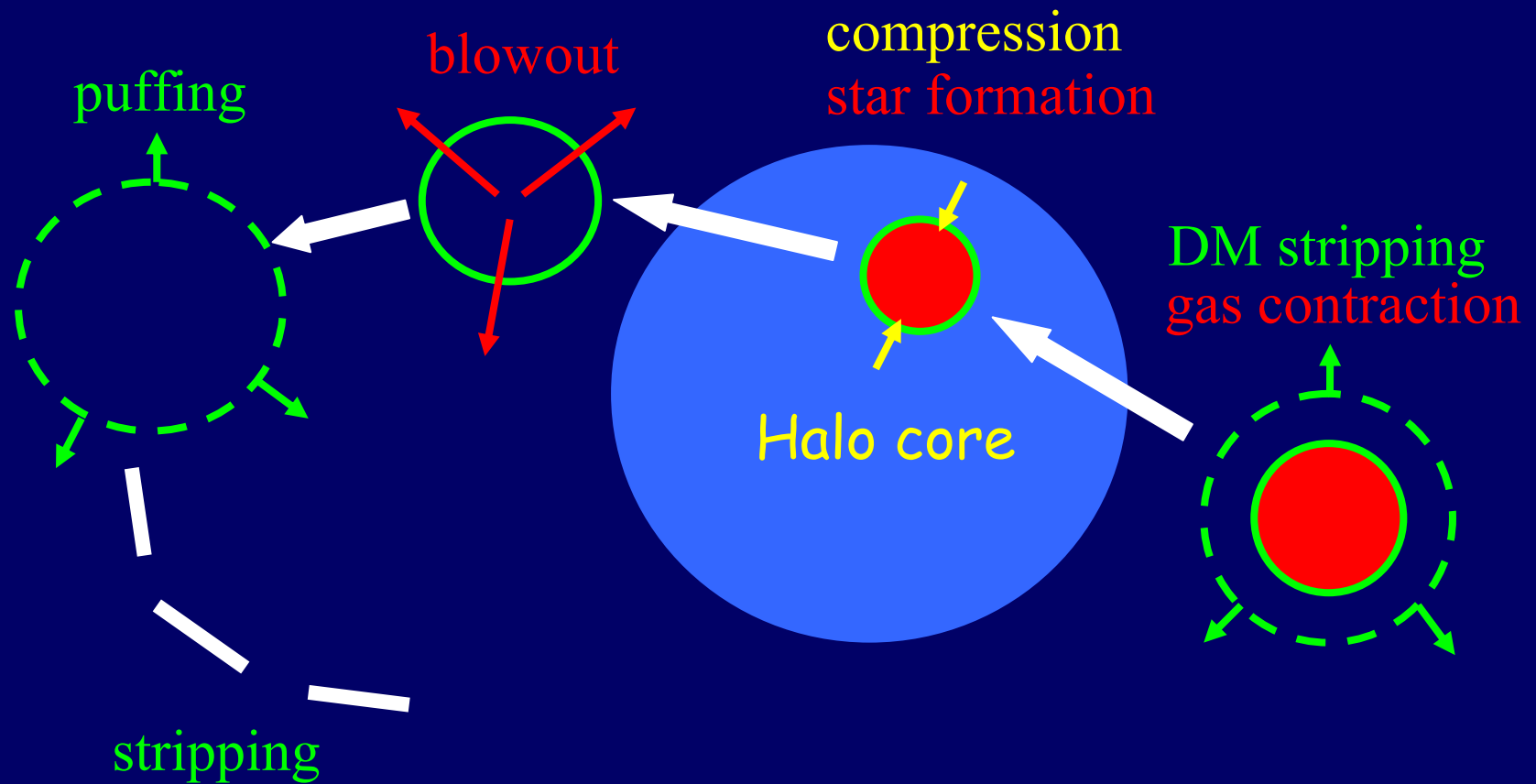
Instant blowout:
by supernova feedback



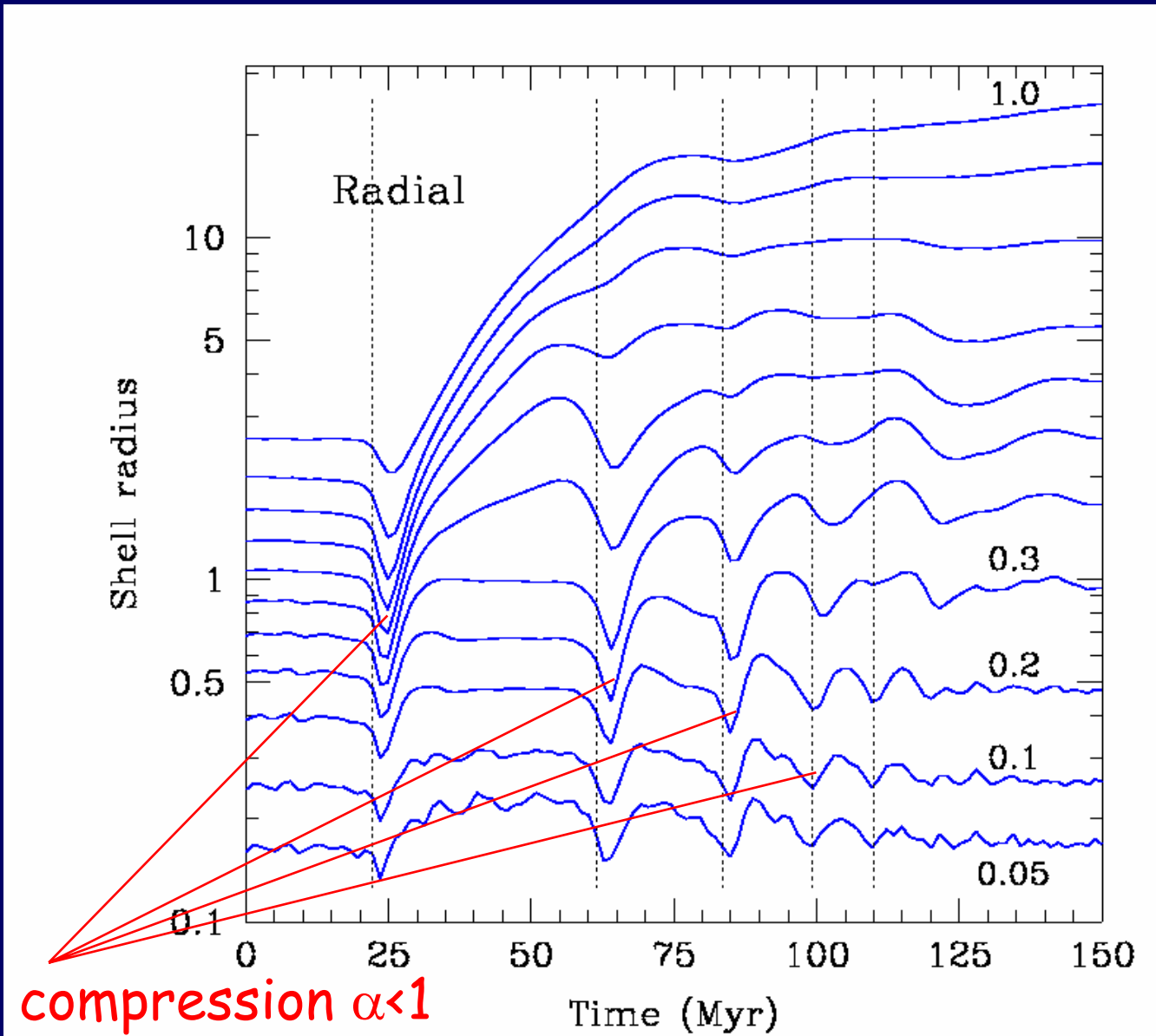
only 1/6 in density (Gnedin & Zhao 02)
not enough in big galaxies?

Enough in satellites?

Satellite disruption by stimulated feedback



Compression in core



Summary: Core

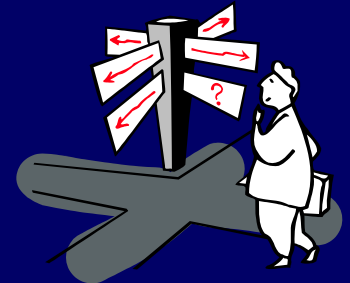
Feedback may lead to a core
by puffing small satellites

Caveats

- Cusps (though flatter) form also in simulations where satellites are suppressed
 - Cores detected in big galaxies and clusters (?)
- Puffing-up of satellite halos is **necessary** for cores, but perhaps **not sufficient**

Other scenarios for core formation

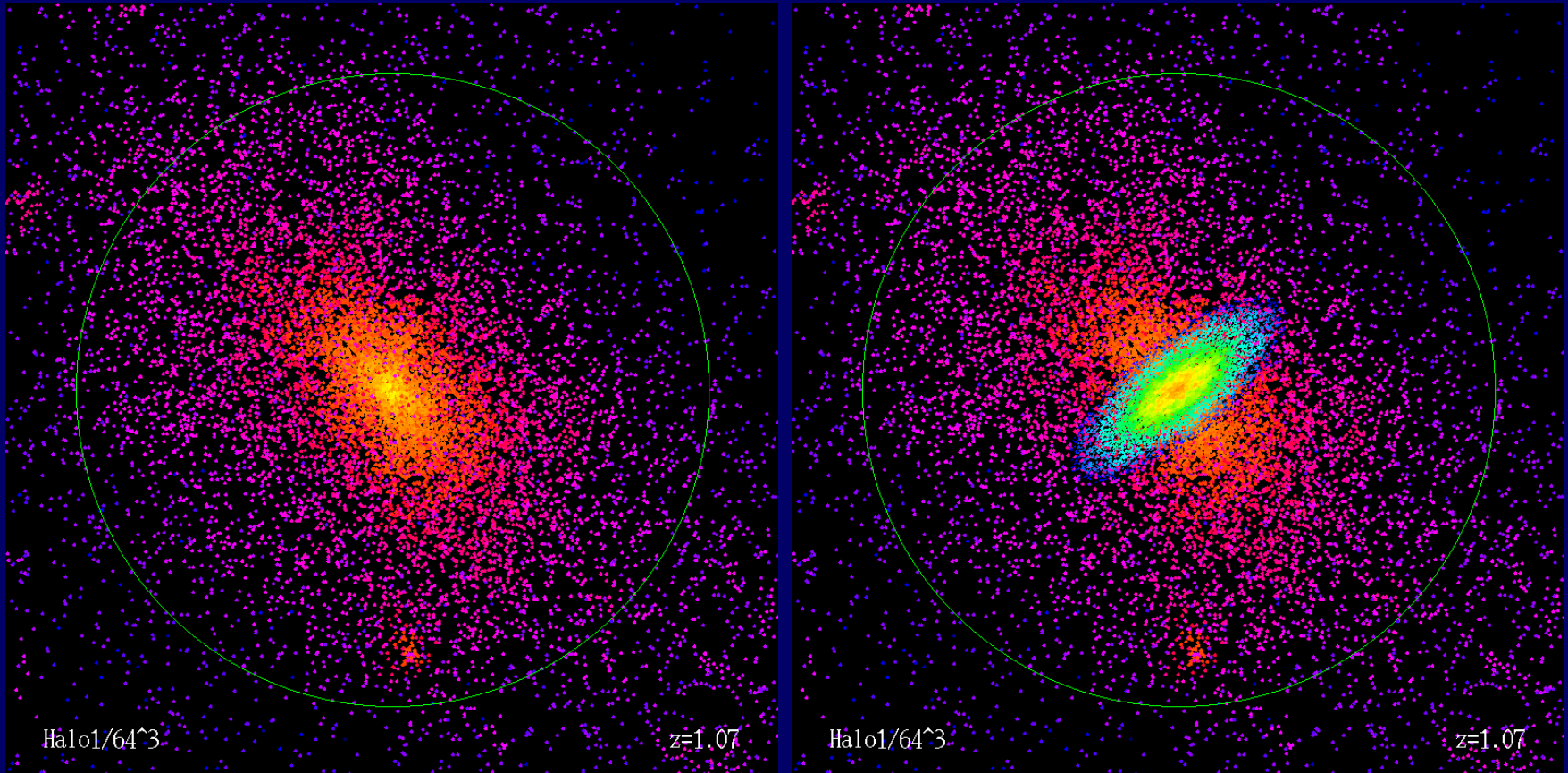
- Warm dark matter, Interacting dark matter
→ suppress satellites
- Disruption of satellites by a massive black hole
(Merritt & Cruz 01)
- Angular-momentum transfer from a big bar
to the halo core (Weinberg & Katz 02)
- Delicate resonant tidal reaction of halo-core orbits
if the system is noise-less (Katz & Weinberg 02)
- Heating of the cusp by merging clouds
(El-Zant, Shlosman & Hoffman 02)



Origin of Core: Disk in Triaxial Halo

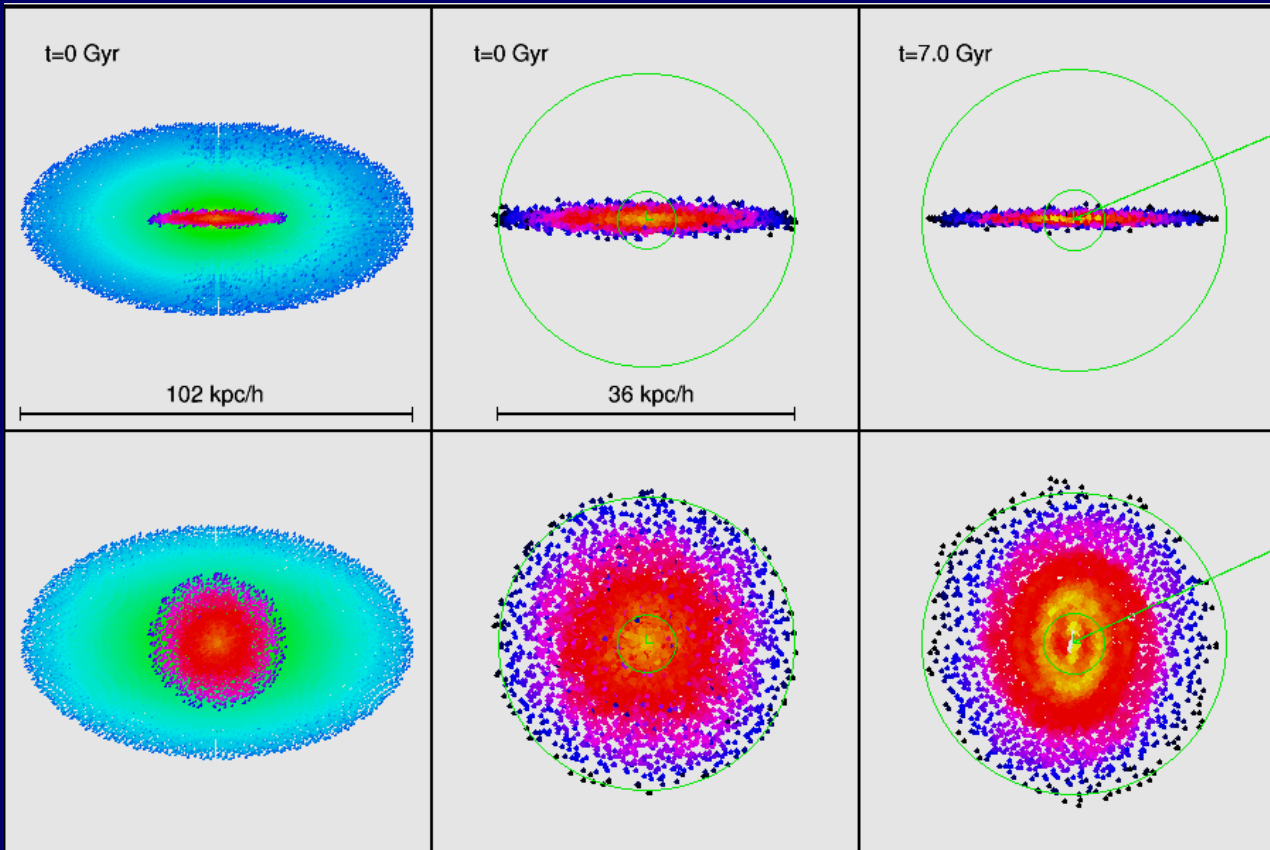
Disk Rotation curve is NOT $V^2=GM(r)/r$
Hayashi, Navarro et al.

Disks in realistic dark matter halos



Massless isothermal gaseous disk in the non-spherical DM halo potential tracks the closed orbits within this potential

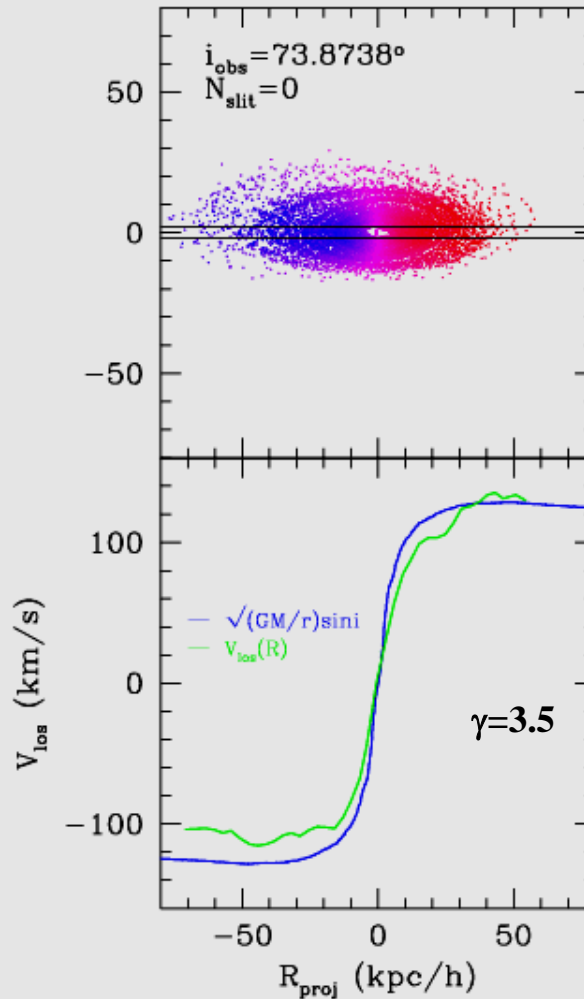
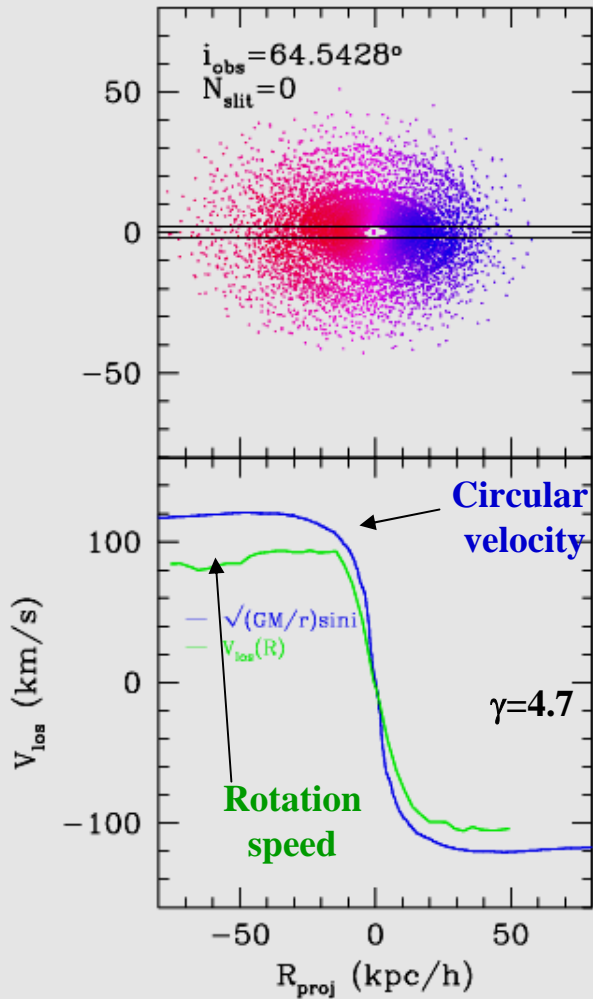
Dynamics of a Gaseous Disk



Closed orbits in triaxial potentials are not circular, and not limited to a plane.

High γ ?

Disks in triaxial dark matter halos



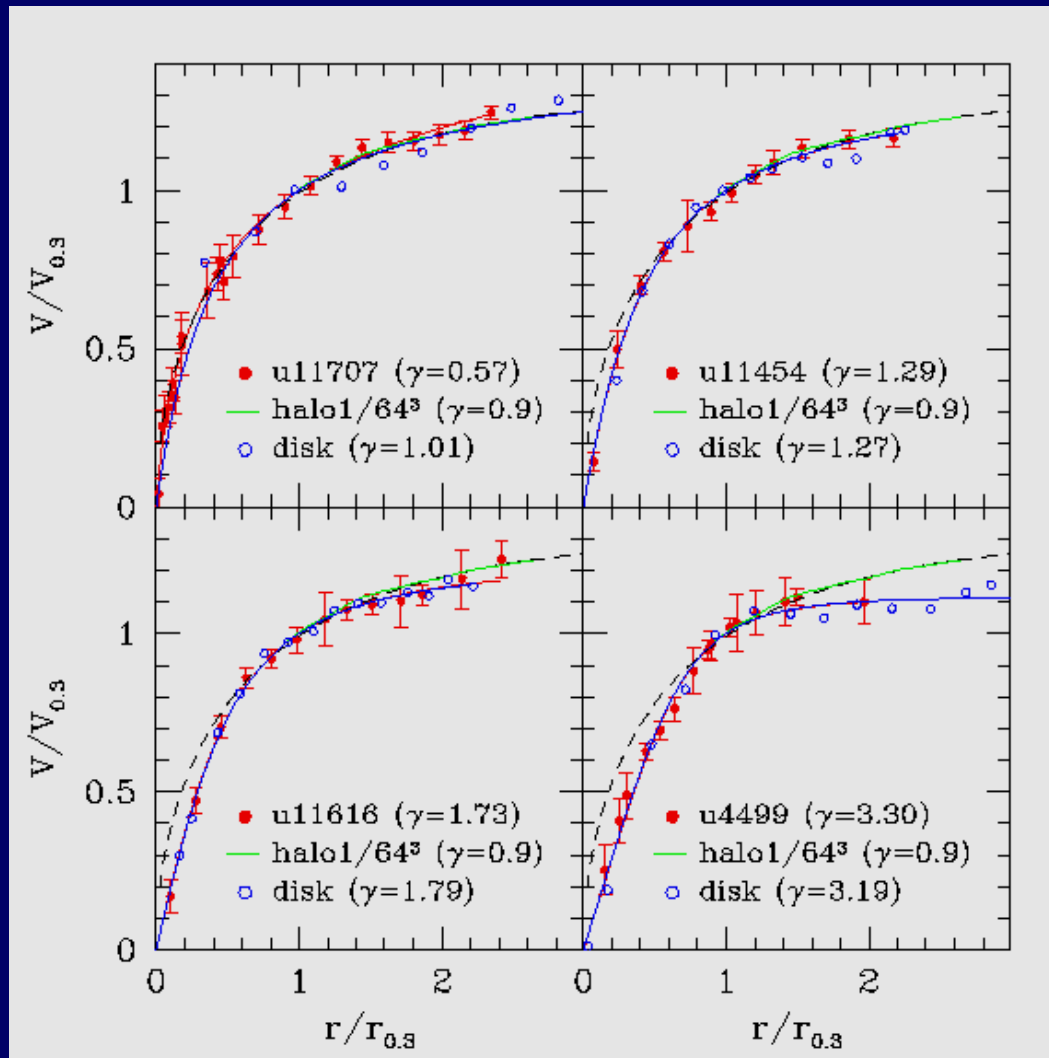
Inferred rotation speeds may differ significantly from actual circular velocity.

Inclination: 50 degrees

67 degrees

Scaled Rotation Curves: disk in CDM halo vs LSBs

Scaled Rotation Speed

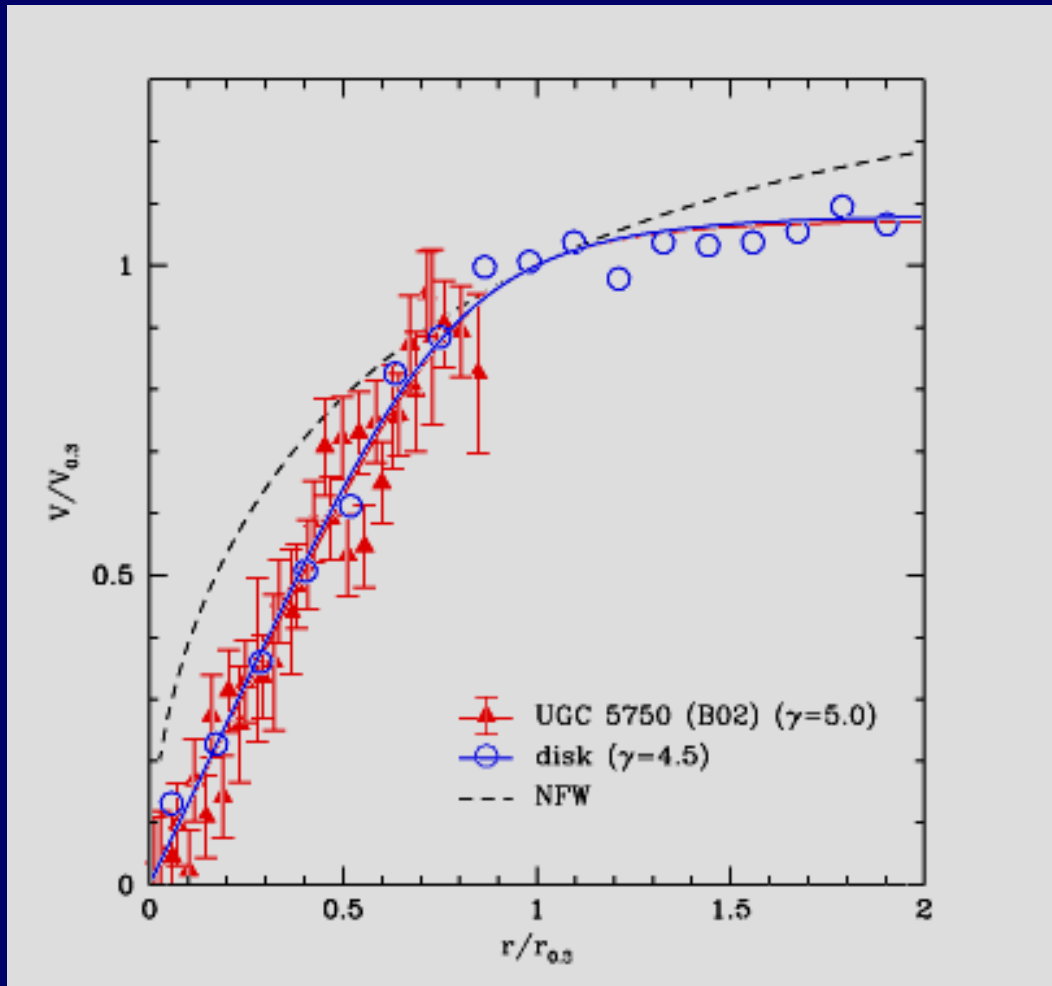


Scaled radius

All LSB rotation curve shapes may be accounted for by various projections of a disk in a single CDM halo

Scaled LSB rotation curves: a representative sample

Rotation Speed



Radius

LSB rotation curve shapes may be accounted for by various projections of a disk in a single CDM halo

Triaxiality in the halo potential may be enough to explain the "cusp-core" discrepancy.

Hayashi et al 2003