Weak Lensing

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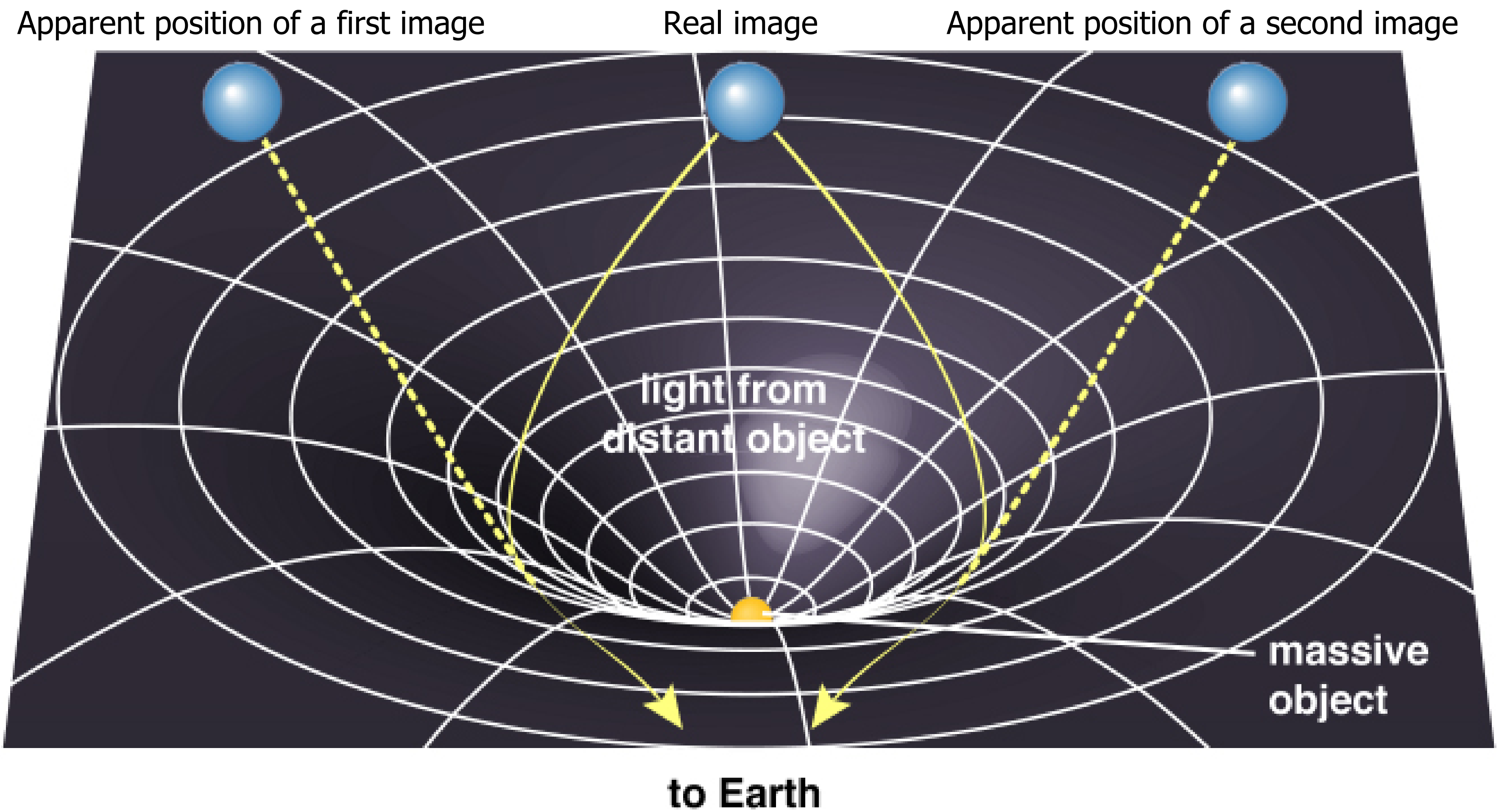
I. Gravitational Lensing:

concepts, definitions, applications to clusters of galaxies

General relativity:

curvature of space time locally modified by mass condensation



Deflection of light, magnification, image multiplication distortion of objects : directly **depend on the amount of matter**

Gravitational lensing effect is **achromatic** (photons follow geodesics regardless their energy)

Lensing : important steps

1804: Soldner works on the deflection of light by gravity

1915: General Relativity

1919: Deflection angle of stars behind the Sun

1936: Einstein's work on lensing by stellar objects

1937: F. Zwicky visions on « extragalactic nebulae » as lenses

1964: Refsdal: time delay and H_0

1979: Multiple image of quasars (0957+561)

1987: Giant arcs in clusters of galaxies (A370, A2218, CL244)

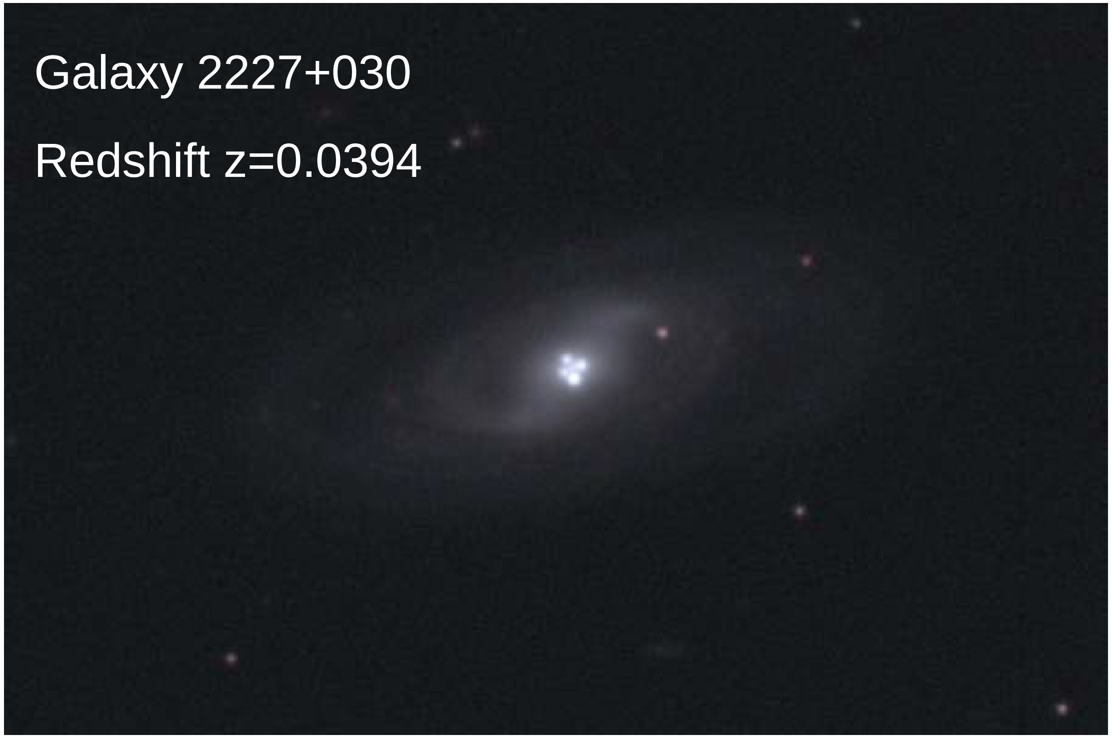
1987: First Einstein ring

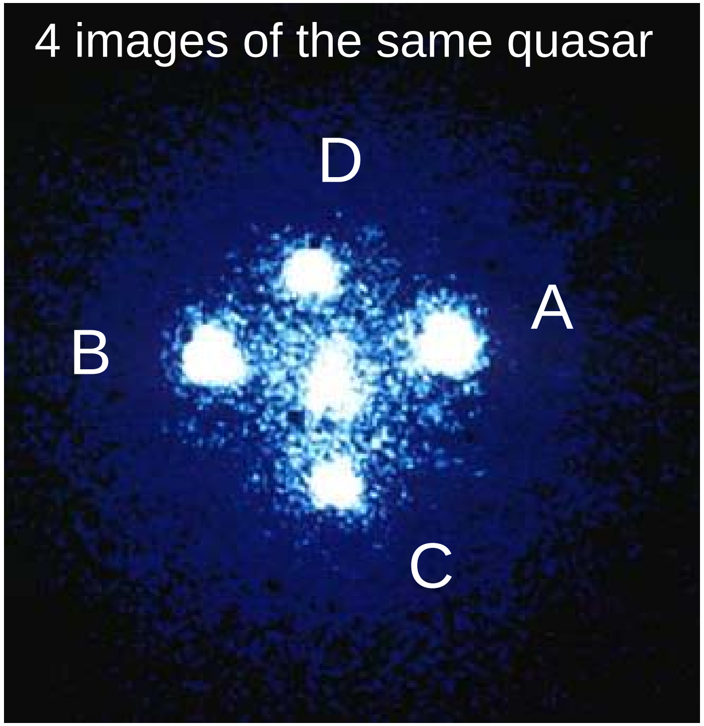
1993: EROS and MACHO microlensing experiments

2000: Detection of cosmic shear signal

2006: The « Bullet » cluster and existence of dark matter

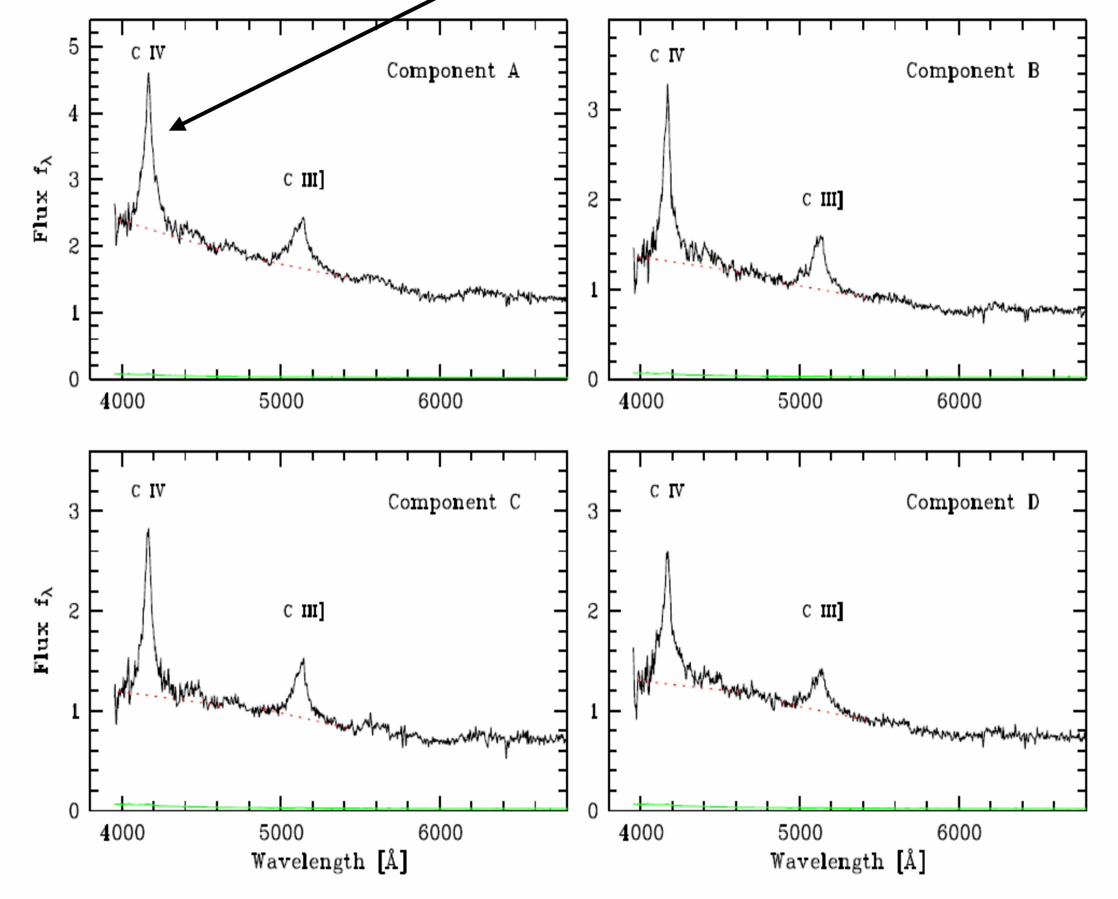
The « Einstein Cross»

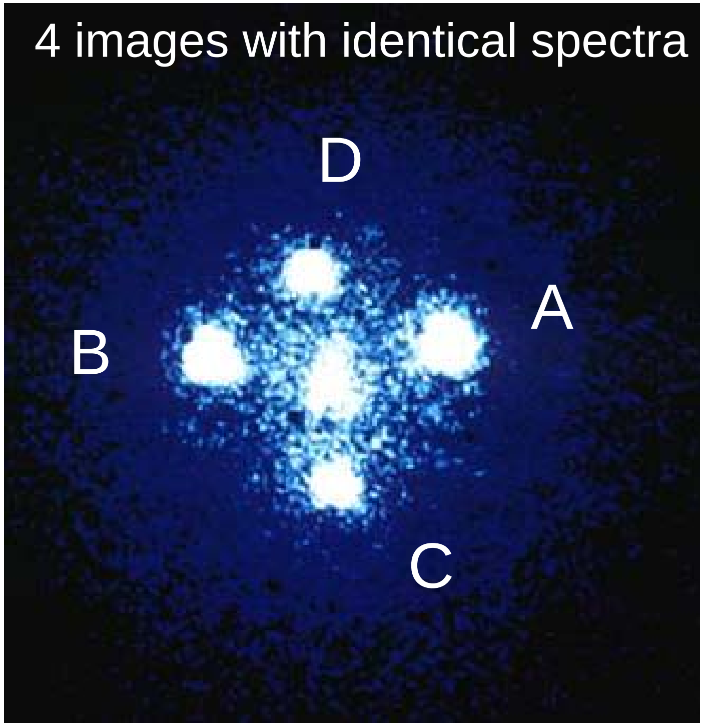


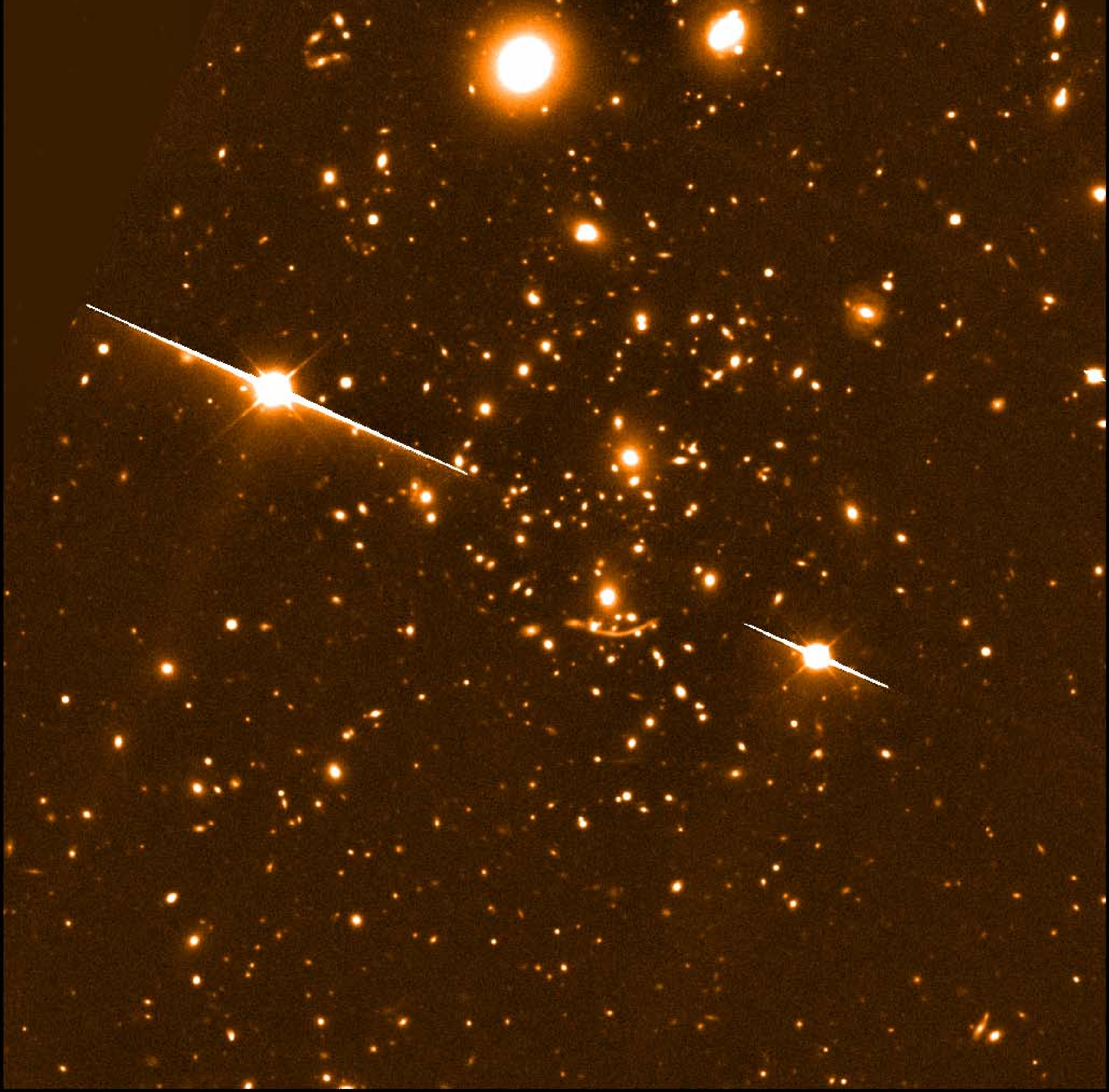


The « Einstein Cross»

CIV emission line at 154.9 nm observed 417.6 nm :z = 1.695



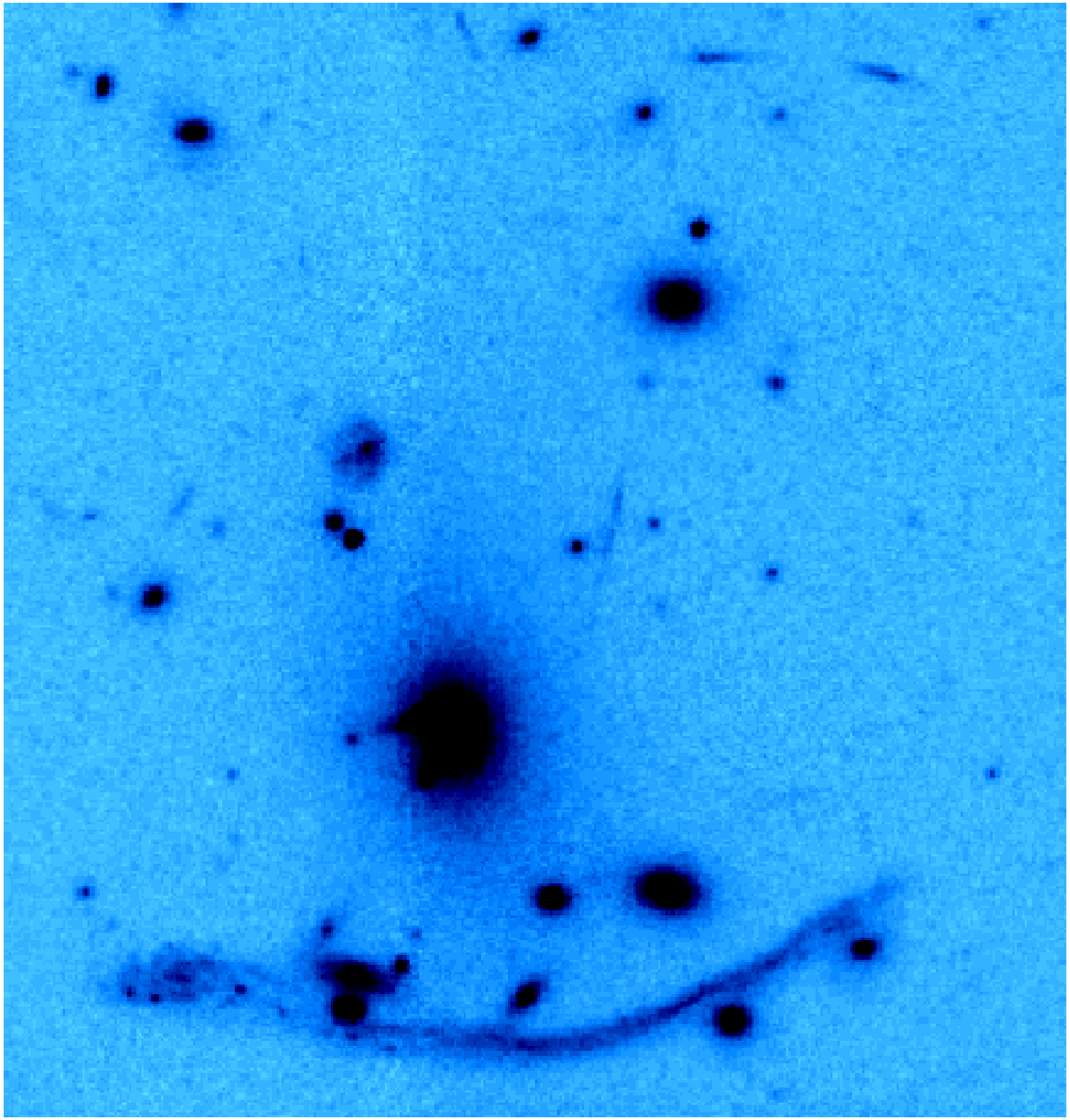




First giant arc discovered in Abell 370

$Z_{cluster}$ =0.374

Z_{arc}=0.720



Gravitational Lensing

- Fundamental assumptions over the lecture
  - Weak field limit v^2/c^2
  - Stationnary field t_{dyn}/t_{cross}
  - Thin lens approximation L_{lens}/L_{bench}
  - Transparent lens
  - Small deflection angle

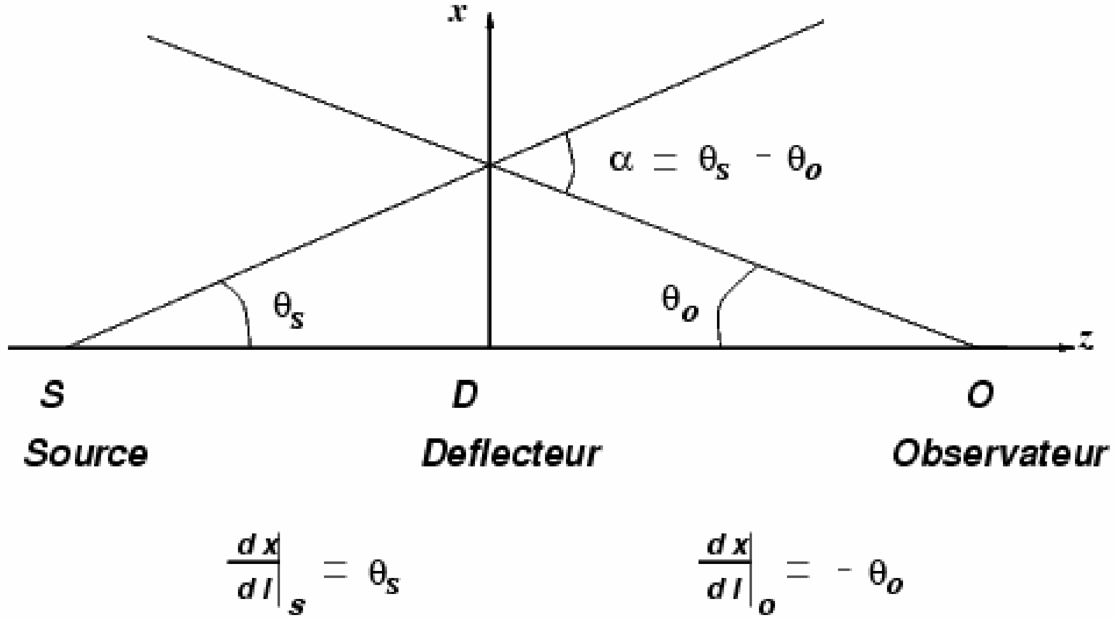


Figure 1: Configuration of the gravitational optical bench.

Deflection Angle

Deflection Angle

In the weak field approximation, the metric writes

$$\mathrm{d}s^2 = c^2\left(1+\frac{2\Phi}{c^2}\right)\mathrm{d}t^2 - \left(1-\frac{2\Phi}{c^2}\right)\mathrm{d}l^2\,. \qquad (1)$$

For a photon $\mathrm{d}s^2 = 0$:

$$\mathrm{d}t = \frac{1}{c}\left(\frac{1-\frac{2\Phi}{c^2}}{1+\frac{2\Phi}{c^2}}\right)^{1/2}\mathrm{d}l \approx \frac{1}{c}\left(1-\frac{2\Phi}{c^2}\right)\mathrm{d}l\,. \qquad (2)$$

Configuration: see Fig.1 .

We then have $\mathrm{d}l^2 = \mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2$.

- According to Fermat Principle photons only follow optical paths with extrema propagation time.

It then given by Eq. (2) and the paths are those stationnary with respect to a small variation δt. Since

$$ct = \int \left(1 - \frac{2\Phi}{c^2}\right) \mathrm{d}l \tag{3}$$

$$ct = \int \left(1 - \frac{2\Phi}{c^2}\right) \left[\left(\frac{\mathrm{d}x}{\mathrm{d}z}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}z}\right)^2 + 1 \right]^{1/2} \mathrm{d}z\,. \tag{4}$$

Equation (3) corresponds to a light beam propagating in a transparent medium with refraction index :

$$n = \left(1 - \frac{2\Phi}{c^2}\right)\,. \tag{5}$$

That is, taking into account the relation between n and Φ:

$$\boxed{\alpha = -\frac{2}{c^2} \int_S^O \nabla_\perp \Phi \, \mathrm{d}l \; .} \tag{16}$$

Relation with the Projected Mass Density

The gravitational force produced by the mass inside the lens is

$$\vec{F} = G \int \frac{\vec{x'} - \vec{x}}{|\vec{x'} - \vec{x}|^3} \rho\left(\vec{x'}\right) \mathrm{d}\vec{x'} \,. \tag{17}$$

Let us define the gravitational potential as $\Phi\left(\vec{x}\right)$

$$\Phi\left(\vec{x}\right) = -G \int \frac{\rho\left(\vec{x'}\right)}{|\vec{x'} - \vec{x}|} \mathrm{d}\vec{x'} \,, \tag{18}$$

so that

$$\vec{F} = -\vec{\nabla}\Phi \,, \tag{19}$$

Relation with the Projected Mass Density

By subtituting in Eq. (16) on can then express the deflection angle as function of the mass density field:

$$\vec{\alpha} = -\frac{2G}{c^2} \int_S^O \mathrm{d}l \int \mathrm{d}\vec{x'}\, \nabla_{\perp_{\vec{x}}} \frac{\rho\left(\vec{x'}\right)}{\left|\vec{x'} - \vec{x}\right|} \tag{21}$$

$$\vec{\alpha} = -\frac{2G}{c^2} \int \rho\left(\vec{x'}\right) \mathrm{d}\vec{x'} \int_S^O \frac{\left(\vec{x'} - \vec{x}\right)_\perp}{\left|\vec{x'} - \vec{x}]\right|^3} \mathrm{d}l \tag{22}$$

In the *thin lens approximation*, we have

$$\int \rho\left(\vec{x}'\right) \mathrm{d}\vec{x}' = \int \rho\left(\vec{\xi}', z\right) \mathrm{d}\vec{\xi}' \, \mathrm{d}z = \Sigma\left(\vec{\xi}'\right) \tag{23}$$

The relation between the deflection angle and the projected mass density is therefore

$$\alpha(\xi) = \frac{4G}{c^2} \int \frac{(\xi - \xi')\Sigma(\xi')}{|\xi - \xi'|^2} \mathrm{d}\xi' \tag{24}$$

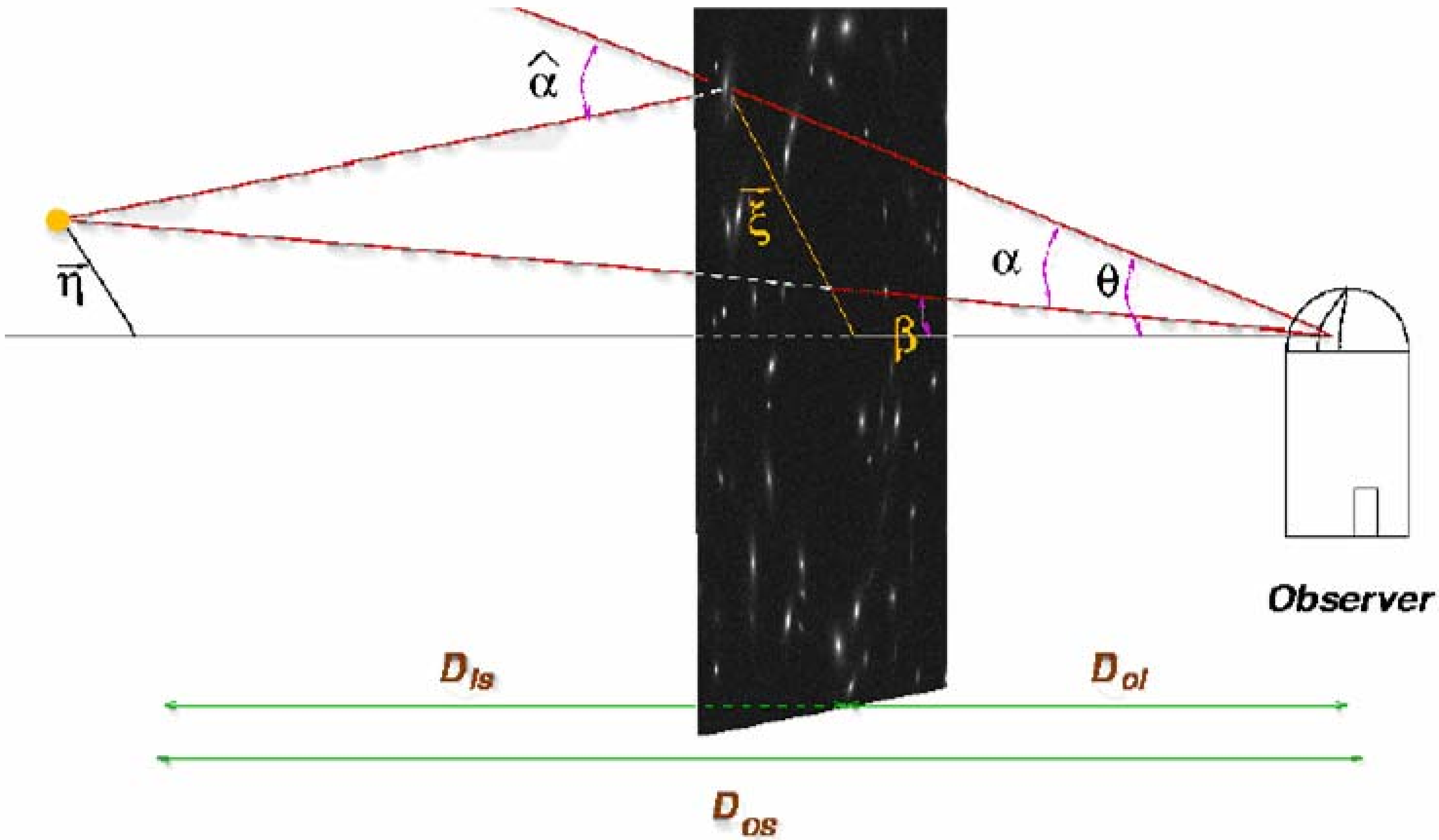
where

- $\Sigma(\xi)$ is the projected mass density,
- ξ is a 2-dimensional vector in the lens plane and
- the integration is done over the lens plane.

For a point mass of mass M: $\Sigma\left(\xi\right) = M\delta\left(\xi\right)$, then

$$\boxed{\alpha\left(\xi\right) = \frac{4GM}{c^2}\frac{\xi}{\left|\xi'\right|^2}} \qquad (25)$$

Useful quantities and terminology



- Lens equation

$$\vec{\eta} = \frac{D_{os}}{D_{ol}}\vec{\xi} - D_{ls}\widehat{\vec{\alpha}}\left(\vec{\xi}\right) \qquad (26)$$

Useful quantities and terminology

- Lens equation

$$\vec{\eta} = \frac{D_{os}}{D_{ol}}\vec{\xi} - D_{ls}\widehat{\vec{\alpha}}\left(\vec{\xi}\right) \tag{26}$$

Setting $\vec{\eta} = D_{os}\vec{\beta}$ and $\vec{\xi} = D_{ol}\vec{\theta}$, one can write the lens eqation in a simple way, using the *scaled deflection angle*:

$$\vec{\beta} = \vec{\theta} - \vec{\alpha}\left(\vec{\theta}\right) \tag{27}$$

- Spherically symmetric lens

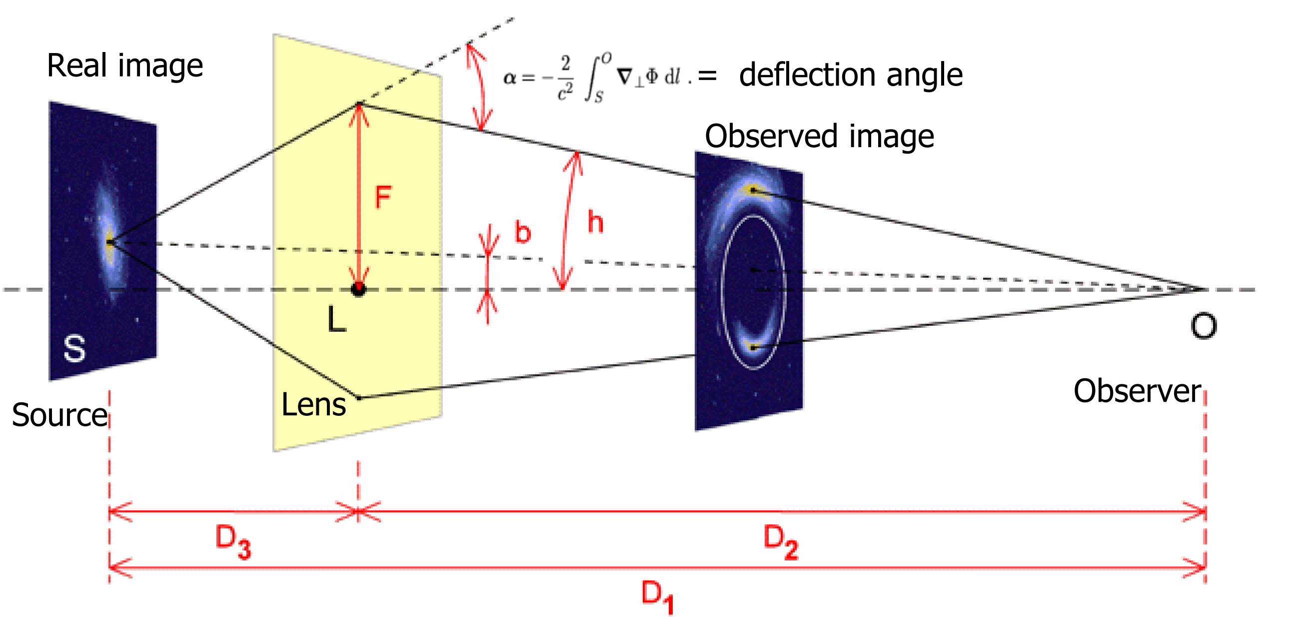
$$\beta = \theta - \frac{D_{ls}}{D_{os}D_{ol}}\frac{4GM\left(\theta\right)}{c^2\theta} \tag{28}$$

where $M(\theta)$ is the mass inside the radius θ.

- Einstein ring For $\vec{\beta} = 0$, the source,lens and observer are perfectly aligned. Formation of *Einstein ring* with radius

$$\theta_E = \left(\frac{D_{ls}}{D_{os} D_{ol}} \frac{4GM(\theta_E)}{c^2} \right)^{1/2} \tag{29}$$

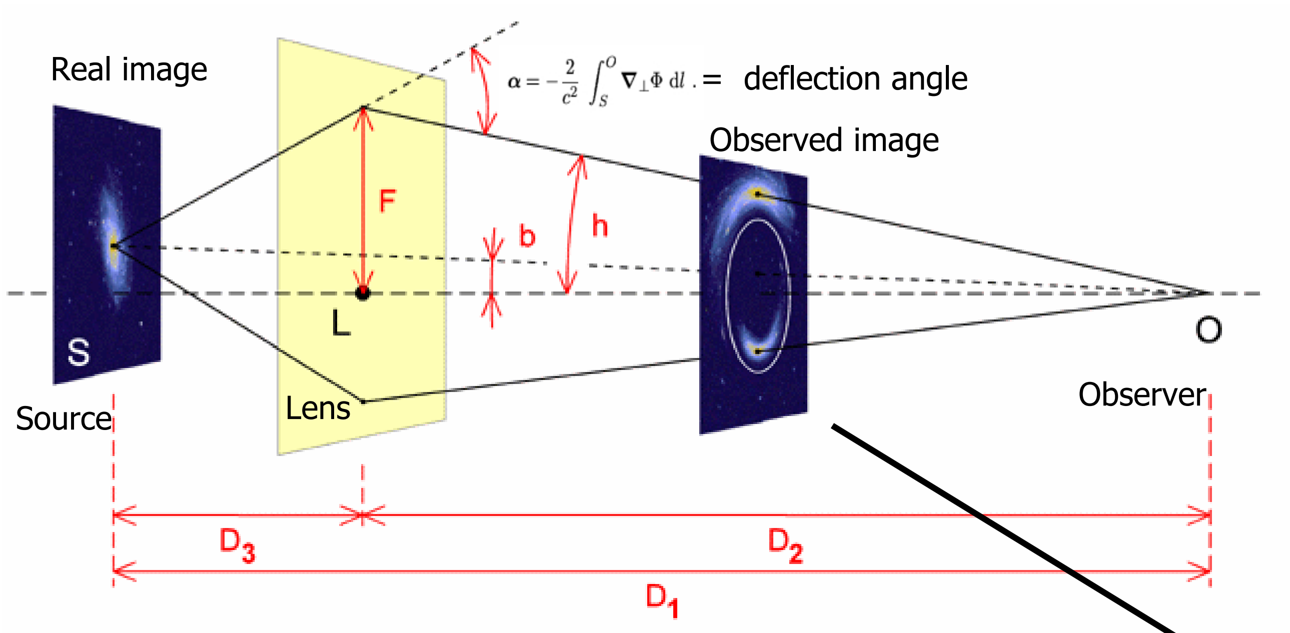
Perfect lens configuration



D_1, D_2, D_3 are cosmological distances: depend on the matter-energy content of the Universe

Source-Lens-Observer perfectly aligned

Perfect lens configuration



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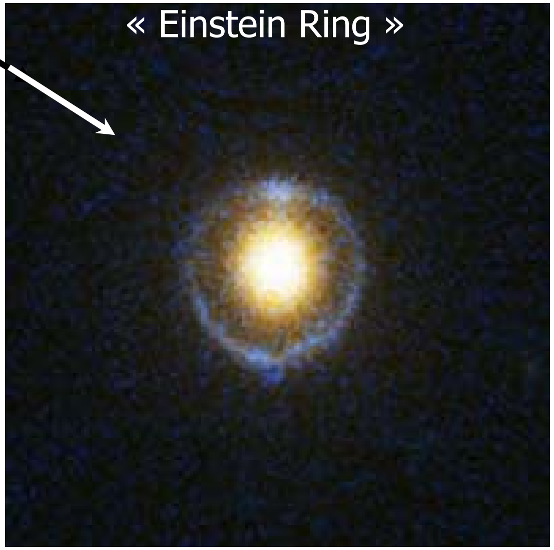


Source-Lens-Observer perfectly aligned

Very rare events

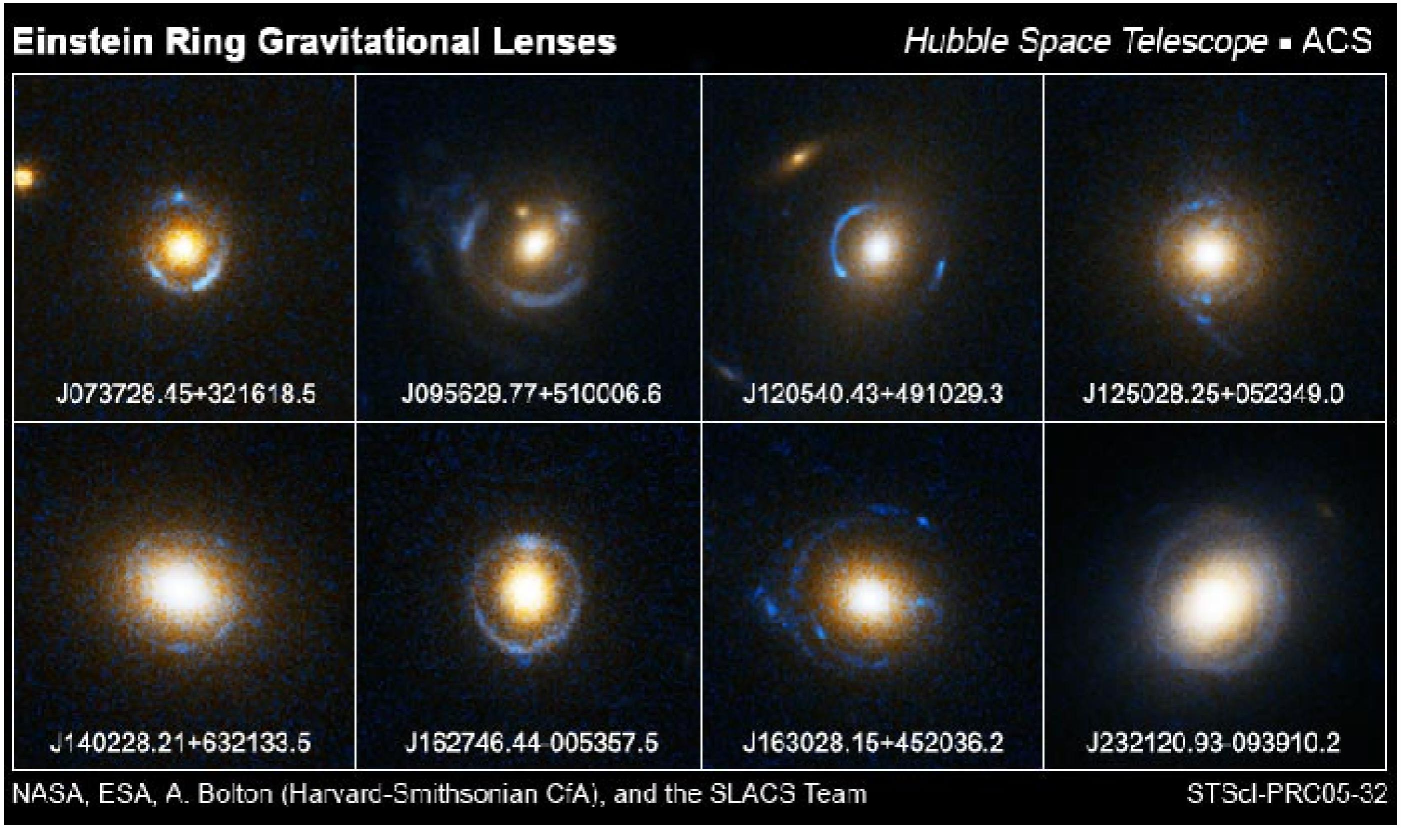


Figure 2: Configuration of the gravitational optical bench.

Einstein Rings:

$$\theta_E = \left(\frac{D_{ls}}{D_{os} D_{ol}} \frac{4GM\left(\theta_E\right)}{c^2} \right)^{1/2} \qquad (29)$$

<u>Typical values:</u>

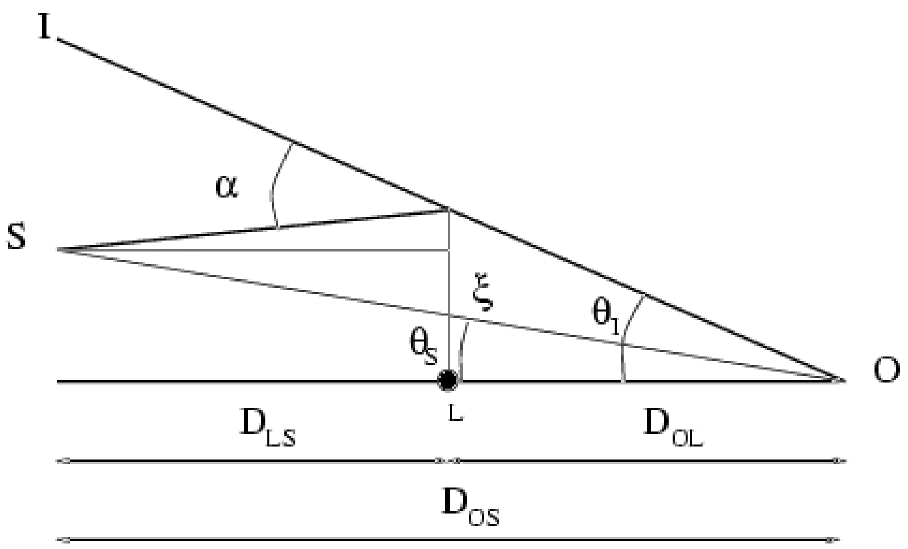
- For a lens of 1 solar mass located at 1 AU and a source a 1 kpc

  θ_E= 0.003 arc-second
- For a lens of 10^{11} solar masse located at 100kpc and a source a 300 kpc

  θ_E= 1 arc-second
- For a lens of 10^{15} solar masses located at 1Gpc and a source a 3 Gpc

  θ_E= 30 arc-second (sensitive to cosmological parameters)

Image multiplicity and shape
Point Mass

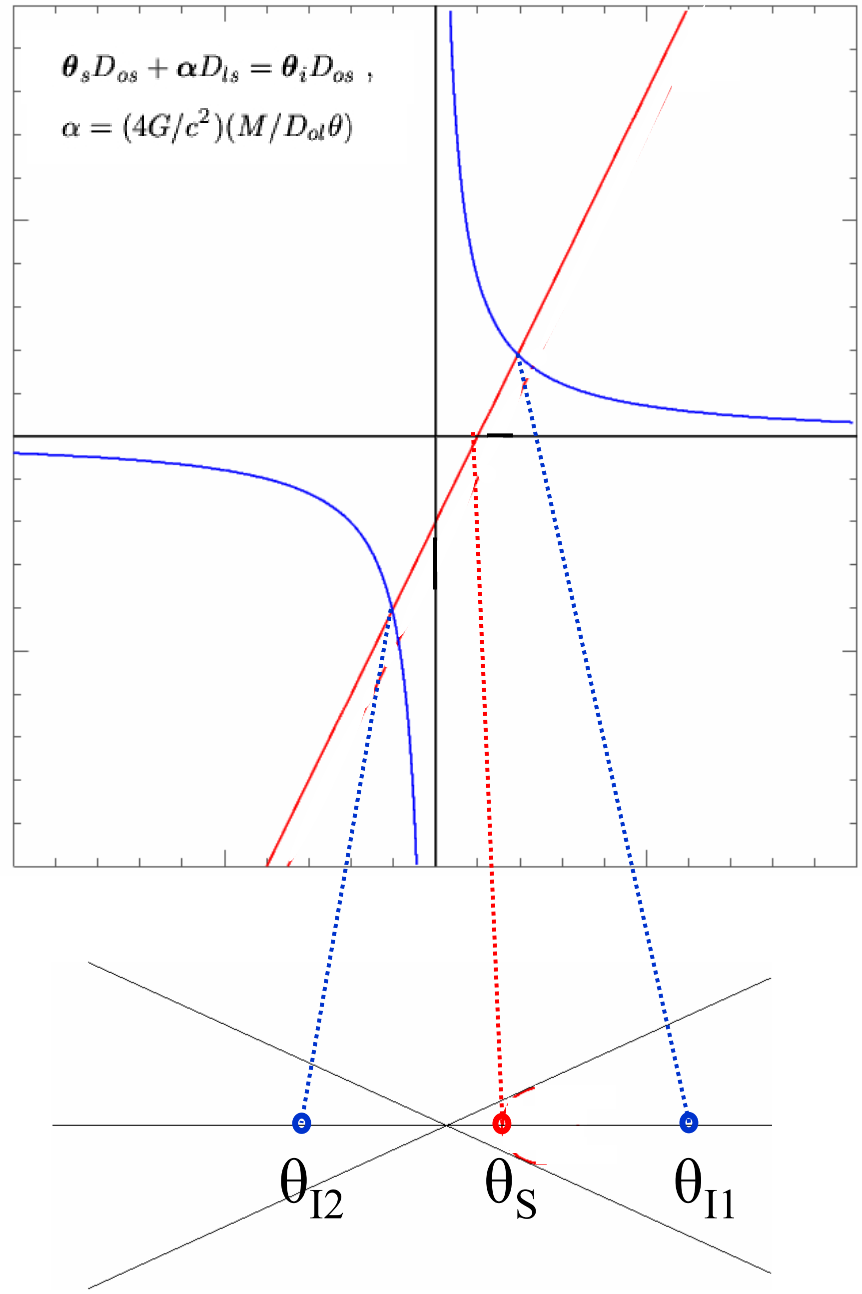


Lens Equation $$\boldsymbol{\theta}_s D_{os} + \boldsymbol{\alpha} D_{ls} = \boldsymbol{\theta}_i D_{os} \ ,$$

Deflection angle $$\alpha = (4G/c^2)(M/D_{ol}\theta)$$

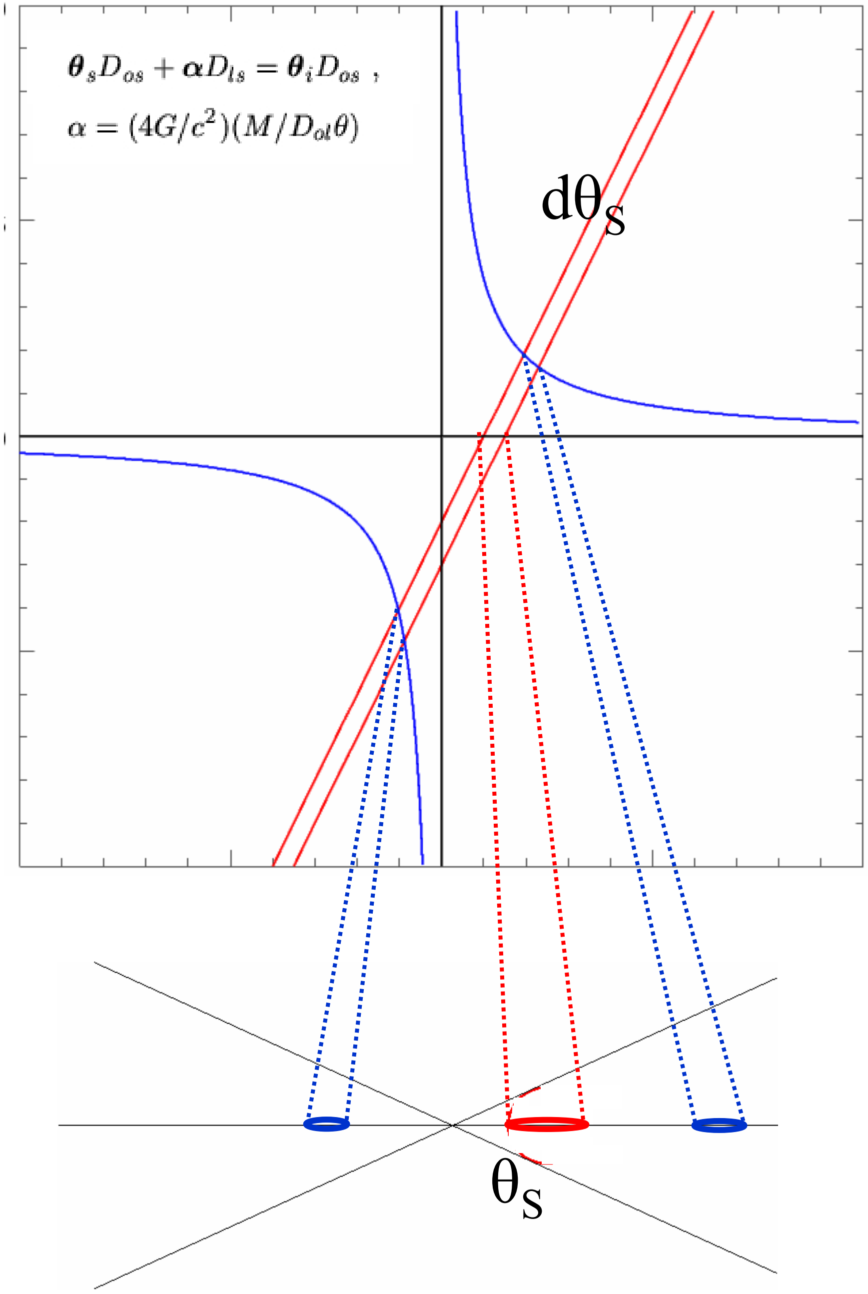
Lens: point mass

Source: point source



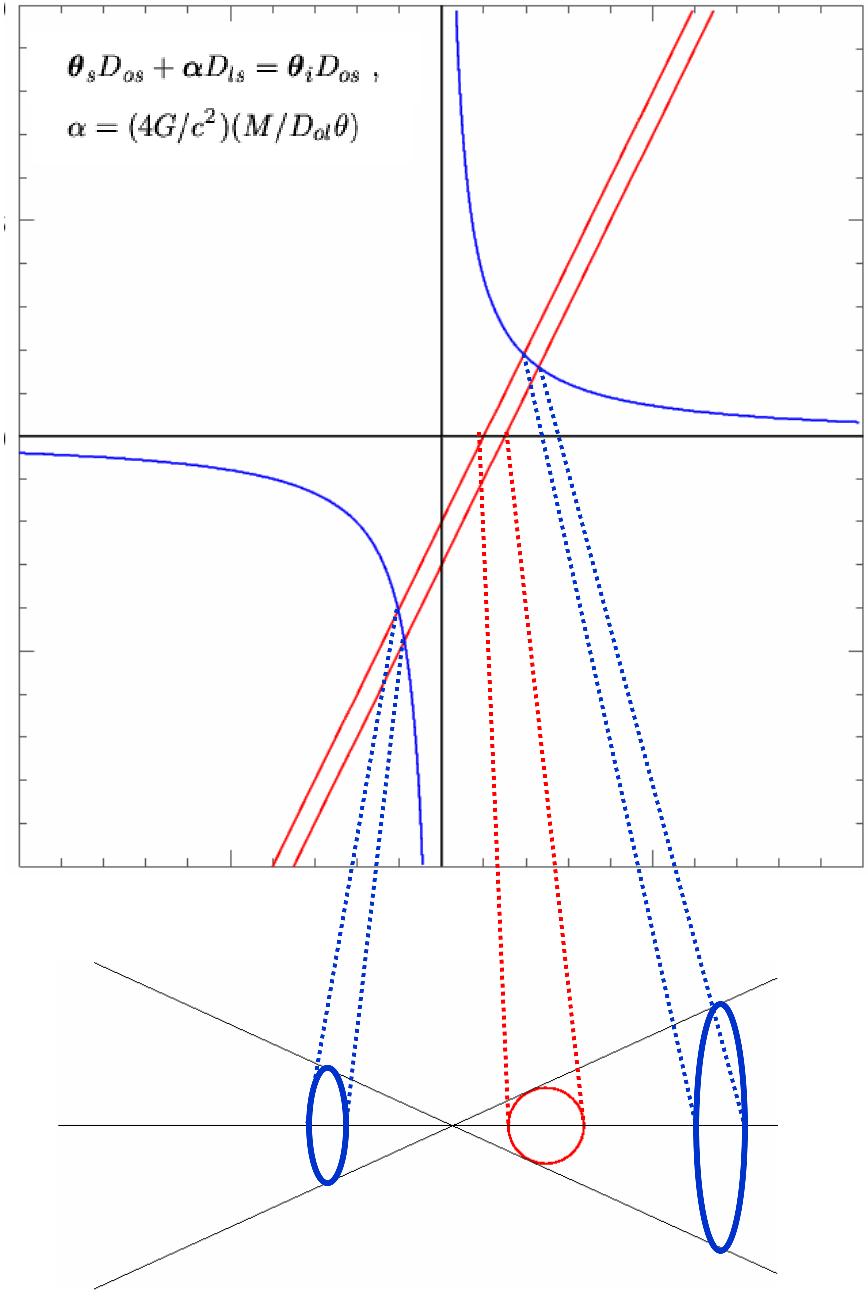
Lens : point mass

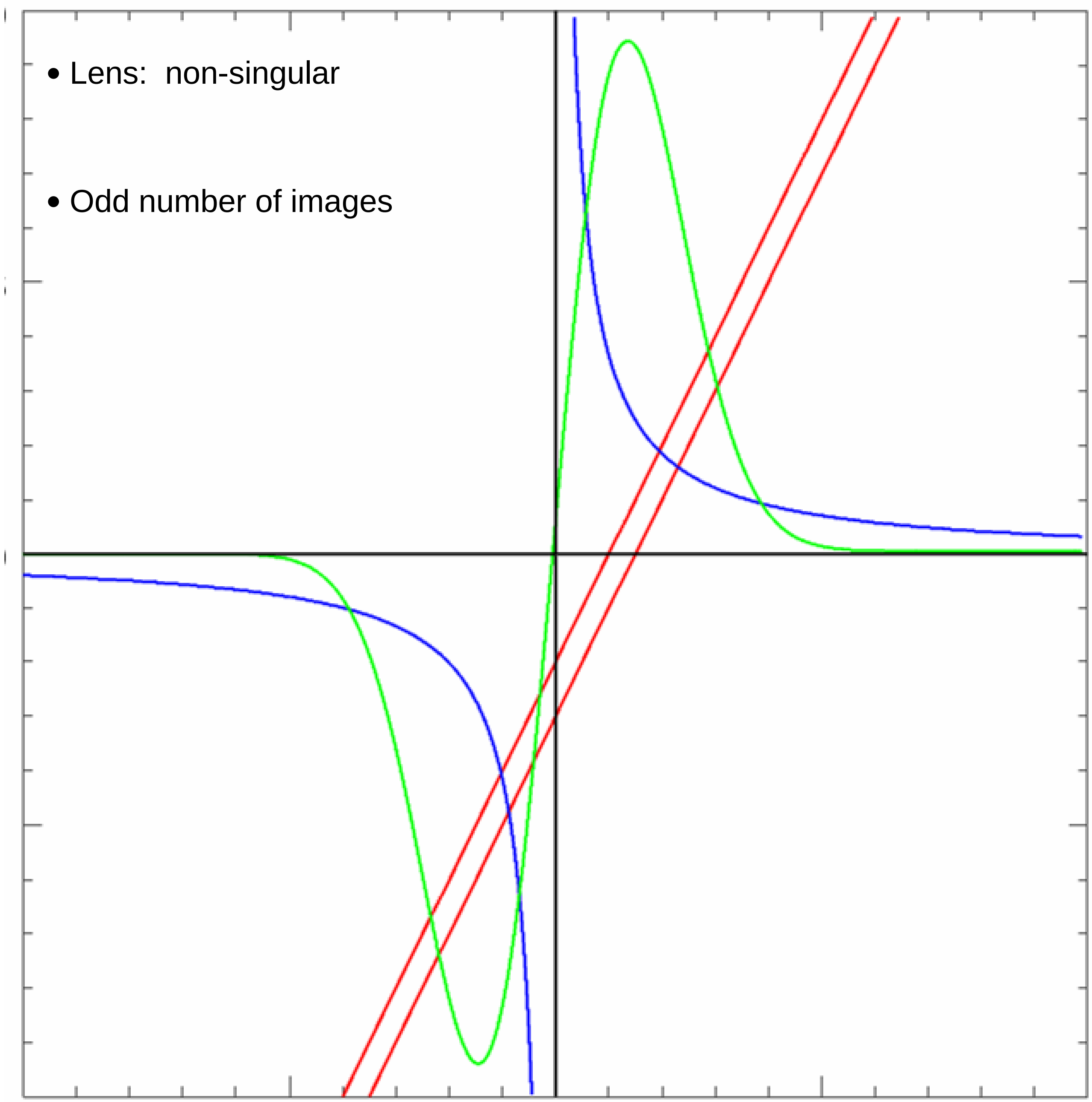
Source: line (1-D extended)



Lens: point mass

Source: ellipse (2-D extended)




• Lens: non-singular
• Odd number of images

- Convergence and critical density The gravitational convergence is a dimensionless surface masse density:

$$\kappa\left(\vec{\theta}\right) = \frac{\Sigma\left(D_{ol}\vec{\theta}\right)}{\Sigma_{cr}} \tag{30}$$

where Σ_{crit} is the *critical surface mass density*:

$$\Sigma_{cr} = \frac{c^2}{4\pi G} \frac{D_{os}}{D_{ol} D_{ls}} \tag{31}$$

that defined a "strength" of the lens. Strong lensing cases have $\Sigma > \Sigma_{cr}$

Example: lensing-cluster

- Typical value: cluster of galaxies of 10^{15} solar masses at 1Gpc and sources at 2 Gpc :

$$\Sigma_{crit} = 0.5 \text{ g/cm}^2$$

- Typical mass density of a 1Mpc diameter cluster, based on its galaxy light distribution:

$$\Sigma = 0.02 \text{ g/cm}^2$$

- Deflection angle and κ

$$\vec{\alpha}\left(\vec{\theta}\right) = \frac{1}{\pi}\int \kappa\left(\vec{\theta}'\right)\frac{\vec{\theta}-\vec{\theta}'}{|\vec{\theta}-\vec{\theta}'|^2}\,\mathrm{d}^2\theta' \qquad (32)$$

Using $\nabla \ln|\vec{\theta}| = \vec{\theta}/|\vec{\theta}|^2$, we derive

$$\boxed{\vec{\alpha} = \vec{\nabla}\psi} \qquad (33)$$

and

- The effective potential

$$\psi(\vec{\alpha}) = \frac{1}{\pi}\int \kappa(\vec{\alpha})\ \ln\left|\vec{\theta} - \vec{\theta}'\right|\ \mathrm{d}^2\theta' \tag{34}$$

Using $\nabla^2 \ln|\vec{\theta}| = 2\pi\delta_D\left(\vec{\theta}\right)$, where δ_D is the 2-dimension Dirac delta function:

$$\boxed{\nabla^2\psi = 2\kappa} \tag{35}$$

which expresses the Poisson equation in 2 dimensions.

Magnification and distortion

- Jacobian of the lens mapping. Differentiting the lens equation

$$A\left(\vec{\theta}\right) = \frac{\partial \vec{\beta}}{\partial \ \theta} = \left(\delta_{ij} - \frac{\partial^2 \psi\left(\vec{\theta}\right)}{\partial \theta_i \partial \theta_j}\right) = M^{-1} \quad (36)$$

- Convergence, Shear

$$\begin{cases} \kappa = \frac{1}{2}\left(\psi_{,11} + \psi_{,22}\right) \\ \gamma_1\left(\vec{\theta}\right) = \frac{1}{2}\left(\psi_{,11} - \psi_{,22}\right) = \gamma\left(\vec{\theta}\right)\cos\left[2\varphi\left(\vec{\theta}\right)\right] \\ \gamma_2\left(\vec{\theta}\right) = \psi_{,12} = \gamma\left(\vec{\theta}\right)\sin\left[2\varphi\left(\vec{\theta}\right)\right] \end{cases} \quad (37)$$

- Magnification, Convergence, Shear

$$A = \mathscr{M}^{-1} = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix} \quad (38)$$

$$\mathscr{M}^{-1} = (1-\kappa)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \gamma \begin{pmatrix} \cos(2\varphi) & \sin(2\varphi) \\ \sin(2\varphi) & -\cos(2\varphi) \end{pmatrix} \quad (39)$$

where $\gamma = \gamma_1 + \mathrm{i}\gamma_2 = |\gamma| e^{2\mathrm{i}\varphi}$

- Amplification amplitude

$$\mu = (\det A)^{-1} = \frac{1}{\left[(1-\kappa)^2 - |\gamma|^2\right]} \quad (40)$$

- Eigenvalues of $\mathscr{M}^{-1}$:

$$1-\kappa+\gamma \ , \ \ 1-\kappa+\gamma \quad (41)$$

- Image and source From the magnification matrix,
  - κ expresses an isotropic magnification. It transforms a circle into a larger/smaller circle.
  - γ is an anisotropic magnification. It transforms a circle into an ellipse with minor and major axes :

$$b = (1-\kappa+\gamma)^{-1} \quad , \quad a = (1-\kappa-\gamma)^{-1} \tag{42}$$

- Reduced shear Let us write the magnification matrix as:

$$A = \mathscr{M}^{-1} = (1-\kappa)\begin{pmatrix} 1-g_1 & -g_2 \\ -g_2 & 1+g_1 \end{pmatrix} \tag{43}$$

where

$$g\left(\vec{\theta}\right) = \frac{\gamma\left(\vec{\theta}\right)}{1-\kappa\left(\vec{\theta}\right)} = g_1 + \mathrm{i}g_2 = |g|e^{2\mathrm{i}\varphi} \qquad (44)$$

is the *reduced shear*. It directly provides the image ellipicity induced by lensing on a circular source:

$$\frac{b}{a} = \frac{1-|g|}{1+|g|} \qquad (45)$$

as well as the orientation of the major axis, φ.

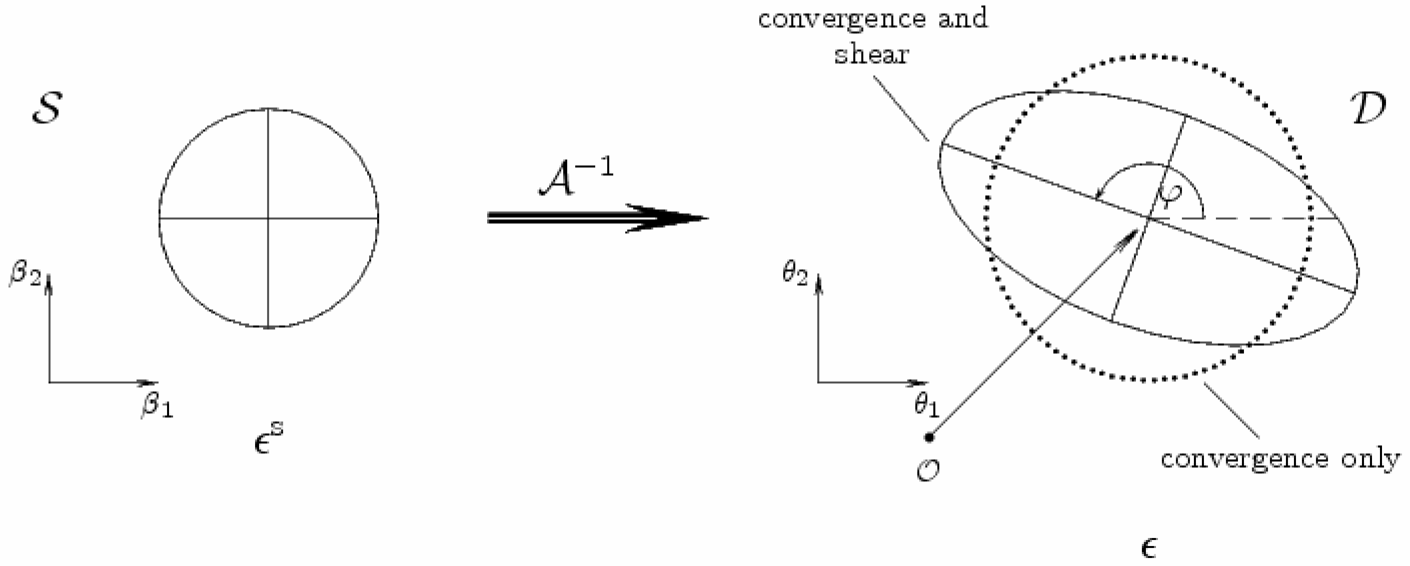


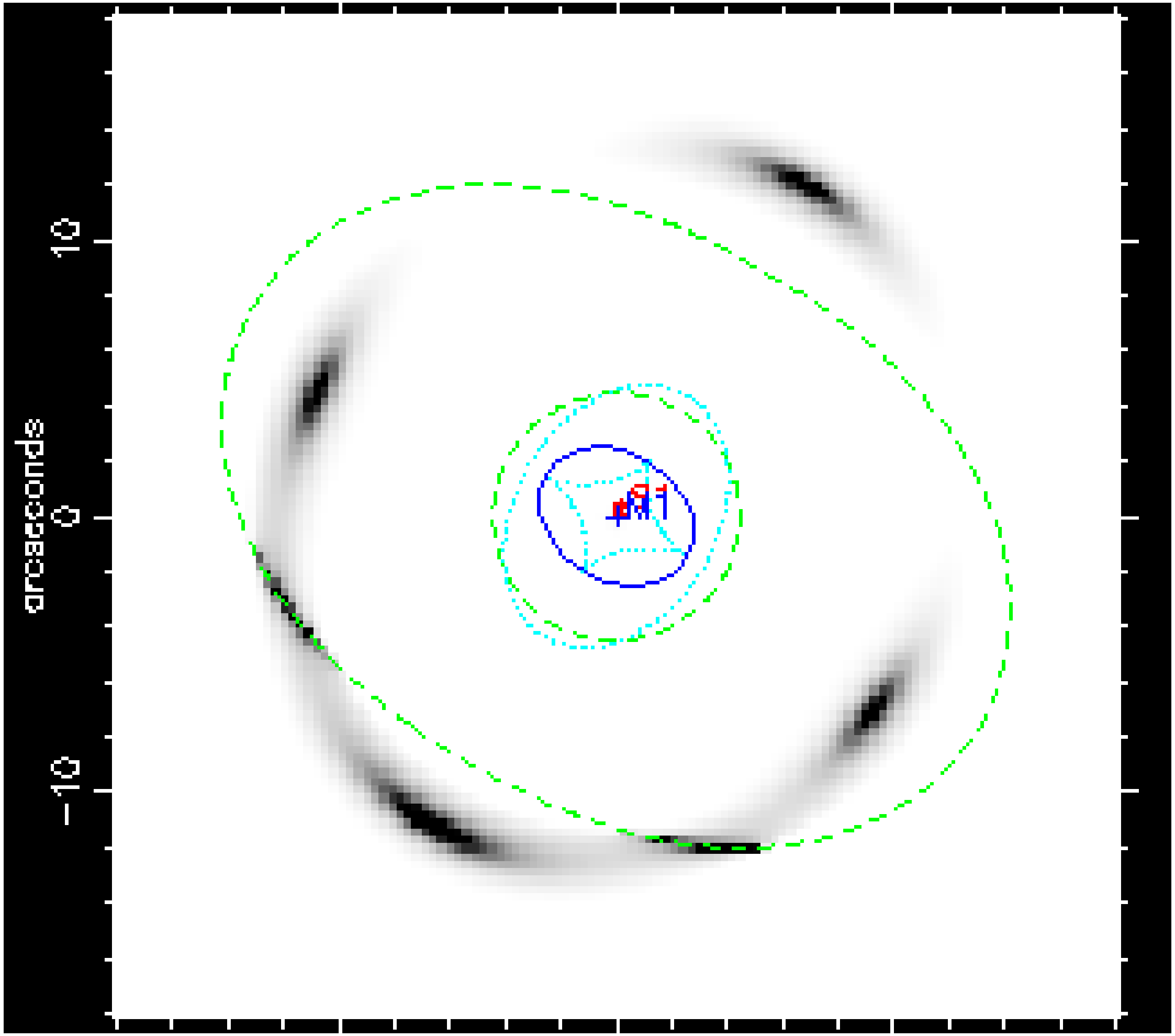
Figure 3: Transformation of a circular source into a ellipse by gravitational lensing.

- Caustics and critical lines

  - critical lines corresponds to positions in the lens plane with $\det A = 0$

- the corresponding positions on the source plane are the caustic lines
- the positions of source points with respect to a caustic lines define de number of image multiplication and the source margnification
- when a source crosses a caustic line, its amplitication is almost infinity, and image pairs are formed

Caustic and Critical Lines



Caustic and Critical Lines

The locally linearised lens equation

- Gravitational lensing conserves the object surface brightness This is a direct consequence of the Liouville Theorem and the fact that gravitational lensing is achromatic (photon energy unchanged in the weak field approximation).
  - The volume of a bundle in phase space $V = V_x \times V_p$ as function of time τ is such that $\frac{\mathrm{d}V}{\mathrm{d}\tau} = 0$
  - Astronomer observing using a filter that only select photons with energy between p^0 and $p^0 + \Delta p^0$ and arriving from the $z-$direction and located inside the solid angle $\Delta\Omega$

- Assume the collecting area of the instrument in A and it lies in the $x-y$ plane perpendicular to the incoming beam alon z
- Assume the number oif photons that cross the aera A in a time interval δt is δN
- then the δN photons, juts before the time interval begins, lie in the cylinder $\delta z = \delta t$ and
  * The 3-D spatial volume is $V_x = A \times \delta t$
  * The 3-D momentum volume is $V_p = \left(p^0\right)^2 \times \Delta p^0 \times \Delta\Omega$
- The number density of photons in space phase is

$$n_\gamma = \frac{\delta N}{V_x V_p} = \frac{\delta N}{A\delta t \ \left(p^0\right)^2 \ \Delta p^0 \ \Delta\Omega} = \frac{\delta N}{A\delta t \ h^3 \ \nu^2 \ \Delta\Omega\Delta\nu} \quad (46)$$

- In astronomy, the specific intensity of a photon, I_ν is

$$I_\nu = \frac{h\nu}{A\ \delta t\ \Delta\nu\ \Delta\Omega} \tag{47}$$

- Therefore

$$n_\gamma = h^{-4}\, I_\nu\, \nu^3 \tag{48}$$

- Since n_γ is conserved (Liouville) and gravitational lensing is an achromatic effect, the surface brightness is conserved (Etherington Theorem)

- Translation for a lensed source

$$I\left(\vec{\theta}\right) = I^s\left[\vec{\beta}\left(\vec{\theta}\right)\right] \tag{49}$$

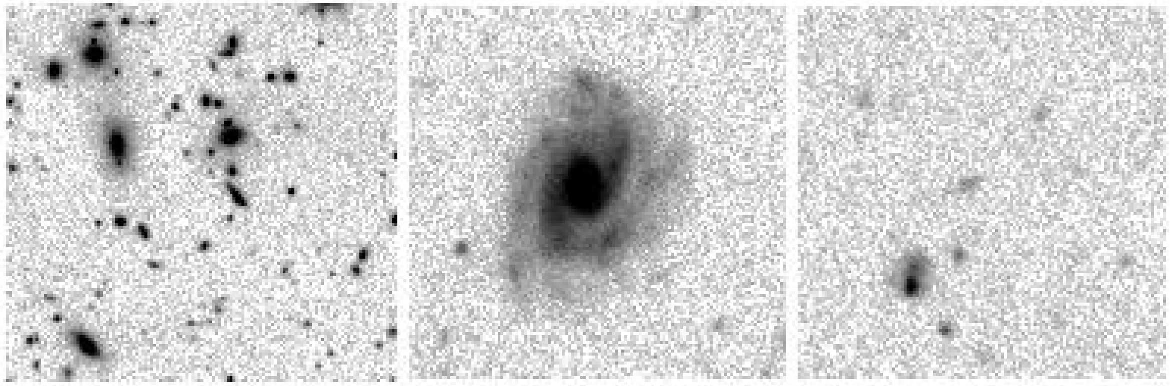
- Assumption of the locally linearised lensing Source vs. Length scale of the Lens: the typical size of the lensed source is smaller than the variation scale of the lens. Hence, the magnification is constant over the source and

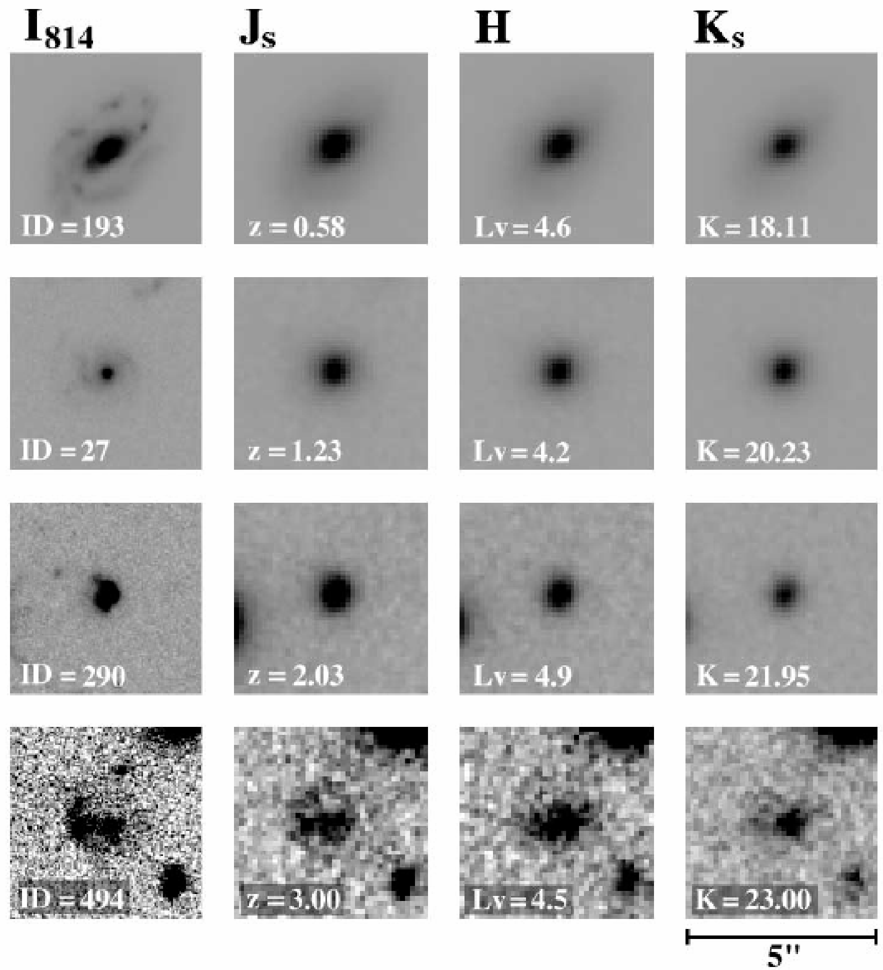
- $\Longrightarrow$ Linearisation:

$$I\left(\vec{\theta}\right) = I^s\left[\vec{\beta}\left(\vec{\theta}_0\right) + A\left(\vec{\theta}_0\right)\,\left(\vec{\theta} - \vec{\theta}_0\right)\right] \tag{50}$$

- $\Longrightarrow$ an elliptical image of a source is transformed into an ellipse. A circular source is transformed into an ellipse with major axis $(1-\kappa-\gamma)^{-1}$ and minor axis $(1-\kappa+\gamma)^{-1}$.

The shape of galaxies



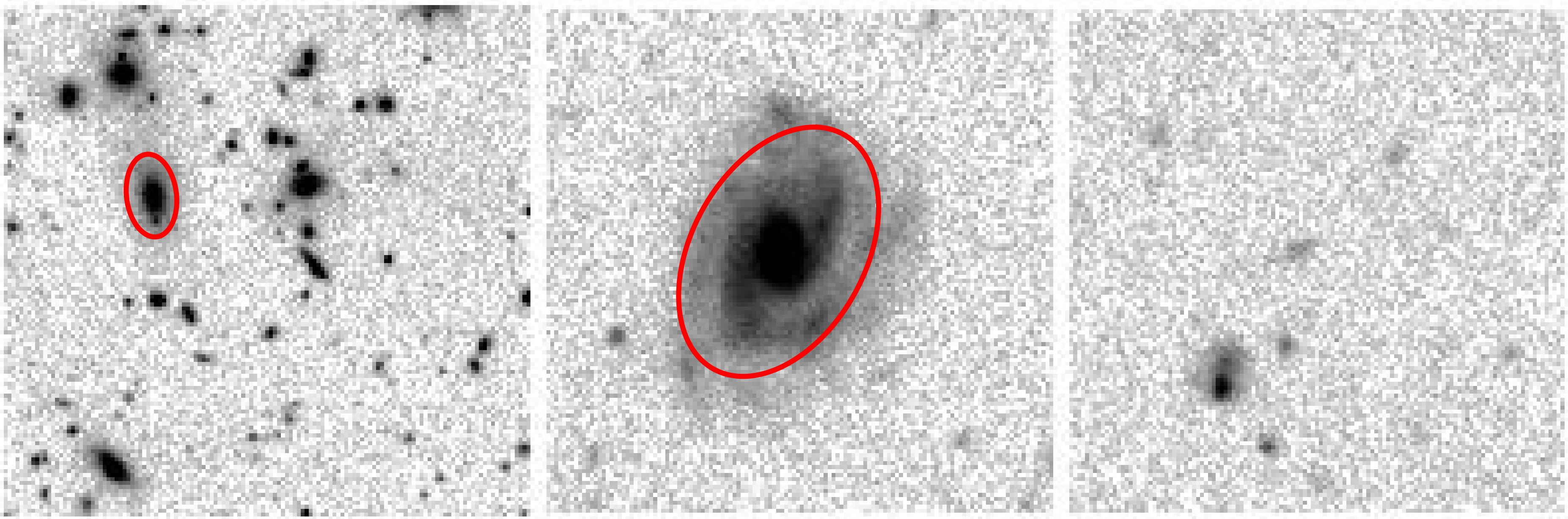


Change

- with PSF
- with image sampling
- with optical aberrations
- with filter
- with morphological tpe
- with redshift (shift+evolution)
- with signal-to-noise ratio

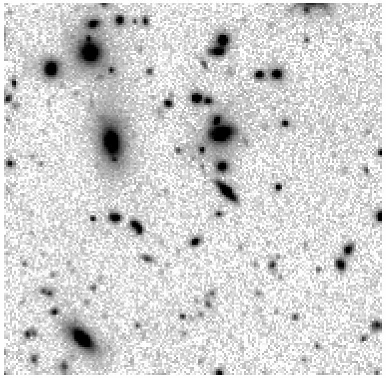
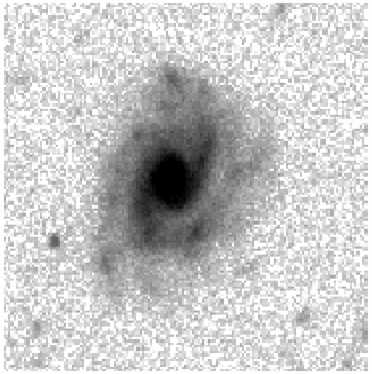
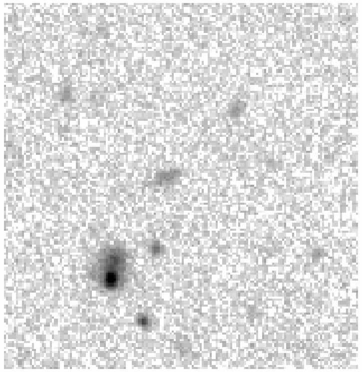
The shape of galaxies

To first order: all galaxies are ellipses, more or less noisy



The shape is weighted by the galaxy surface brightness

The shape of galaxies

- Characterisation of the shape of galaxies. Galaxies are ellipses.
- The second moments provide the shape of the galaxies
- Critical assumption for nearby and very distant galaxies

From Source to Image Shape

Assume $I\left(\vec{\theta}\right)$ is the surface brigthness in an image. The centroid of the image can be defined as

- Centroid of a galaxy:

$$\vec{\theta}_0 = \frac{\int \mathrm{d}^2\theta \;\; \vec{\theta} \;\; I\left(\vec{\theta}\right) \;\; q_I\left[I\left(\vec{\theta}\right)\right]}{\int \mathrm{d}^2\theta \;\; I\left(\vec{\theta}\right) \;\; q_I\left[I\left(\vec{\theta}\right)\right]} \tag{51}$$

  where q_I is a weight function (*e.g.* the Heaviside function).

- Second moments On can then define the second brightness momment tensor

$$Q_{ij} = \frac{\int \mathrm{d}^2\theta \;\; (\theta_i - \theta_{0,i}) \; (\theta_j - \theta_{0,j}) \;\; I\left(\vec{\theta}\right) \; q_I\left[I\left(\vec{\theta}\right)\right]}{\int \mathrm{d}^2\theta \;\; I\left(\vec{\theta}\right) \; q_I\left[I\left(\vec{\theta}\right)\right]} \tag{52}$$

So, a circular object has

$$Q_{11} = Q_{22} \text{ and } Q_{12} = 0 \tag{53}$$

Likewise, for the source, taking into account the surface brightness conservation:

$$Q^S_{ij} = \frac{\int \mathrm{d}^2\beta \;\; (\beta_i - \beta_{0,i}) \; (\beta_j - \beta_{0,j}) \;\; I^S\left(\vec{\theta}\right) \; q_I\left[I^S\left(\vec{\beta}\right)\right]}{\int \mathrm{d}^2\,\beta \; I^S\left(\vec{\theta}\right) \; q_I\left[I^S\left(\vec{\beta}\right)\right]} \tag{54}$$

Then using lensing equation and the locally linearised lens aquation, we have

$$\mathrm{d}^2\beta = \mathrm{det}A \; \mathrm{d}^2\theta \;\; \text{and} \;\; \vec{\beta} - \vec{\beta}_0 = A\left(\vec{\theta} - \vec{\theta}_0\right) \tag{55}$$

we then find the

- Relation between source and image shapes

$$Q^S = A\left(\vec{\theta}_0\right) \; Q \, A^T\left(\vec{\theta}_0\right) \tag{56}$$

Ellipticities

- Current definition(s) of ellipticities:

$$\varepsilon = \frac{1-\frac{b}{a}}{1+\frac{b}{a}}\, e^{2i\phi} \;\;;\;\; \chi = \frac{1-\left(\frac{b}{a}\right)^2}{1+\left(\frac{b}{a}\right)^2}\, e^{2i\phi} \tag{57}$$

with

$$a = (1-\kappa-\gamma)^{-1} \;\text{ and }\; b = (1-\kappa+\gamma)^{-1} \tag{58}$$

ε is directly the reduced shear

$$|g| = |\varepsilon| = \frac{|\gamma|}{1-\kappa} \tag{59}$$

- Relations with second moments:

$$\varepsilon = \frac{Q_{11} - Q_{22} + 2iQ_{12}}{Q_{11} + Q_{22} + 2\left(Q_{11}Q_{22} - Q_{12}^2\right)^{1/2}} \tag{60}$$

$$\chi = \frac{Q_{11} - Q_{22} + 2iQ_{12}}{Q_{11} + Q_{22}} \tag{61}$$

so,

$$\varepsilon = \frac{\chi}{1 + (1 - |\chi|^2)^{1/2}} \text{ and } \chi = \frac{2\varepsilon}{1 + |\varepsilon|^2} \tag{62}$$

From Ellipticity to Shear

$$Q^S = A\,Q\,A \tag{63}$$

$$Q^S = [1-\kappa]^2 \begin{pmatrix} 1-g_1 & -g_2 \\ -g_2 & 1+g_1 \end{pmatrix} \times \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{12} & Q_{22} \end{pmatrix} \times \begin{pmatrix} 1-g_1 & -g_2 \\ -g_2 & 1+g_1 \end{pmatrix} \tag{64}$$

Hence

$$\frac{Q^S_{11}}{(1-\kappa)^2} = (1-g_1)^2 Q_{11} - 2g_2\,(1-g_1)\,Q_{12} + g_2^2 Q_{22} \tag{65}$$

$$\frac{Q^S_{22}}{(1-\kappa)^2} = g_2^2 Q_{11} - 2g_2\,(1+g_1)\,Q_{12} + (1+g_1)^2\,Q_{22} \tag{66}$$

$$\frac{Q^S_{12}}{(1-\kappa)^2} = -g_2^2(1-g_1)\,Q_{11} + \left(1-g_1^2+g_2^2\right)Q_{12} - g_2(1+g_1)^2\,Q_{22} \tag{67}$$

Therefore

$$\frac{\chi^S}{(1-\kappa)^2}: \tag{68}$$

Source terms

$$\frac{Q^S_{11}-Q^S_{22}+2\mathrm{i}Q^S_{12}}{(1-\kappa)^2} = \tag{69}$$

Image terms

$$-2g_1\left(Q_{11}+Q_{22}\right)+\left(1+g_1^2+g_2^2\right)\left(Q_{11}+Q_{22}\right) \tag{70}$$

$$-4g_1g_2Q_{12}-2\mathrm{i}g_2\left(Q_{11}+Q_{22}\right)Q_{12} \tag{71}$$

$$+2\mathrm{i}g_1g_2\left(Q_{11}-Q_{22}\right)+2\mathrm{i}\left(1+g_1^2-g_2^2\right)Q_{12} \tag{72}$$

From ellipticity to Shear

$$\frac{Q_{11}^S + Q_{22}^S}{(1-\kappa)^2} = (Q_{11} + Q_{22}) \times \left(1 + |g|^2 - 2g_1\chi_1 - 2g_2\chi_2\right) \quad (79)$$

We can now express the elliptcy of the source as function of the ellipticity of the image and the gravitational shear:

$$\chi^S = \frac{-2g + \chi + g^2\chi^*}{1 + |g|^2 - 2\Re(g\chi^*)} \quad (80)$$

and since

$$\varepsilon = \frac{\chi}{1 + (1 - |\chi|^2)^{1/2}} \tag{81}$$

we have

$$\varepsilon^s = \begin{cases} \frac{\varepsilon^i - g}{1 - g^* \varepsilon^i} & \text{for } |g| < 1 \\ \frac{1 - g\left(\varepsilon^i\right)^*}{(\varepsilon^i)^* - g^*} & \text{for } |g| > 1 \end{cases} \tag{82}$$

The Weak Lensing Regime

- From ellipticity to shear

$$\varepsilon^s = \begin{cases} \frac{\varepsilon^i - g}{1 - g^* \varepsilon^i} & \text{for } |g| < 1 \\ \frac{1 - g(\varepsilon)^*}{(\varepsilon)^* - g^*} & \text{for } |g| > 1 \end{cases} \tag{83}$$

- Weak Lensing regime : $\kappa << 1, \ |\gamma| << 1 \ |g|^2 \simeq 0$ Therefore

$$g = \frac{\gamma}{1-\kappa} \simeq \gamma \tag{84}$$

In that case

$$\chi \simeq \chi^S + 2\left(g - \chi^S \Re\left(g\chi^*\right)\right) \tag{85}$$

In we decompose one component , for example

$$\chi_1 = \chi_1^S + 2g_1\left(1 - \chi_i^S\chi_1\right) - 2g_2\chi_1^S\chi_2 \tag{86}$$

So, we can write the relation as follow:

$$\chi_i = \chi_i^S + 2\left(\delta_{ij} - \chi_i^S\chi_j\right)\gamma_j \tag{87}$$

- Sources orientation is isotropically distributed ←

$$\langle\chi^S\rangle \simeq 0 \tag{88}$$

which implies!

$$\boxed{\langle\chi\rangle \simeq 2\gamma = 2g} \tag{89}$$

Therefore, the image ellipticity of galaxies provide an unbiased estimate of the gravitational shear.

Tangential and Cross Components of the Shear:

- Cartesian coordinates:

$$\gamma = \gamma_1 + i\gamma_2 = |\gamma| e^{2i\varphi} \tag{90}$$

The tangential and cross components of the shear with respect to a direction ϕ is

$$\gamma_t = -\Re\left[\gamma e^{-2i\phi}\right] \;\; ; \;\; \gamma_x = -Im\left[\gamma e^{-2i\phi}\right] \tag{91}$$

Likewise, one can define the tangential and cross image ellipticity components, ε_t and ε_x.

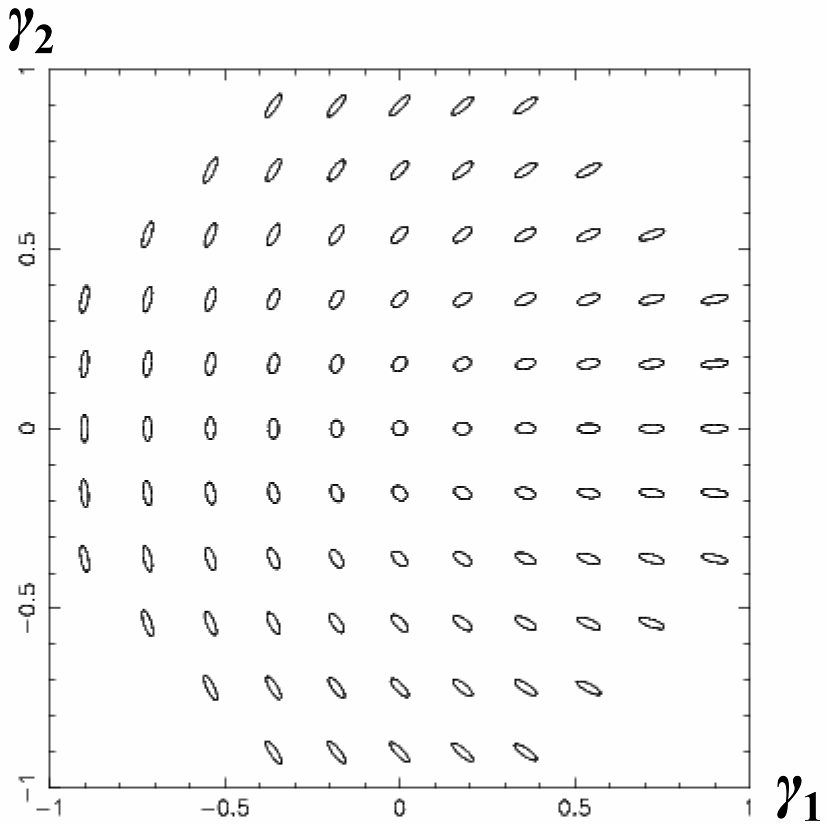
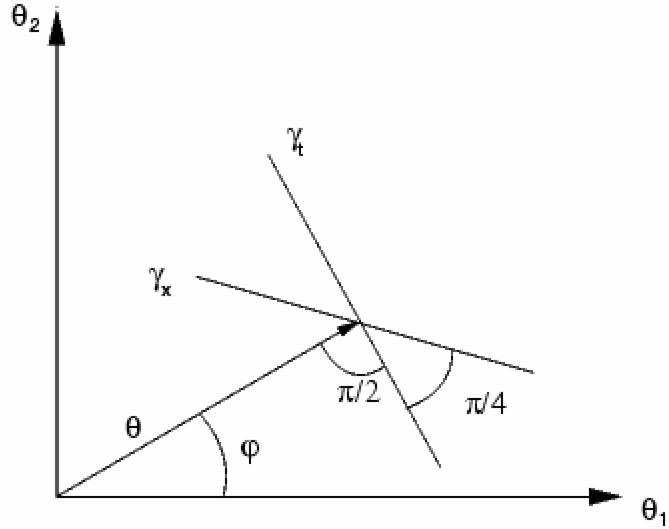
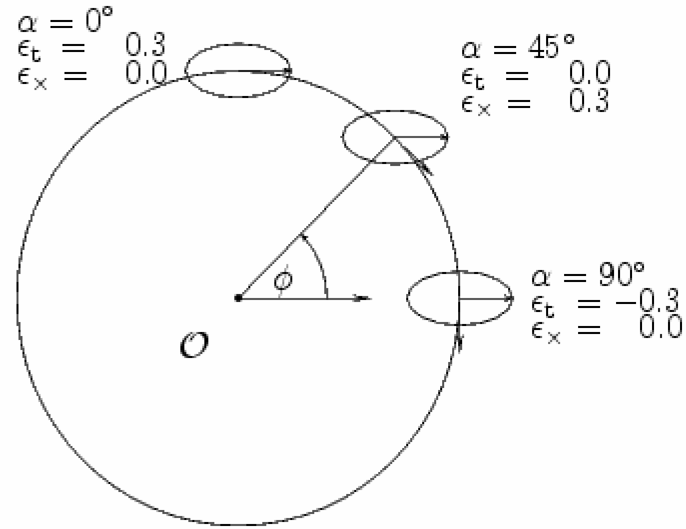


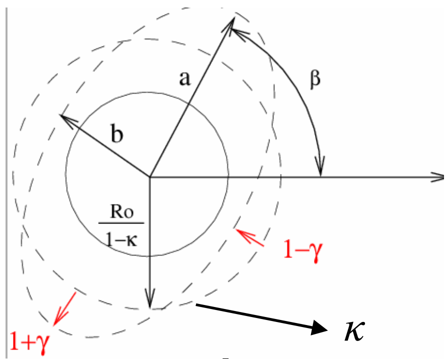
Figure 4: Orientation of the ellipses given by the Cartesian coordinates γ_1 (x-axis) and γ_2 (y-axis), with the polar angle ranging from 0 ti 2π.





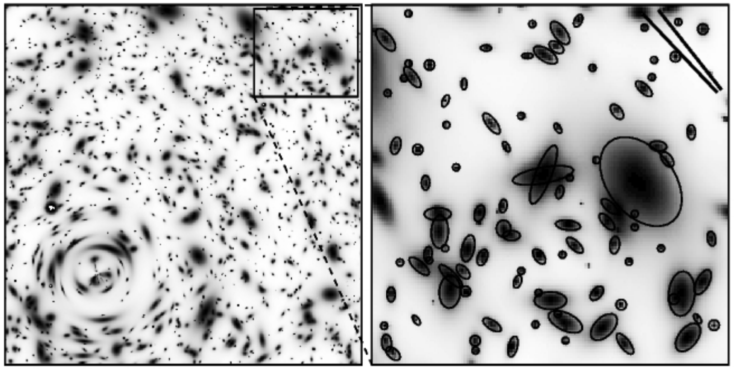
With these definition, for a spherical mass overdensity, the shear will be tangential with $\gamma_t > 0$ and $\gamma_x = 0$.

Summary



$$\delta = \frac{2\gamma\,(1-\kappa)}{(1-\kappa)^2 + |\gamma|^2} = \frac{a^2-b^2}{a^2+b^2}$$

$\delta \sim 2\gamma$ (weak lensing regime)



$$M_{ij} = \frac{\int I(\boldsymbol{\theta})\,\theta_i\,\theta_j\,\mathrm{d}^2\theta}{\int I(\boldsymbol{\theta})\,\mathrm{d}^2\theta}$$

$$\frac{a^2-b^2}{a^2+b^2}$$

PSF anisotropy correction

Derived from star shape analysis.
Image quality of primary importance for weak lensing

$= \varepsilon_s + \varepsilon_i$ + noise + systematics....

Reliability of results: depends on PSF analysis

Assume sources orientation is isotropic:

Weak lensing regime : $\delta \sim 2\gamma = \langle \varepsilon_{Shear} \rangle_\theta$ + noise

Mass Maps from Ellipticities

We have shown that

$$\alpha(\theta) = \frac{1}{\pi} \int \kappa(\theta') \frac{\theta - \theta'}{|\theta - \theta'|^2} \, d\theta^2 \tag{92}$$

or,

$$\psi(\theta) = \frac{1}{\pi} \int \kappa(\theta') \, \ln|\theta - \theta'| \, d\theta^2 \tag{93}$$

where the amplification matrix writes

$$A(\theta) = \frac{\partial \beta}{\partial \theta} = \left(\delta_{ij} - \frac{\partial^2 \psi(\theta)}{\partial \theta_i \theta_j} \right) \tag{94}$$

or, as function of the convergence and shear components

$$\begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix} \tag{95}$$

Let set

$$\begin{cases} \gamma & = \gamma_1 + i\gamma_2 = |\gamma| e^{2i\psi} \\ \gamma_1 & = \dfrac{1}{2}\left(\psi_{,11} + \psi_{,22}\right) \\ \gamma_2 & = \psi_{,12} \end{cases} \tag{96}$$

then

$$\gamma = \left(\frac{\partial_1^2 - \partial_2^2}{2} + i\partial_1\partial_2\right)\psi\left(\theta\right) \tag{97}$$

So, we can express γ as function of κ as follows :

$$\gamma(\theta) = \frac{1}{\pi} \int \kappa(\theta') F(\theta - \theta') \, \mathrm{d}\theta^2 \tag{98}$$

with

$$F(\theta) = \frac{\theta_1^2 - \theta_2^2 + 2i\theta_1\theta_2}{|\theta|^4} = F_1(\theta) + iF_2(\theta) \tag{99}$$

Let write this relation in Fourier space:

$$\kappa(\theta) = \frac{1}{(2\pi)^2} \int \hat{\kappa}(\mathbf{k}) e^{i\mathbf{k}.\theta} \, \mathrm{d}^2k \tag{100}$$

Equation (98) is formally identical to a convolution, so the relation in Fourier space is simply:

$$\hat{\gamma}(\mathbf{k}) = \frac{1}{\pi} \hat{\kappa}(\mathbf{k}) \hat{F}(\hat{k}) \tag{101}$$

with

$$\hat{F}\left(\mathbf{k}\right)=\pi\frac{k_1^2-k_2^2+2ik_1k_2}{\left|\mathbf{k}\right|^2} \tag{102}$$

Therefore

$$\hat{F}\left(\hat{k}\right)\ \hat{F}^*\left(\hat{k}\right)=\pi^2 \tag{103}$$

so

$$\hat{F}^{-1}\left(\hat{k}\right)=\frac{1}{\pi^2}\hat{F}^*\left(\hat{k}\right) \tag{104}$$

So $\hat{\kappa}$ writes:

$$\hat{\kappa}\left(\hat{k}\right)=\frac{1}{\pi}\hat{\gamma}\left(\mathbf{k}\right)\ \hat{F}^*\left(\hat{k}\right) \tag{105}$$

or

$$\hat{\kappa}=k^{-2}\ \left[\left(k_1^2-k_2^2\right)\hat{\gamma}_1+2k_1k_2\hat{\gamma}_2\right] \tag{106}$$

By inverting this relation, one can then reconstruct the κ field:

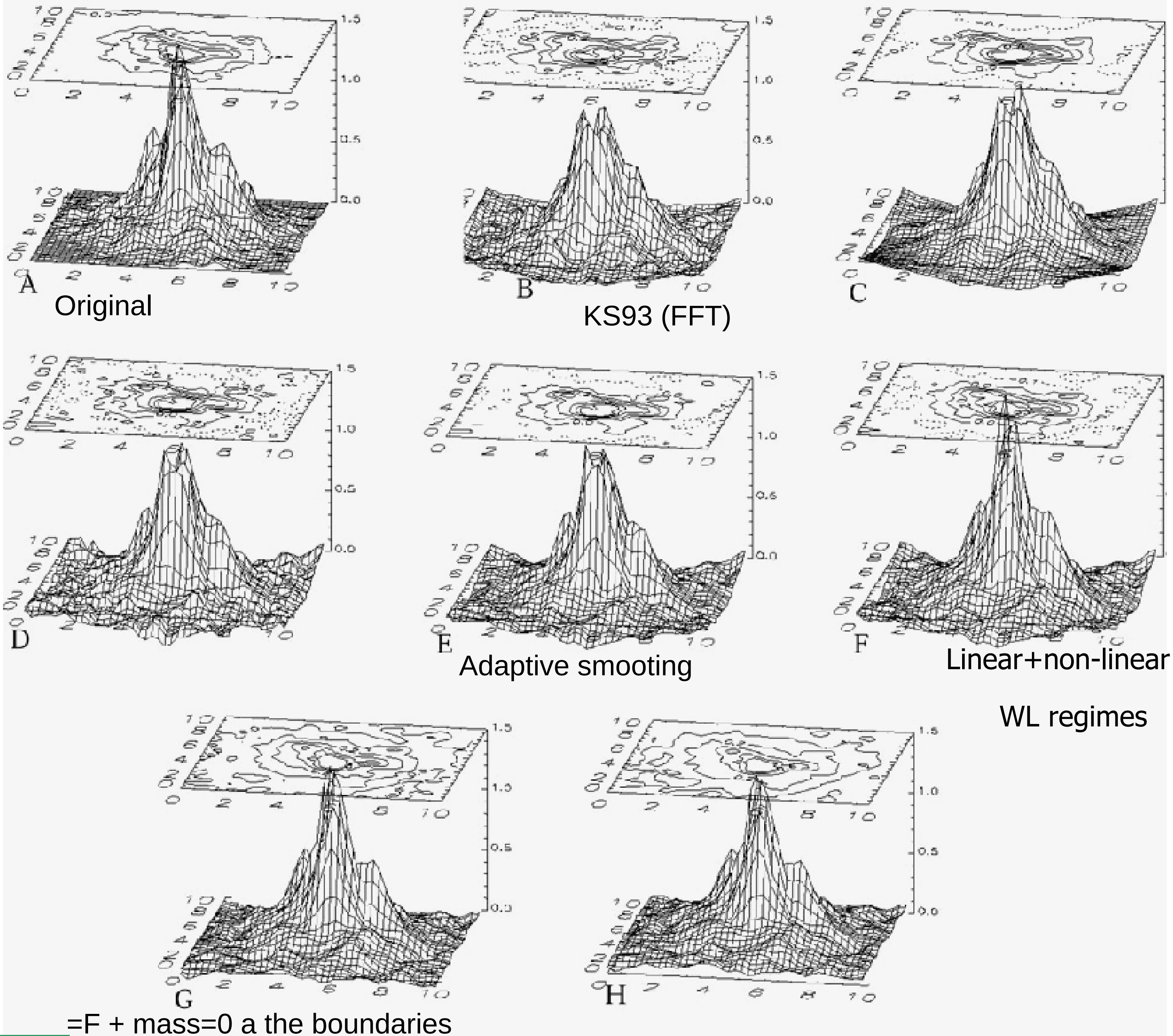
$$\boxed{\kappa(\theta) = \frac{1}{\pi}\int \hat{F}^{*}(\theta - \theta')\,\gamma(\theta')\;\mathrm{d}\theta^{2} + \kappa_{0}} \qquad (107)$$

where the real part is the matter field. *Application:*

$$\Sigma(\theta) - \Sigma_{0} = \Sigma_{critic}\,\frac{1}{\pi}a^{2}\sum_{i,j}\Re\left(\hat{F}^{*}(\theta - \theta_{i,j})\,\bar{\varepsilon}(\theta_{i,j})\right) \qquad (108)$$

where a is the distance between grid points

Mass reconstruction algorithms



Cluster of galaxies Abell 1698, z~ 0.2 ; HST/ACS

Cluster of galaxies Abell 1698, z~ 0.2 ; HST/ACS

Gravitational lensing quantities

Cluster of galaxies Abell 1698, z~ 0.2 ; HST/ACS

$$A^{-1} = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix}$$

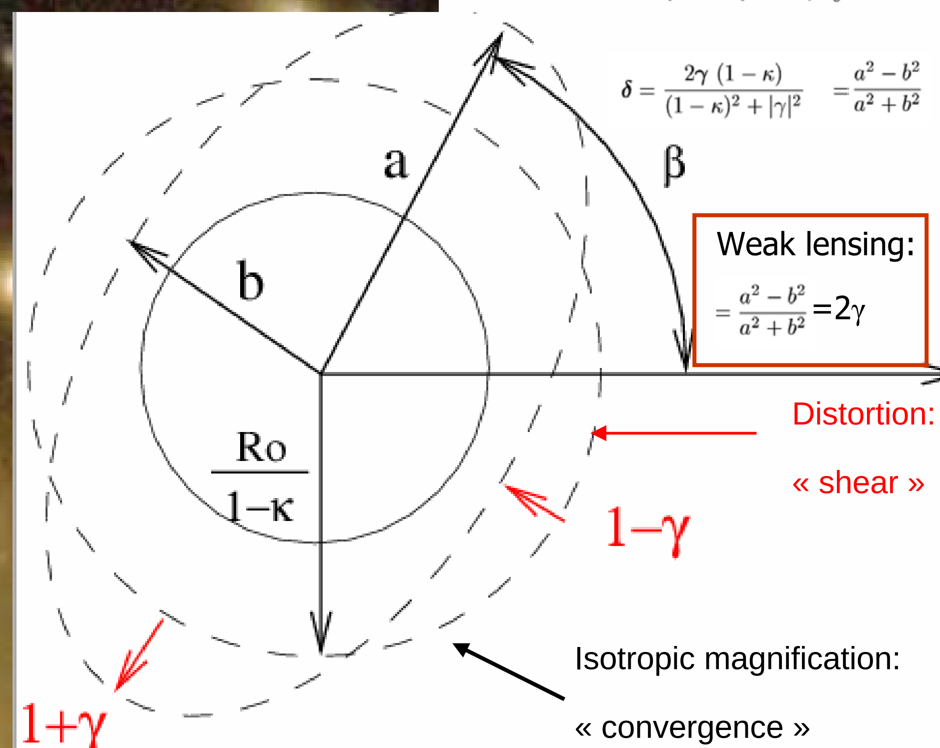
Magnification matrix

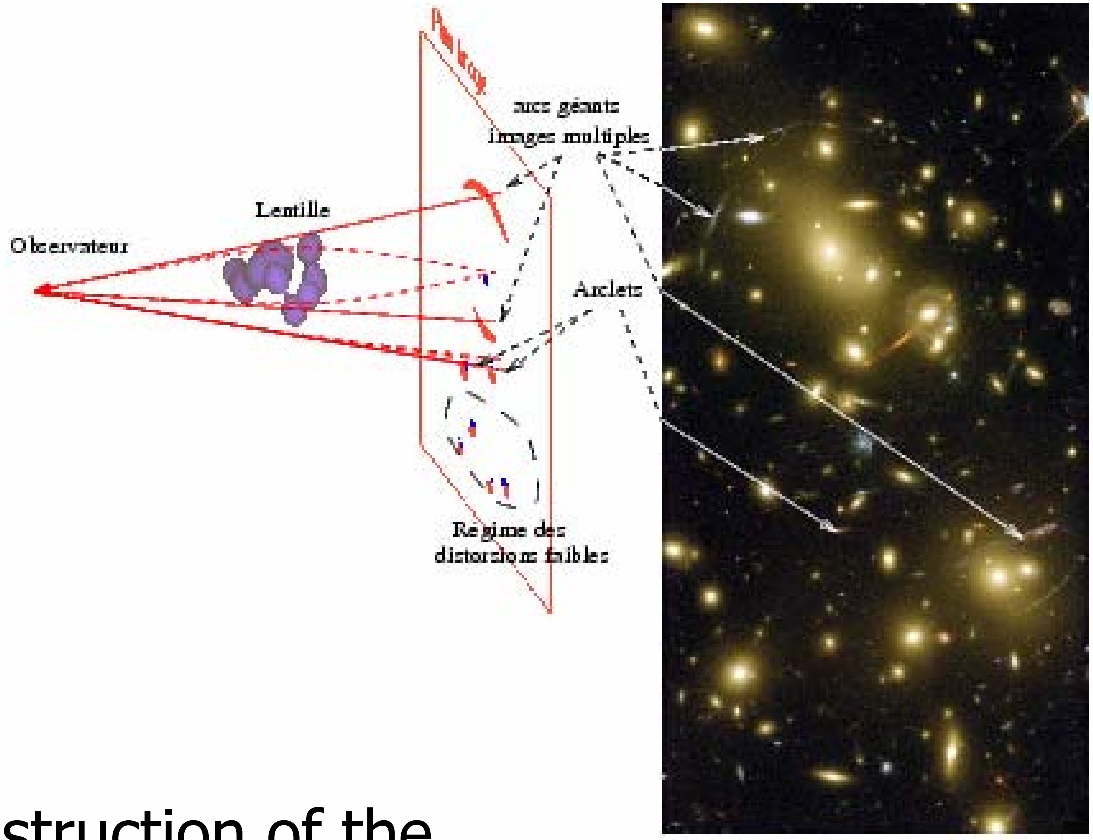
$$\Longrightarrow A^{-1} = (1-\kappa)I - \gamma J_\theta$$

$$\delta = \frac{2\gamma\,(1-\kappa)}{(1-\kappa)^2+|\gamma|^2} \quad = \frac{a^2-b^2}{a^2+b^2}$$

Weak lensing:

$$= \frac{a^2-b^2}{a^2+b^2} = 2\gamma$$

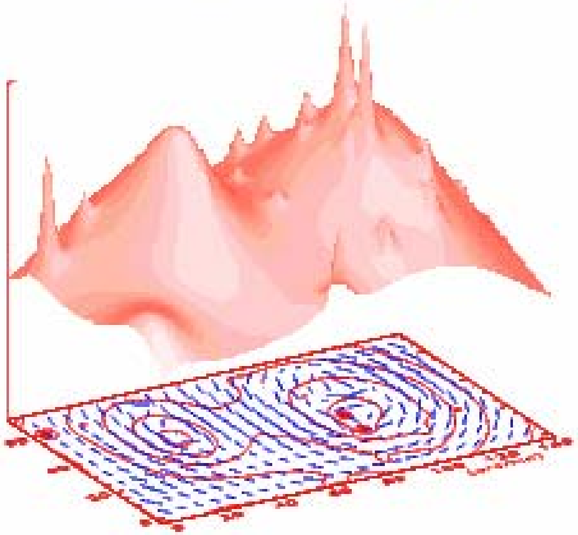




Mass reconstruction of the

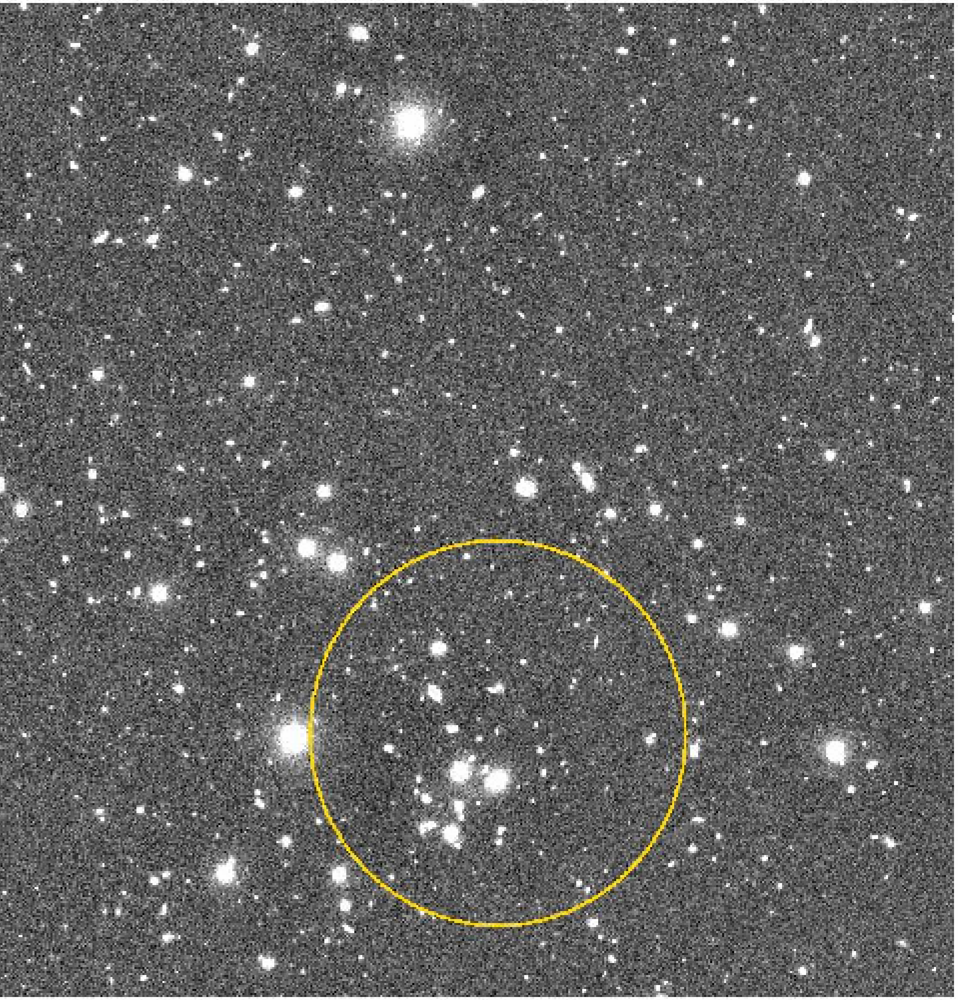
Lensing cluster Abell 2218

(z=0.18)

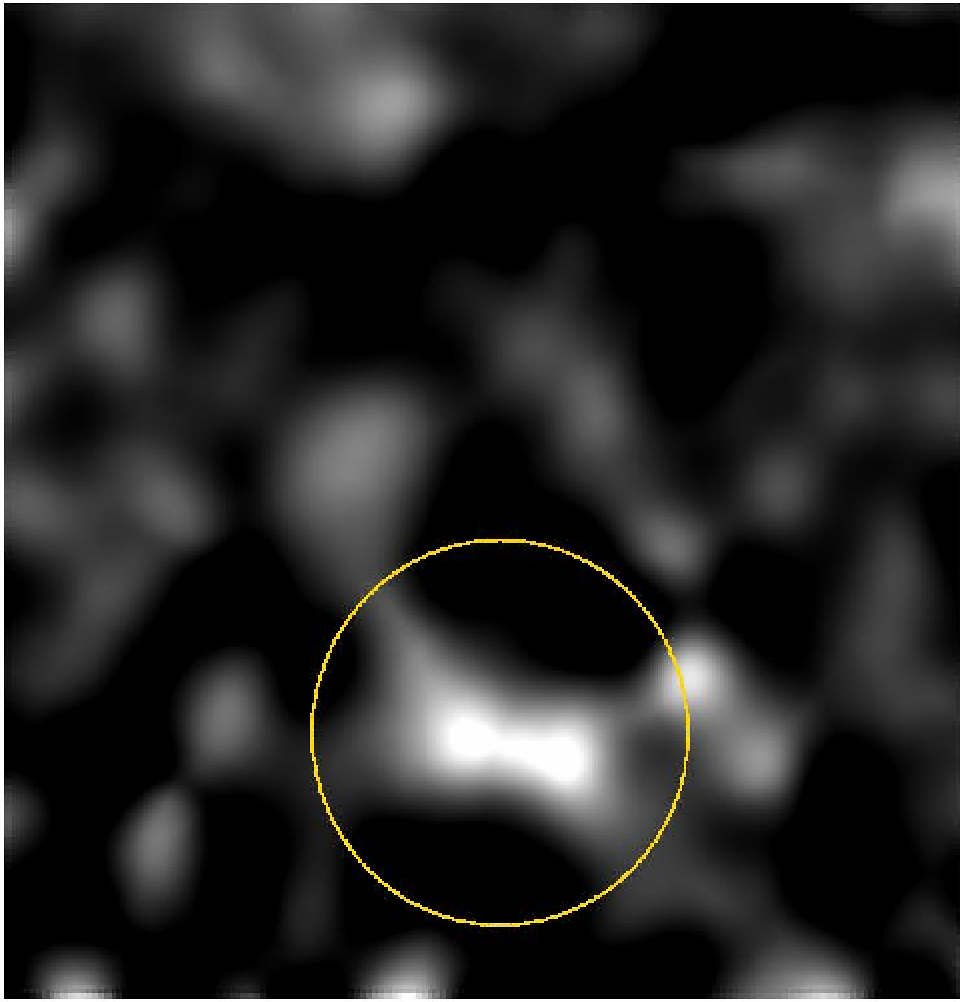


HST image

Blind mass reconstruction of an VLT/FORS1 field



VLT I-band Image: 36 mn exposure



Dark Matter reconstruction

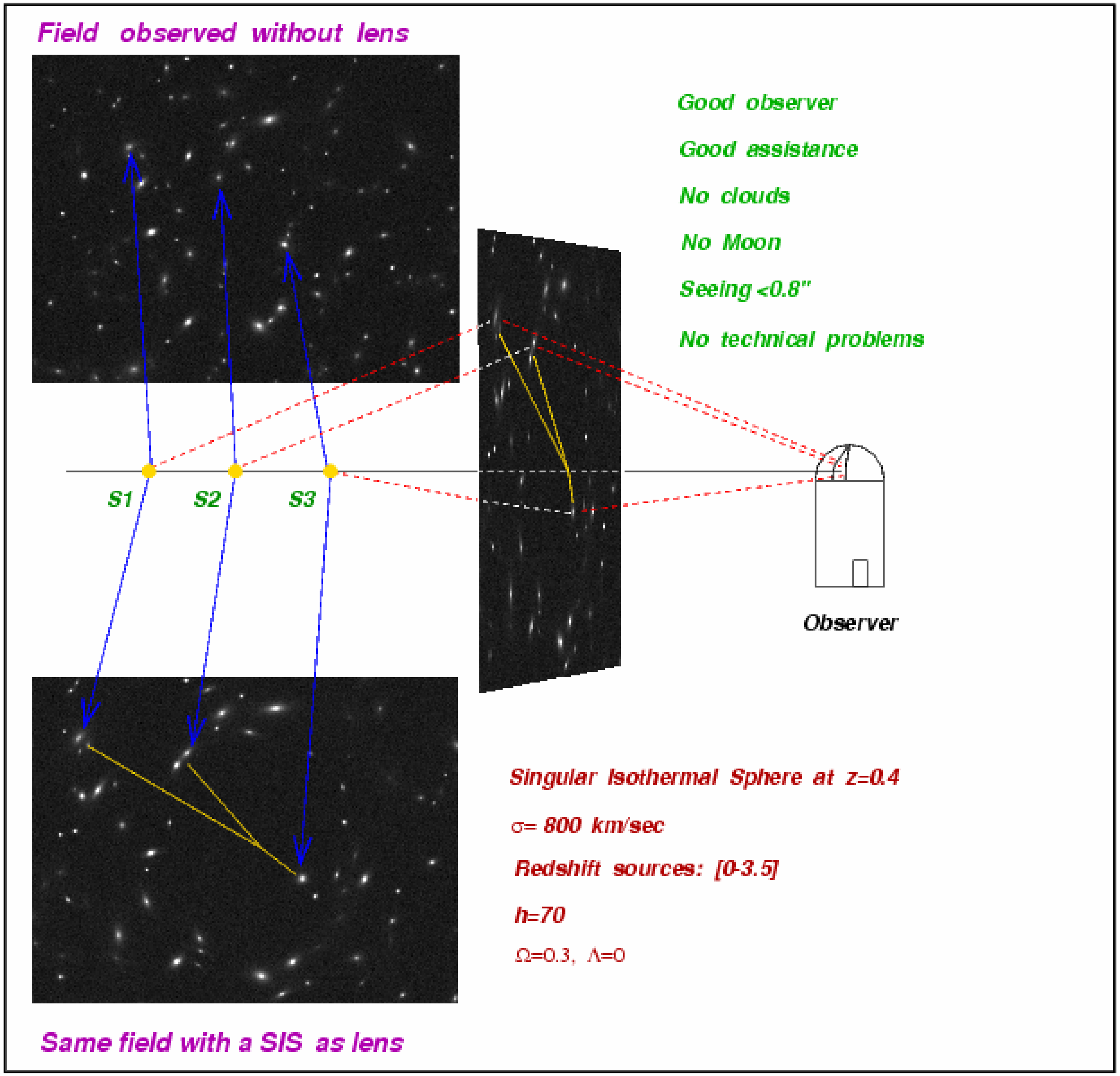
Getting the Absolute Mass

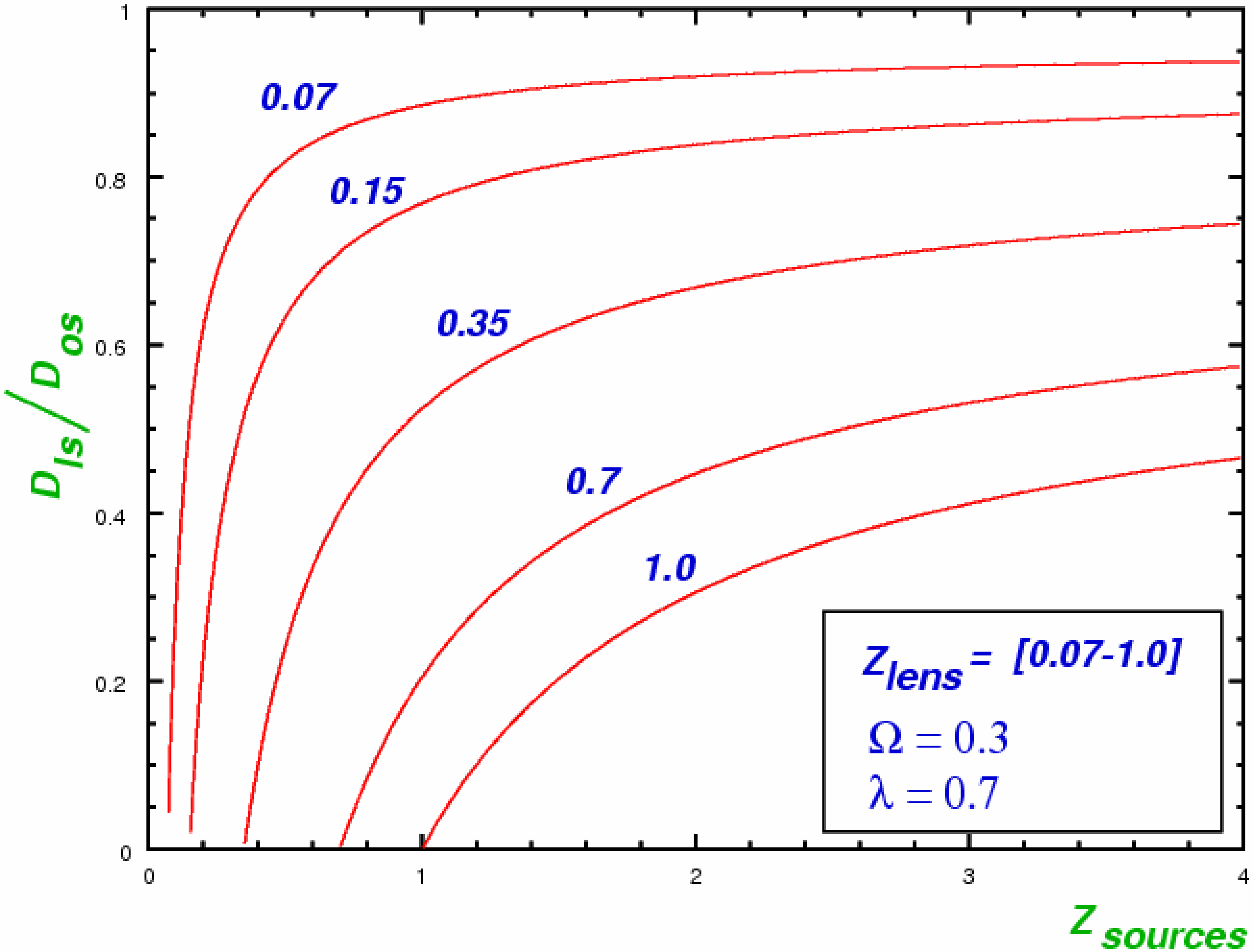
- The mass reconstruction provides the shape of the projeted mass distribution but not the absolute scale.
- Need the redshift of the sources:

$$\kappa\left(\vec{\theta},z\right)=\frac{\Sigma\left(\vec{\theta}\right)}{\Sigma_{critic}\left(z\right)}=\Sigma\left(\vec{\theta}\right)\frac{4\pi G}{c^2}D_{ol}\left[\frac{D_{ls}}{D_{os}}\right] \quad (109)$$

$$\gamma\left(\vec{\theta},z\right)=\frac{4\pi G}{c^2}D_{ol}\left[\frac{D_{ls}}{D_{os}}\right]\gamma\left(\vec{\theta}\right) \quad (110)$$

- Very sensitive to the redshift distribution for high-z lenses


Field observed without lens
Good observer
Good assistance
No clouds
No Moon
Seeing <0.8"
No technical problems
S1
S2
S3
Observer
Singular Isothermal Sphere at z=0.4
σ= 800 km/sec
Redshift sources: [0-3.5]
h=70
Ω=0.3, Λ=0
Same field with a SIS as lens



0.07
0.15
0.35
0.7
1.0
Z lens = [0.07-1.0]
Ω = 0.3
λ = 0.7
D ls / D os
Z sources
0
0.2
0.4
0.6
0.8
1
0
1
2
3
4

Application. the Singular Isothermal Sphere

- Mass density:

$$\rho(r) = \frac{\sigma_v^2}{2\pi G r^2} \tag{111}$$

- Projected mass density:

$$\Sigma(\xi) = \frac{\sigma_v^2}{2G\,\xi} \tag{112}$$

- Mass inside ξ

$$M(\xi) = \int_0^{\xi} 2\pi\xi'\,\Sigma(\xi')\,\mathrm{d}\xi' \tag{113}$$

- Angular Einstein radius

$$\theta_E = 4\pi \left(\frac{\sigma_v}{c}\right)^2 \frac{D_{ls}}{D_{os}} \tag{114}$$

- Lensing equation

$$\vec{\beta} = \vec{\theta} - \theta_E \frac{\vec{\theta}}{|\vec{\theta}|} \tag{115}$$

- Lensing properties

$$\kappa(\theta) = \frac{\theta_E}{2\theta};\ \bar{\kappa}(\theta) = \frac{\theta_E}{\theta};\ |\gamma(\theta)| = \frac{\theta_E}{2\theta};\ \alpha\left(\vec{\theta}\right) = \theta_E \frac{\vec{\theta}]}{|\vec{\theta}|}|; \tag{116}$$

- Signal to Noise ratio of a mass reconstruction
  - Tangential ellipticity $\varepsilon_t = \Re\left(\varepsilon e^{2i\varphi}\right)$

- Let the weighted mean tangential ellipticity be $\bar{\varepsilon}_t = \sum_i a_i \, \varepsilon_{ti}$

- What is the best weighting that maximize the S/N?
- Answer:

- For an isothermal sphere:

$$E\left(\bar{\varepsilon}_t\right) = \theta_E \sum_i \frac{a_i}{2\theta_i} \qquad (117)$$

- Variance:

$$E\left(\varepsilon^2\right) = \sum_{ij} a_i \, a_i \, E\left(\varepsilon_i \varepsilon_j\right) \qquad (118)$$

that is, using the intrinsic ellipticity distrbution of galaxies, σ_ε:

$$E\left(\varepsilon^2\right) = \sum_{ij} \gamma_t\left(\theta_i\right) \; \gamma_t\left(\theta_j\right) + \delta_{ij} \frac{\sigma_\varepsilon}{2} \qquad (119)$$

(negligible as compared to intrinsic)

So, for an isothermal sphere, the S/N is

$$S/N = \frac{\theta_E \sum_{ij} \frac{a_i}{\theta_i}}{\sqrt{2\sigma_{\varepsilon^2}}\sqrt{\sum_i a_i^2}} \tag{120}$$

- The derivative of S/N with respect to a_i gives the optimal weighting

$$\frac{\partial S/N}{\partial a_i} = \frac{\theta_E}{\sqrt{2\sigma_\varepsilon^2}} \frac{1}{\sqrt{\sum_i a_i^2}} \left[\frac{1}{\theta_i} - \frac{a_i \sum_i \frac{a_i}{\theta_i}}{\sqrt{\sum_i a_i^2}}\right] = 0 \tag{121}$$

- Therefore, the optimal weighting is $a_i \propto 1/\theta_i$

-

$$S/N = \frac{\theta_E}{2\sigma_\varepsilon^2}\sqrt{\sum_i \frac{1}{\theta_i^2}} \tag{122}$$

The sum on the right hand side can be estimated taking the ensemble average inside an annulus centered on the lens:

$$\sqrt{\sum_i \frac{1}{\theta_i^2}} \simeq \bar{n}\langle\frac{1}{\theta}\rangle \tag{123}$$

But

$$\bar{n}\langle\frac{1}{\theta}\rangle = \int_{\theta_1}^{\theta_2} 2\pi \frac{1}{\theta^2}\theta \mathrm{d}\theta = 2\pi \ln\left[\frac{\theta_2}{\theta_1}\right] \tag{124}$$

- Therefore

$$S/N = \frac{\theta_E}{\sigma_\varepsilon}\sqrt{\pi\,\bar{n}}\sqrt{\ln\left[\frac{\theta_2}{\theta_1}\right]} \tag{125}$$

- So let take a cluster if galaxies:

$$S/N = 10\left(\frac{n}{30\ \mathrm{arcmin}^2}\right)^{1/2}\left(\frac{\sigma_\varepsilon}{0.2}\right)^{-1}\left(\frac{\sigma_v}{600\ \mathrm{km/s}}\right)^2\left(\frac{\ln\left[\frac{\theta_2}{\theta_1}\right]}{\ln 10}\right)^{1/2}\left\langle\frac{D_{ls}}{D_{os}}\right\rangle$$

Therefore to increase the S/N

- Go deep: increase *n*
- Observe massive clusters: increase σ^2
- Reduce intrinsic ellipticity dispersion: space better

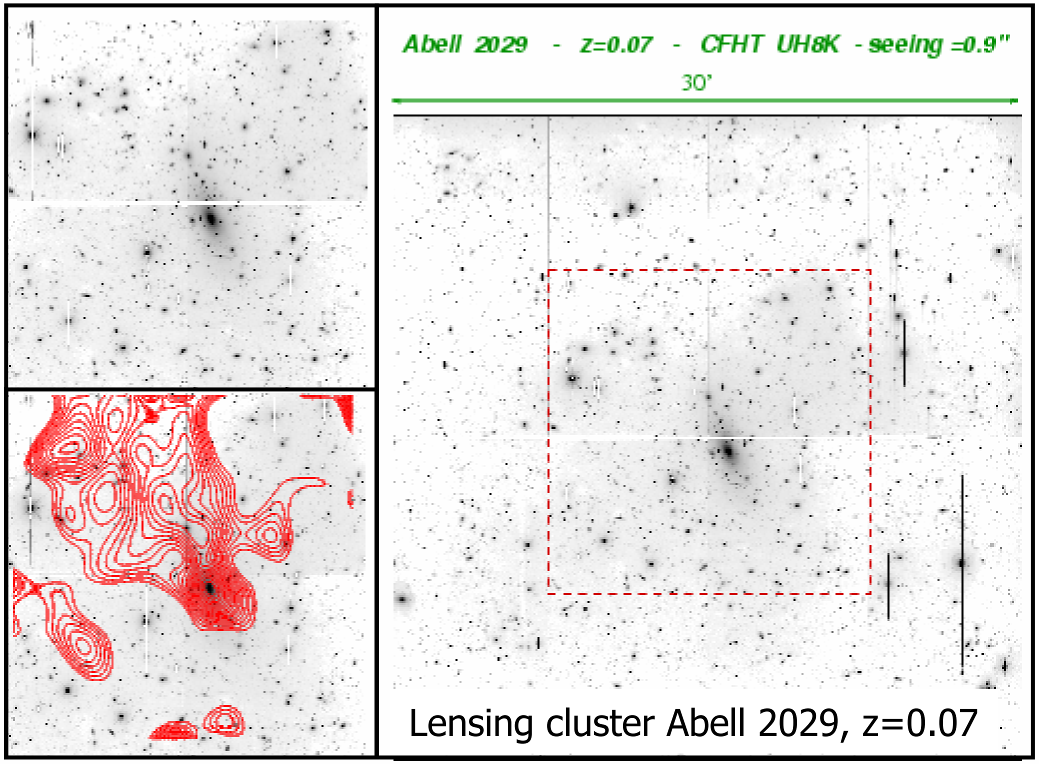
Invariance over Isotropic Expansion (Mass Sheet Degeneracy)

- Changing $a \longrightarrow \lambda a$ and $b \longrightarrow \lambda b$ keeps ε unchanged.
- $\Longleftrightarrow \gamma \longrightarrow \lambda \gamma$ and $\kappa \longrightarrow \lambda \kappa$.
- $\Longleftrightarrow$ changing $\kappa \longrightarrow \kappa' = \lambda \kappa + (1 - \lambda)$: rescaling κ and adding a constant mass density.
- This mass sheet degeneracy results from the fact that the gravitational distortion is sensitive to the gradient of the projected mass density.

- We are only measuring ellipticity modification, no size modification of galaxies

Solve the degeneracy: field of view

- If FOV large: assume $\Sigma(R \longrightarrow \infty) \approx 0$



Lensing cluster Abell 2029, z=0.07

Solve the degeneracy: depletion

- Magnification: increases the flux received from galaxies $\longrightarrow$ more galaxies visible and ALSO magnifies by the same amount the area of the projected lensed sky $\longrightarrow$ decreases the galaxy number density (Broadhurst 1995):

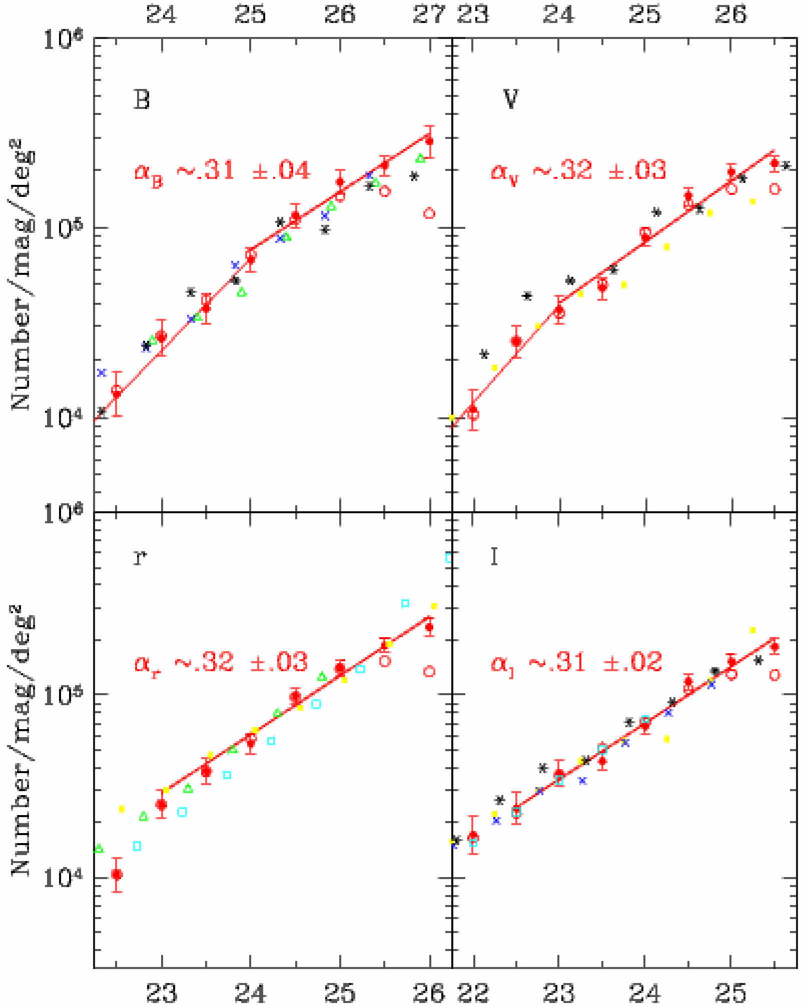
$$N(< m, r) = N_0(< m)\, \mu(r)^{2.5\alpha - 1} \approx N_0\, (1 + 2\kappa)^{2.5\alpha - 1} \tag{127}$$

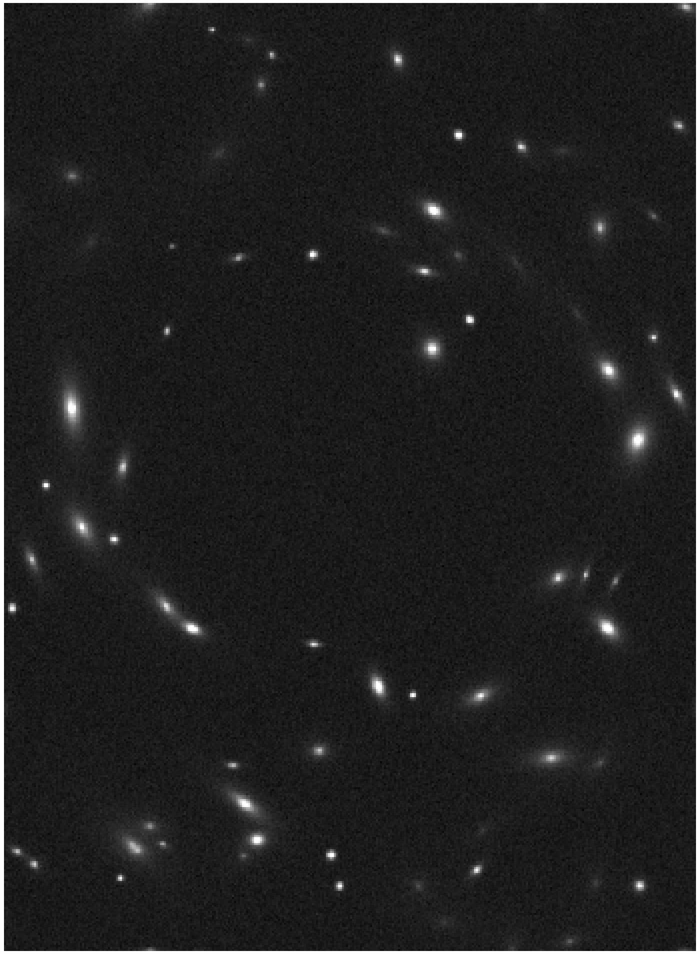
$\mu(r)$ is the magnification, $N_0(< m)$ the intrinsic (unlensed) number density, and

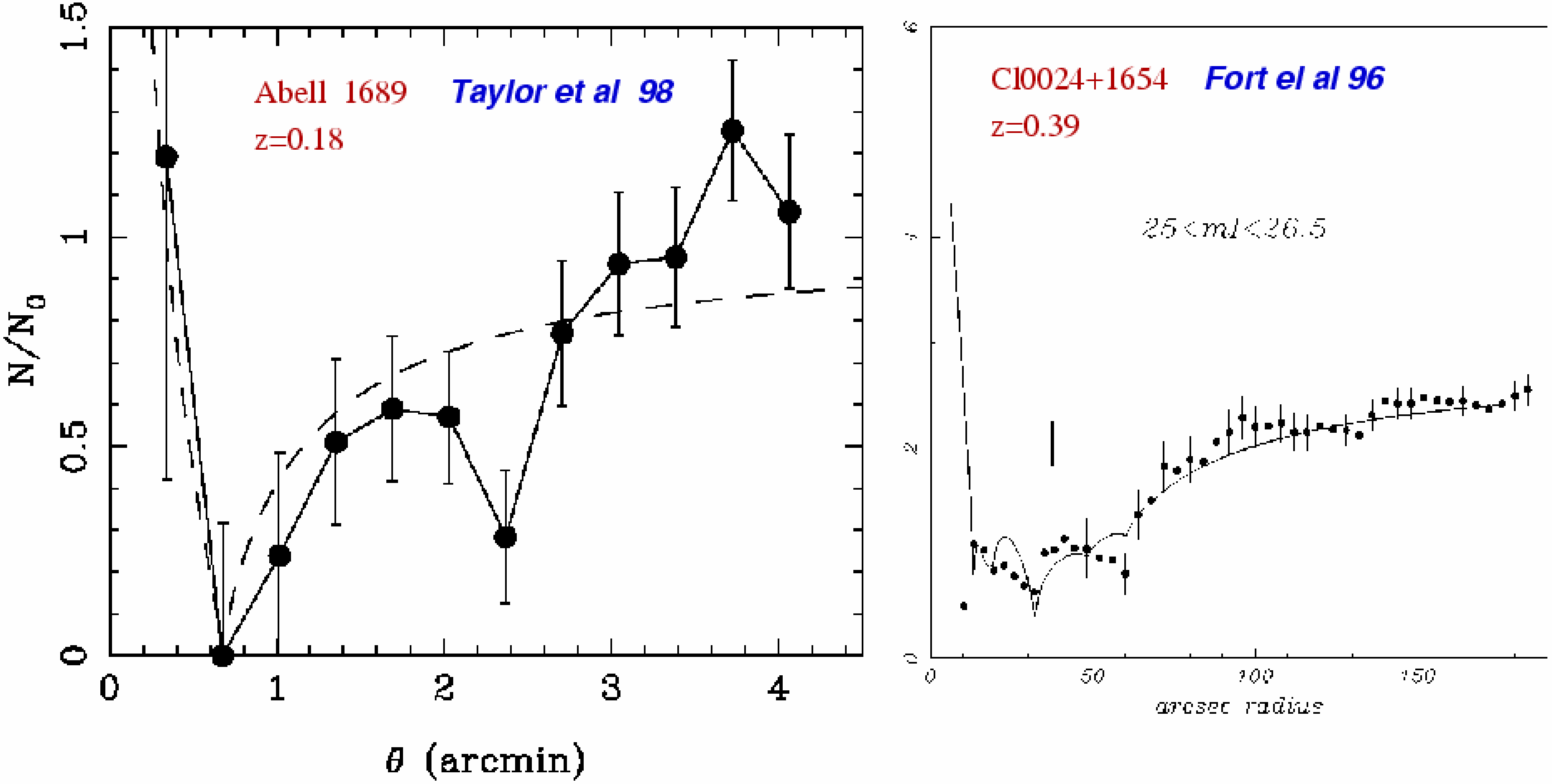
$$\alpha = \frac{\mathrm{d}logN(< m)}{\mathrm{d}m} \tag{128}$$

NTT DEEP FIELD

Arnouts et al 1999






Abell 1689 Taylor et al 98
z=0.18
N/N0
θ (arcmin)
Cl0024+1654 Fort el al 96
z=0.39
25<mI<26.5
arcsec radius

Mean Tangential Shear and κ inside circles

Consider a axially symmetric mass distribution.

Gauss Theorem:

$$\int_0^\theta \mathrm{d}^2\theta \; \vec{\nabla}.\vec{\nabla}\psi = \theta \oint \mathrm{d}\varphi \; \vec{\nabla}\psi.\vec{n} \tag{129}$$

where the left hand side integral is over the circle area of radius θ and ψ is a scalar function.

The circulation on the right hand side is on the circle line of radius θ and $\vec{n}$ is a vector normal to the the circle directed outward.

Since $\nabla^2\psi = 2\kappa$, and $\vec{\nabla}\psi.\vec{n} = \psi_{,\theta}$, we then have

$$m(\theta) = \frac{1}{\pi}\int_0^{\theta} \mathrm{d}^2\theta\ \kappa\left(\vec{\theta}\right) = \frac{\theta}{2\pi}\oint \mathrm{d}\varphi \frac{\partial\psi}{\partial\theta} \tag{130}$$

Differentiating with respect to θ:

$$\frac{\mathrm{d}m}{\mathrm{d}\theta} = \frac{m}{\theta} + \frac{\theta}{2\pi}\oint \mathrm{d}\varphi \frac{\partial^2\psi}{\partial\theta^2} \tag{131}$$

Consider a point on the θ_1-axis. For a axially symmetric distribution we have $\psi_{,\theta\theta} = \psi_{11} = \kappa + \gamma_1 = \kappa$ - γ_t, regardless the coordinate systems. So it must be valid for all φ.

So, denoting

- $\langle\kappa(\theta)\rangle$ the mean surface density on the circle or radius θ

- $\langle\gamma_t(\theta)\rangle$ the mean tangential shear on the circle or radius θ

we have

$$\frac{\mathrm{d}m}{\mathrm{d}\theta} = \frac{m}{\theta} + \theta\left[\langle\kappa(\theta)\rangle - \langle\gamma_t(\theta)\rangle\right] \tag{132}$$

But the mass in the circle, $m(\theta)$, can be derived from the mean mass density inside the circle $\bar{\kappa}(\theta)$:

$$m(\theta) = \theta^2\bar{\kappa}(\theta) = 2\int_0^\theta \mathrm{d}\theta'\theta'\langle\kappa(\theta')\rangle \tag{133}$$

Together with the fact that

$$m(\theta) = 2\int_0^\theta \mathrm{d}\theta'\theta'\,\langle\kappa(\theta'\rangle \Longrightarrow \frac{\mathrm{d}m}{\mathrm{d}\theta} = 2\theta\langle\kappa(\theta)\rangle \tag{134}$$

one find

$$\boxed{\langle\gamma_t\rangle = \bar{\kappa} - \langle\kappa\rangle} \tag{135}$$

Therefore, from the measurement of the tangential shear, averaged over concentric circles, one can determine the azimuthally-averaged mass profiles of lenses,

The Aperture Mass

- Consider the function $U\left(|\theta|\right)$ a compensated filter:

$$\int U\left(\theta\right)\ \theta\ \mathrm{d}\theta = 0 \tag{136}$$

then the perture mass

$$M_{ap}\left(\vec{\theta}_0\right) = \int U\left(\left|\vec{\theta} - \vec{\theta}_0\right|\right)\ \kappa\left(\vec{\theta}\right) \mathrm{d}^2\theta \tag{137}$$

is independent of the mass sheet degenaracy, κ_0.

Consider first

$$M_{ap} = 2\pi \int_0^{\theta_u} \theta' \, U(\theta') \, \langle \kappa(\theta') \rangle \mathrm{d}\theta' \tag{138}$$

that is

$$M_{ap} = 2\pi \left[X(\theta') \langle \kappa(\theta') \rangle \right]_0^{\theta_u} - 2\pi \int_0^{\theta_u} \mathrm{d}\theta' \, X(\theta') \frac{\mathrm{d}\langle \kappa \rangle}{\mathrm{d}\theta'} \tag{139}$$

where θ_u is the radius of the aperture and

$$X(\theta') = \int_0^{\theta} \mathrm{d}\theta' \, \theta' U(\theta') \tag{140}$$

Using this definition together with the compensated nature of U implies that the left hand side term with boundaries vanishes.

We have seen before that

$$\langle \gamma_t \rangle = \bar{\kappa} - \langle \kappa \rangle \Longrightarrow \frac{\mathrm{d}\langle \gamma_t \rangle}{\mathrm{d}\theta'} = \frac{\mathrm{d}\bar{\kappa}}{\mathrm{d}\theta'} - \frac{\mathrm{d}\langle \kappa \rangle}{\mathrm{d}\theta'} \tag{141}$$

So, since

$$\theta^2 \bar{\kappa} = 2 \int_0^{\theta} \mathrm{d}\theta \ \theta \langle \kappa(\theta) \rangle \tag{142}$$

Differentiating this equation gives:

$$\frac{\mathrm{d}\bar{\kappa}}{\mathrm{d}\theta} = -\frac{2}{\theta} \langle \gamma_t \rangle \tag{143}$$

Replacing this in the previous relation we then find

$$2\pi \int_0^{\theta_u} X(\theta) \ \frac{\mathrm{d}\langle \kappa \rangle}{\mathrm{d}\theta} = 2\pi \int_0^{\theta_u} \mathrm{d}\theta X(\theta) \ \mathrm{d}\langle \gamma_t \rangle + 2\pi \int_0^{\theta_u} \mathrm{d}\theta X(\theta) \ \frac{\langle \gamma_t \rangle}{\theta} \tag{144}$$

That is, integrating by parts the first term

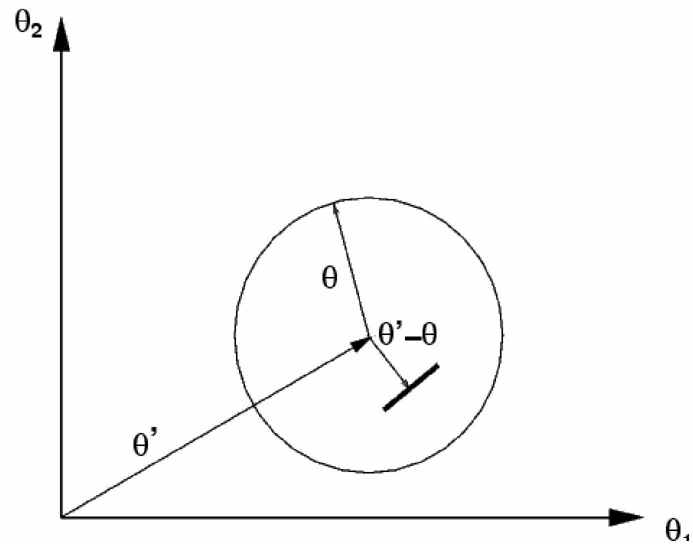
$$M_{ap} = 2\pi \left[X(\theta) \langle \gamma_t(\theta) \rangle \right]_0^{\theta_u} - 2\pi \int_0^{\theta_u} \mathrm{d}\theta \frac{\mathrm{d}X}{\mathrm{d}\theta} \langle \gamma_t \rangle + 2\pi \int_0^{\theta_u} \mathrm{d}\theta \, X(\theta) \ \theta \frac{\langle \gamma_t \rangle}{\theta^2} \tag{145}$$

The boundary term again vanishes. Therefore , if we define the function $Q = 2/\theta^2 - U$ one can then express the mass inside a circular aperture as function of the tangential shear

$$M_{ap}\left(\vec{\theta}_0\right) = \int Q\left(\left|\vec{\theta}\right|\right)\ \gamma_t\left(\vec{\theta};\vec{\theta}_0\right) \mathrm{d}^2\vec{\theta} \qquad (146)$$

where

$$Q(\theta) = \frac{2}{\theta}\int_0^{\theta} \mathrm{d}\theta'\,\theta'\,U(\theta') - U(\theta) \qquad (147)$$



Consequences and pratictal implications

- If U has a finite support, then Q also has a finite support $\Longrightarrow$ the aperture mass can be calculated on a finite dat field, *i.e* from the shear signal in the same circle where $U \neq 0$
- Since the compensated filter leaves M_{ap} invariant to a constant additive mass density, M_{ap} is insensitive to the mass sheet degeneracy

- If $U(\theta) = Cte$ for $0 < \theta < \theta_m$, then $Q = 0$ in the same interval. Therefore the strong lensing regime, where γ deviates from g, can ba avoided by perperly choosing U and Q
- The filter can be optimized: narrow range of frequencies (cosmic shear), detection of (dark) clusters (surveys)
- In the same way as M_{ap} for the tangential component γ_t, one can define a cross component:

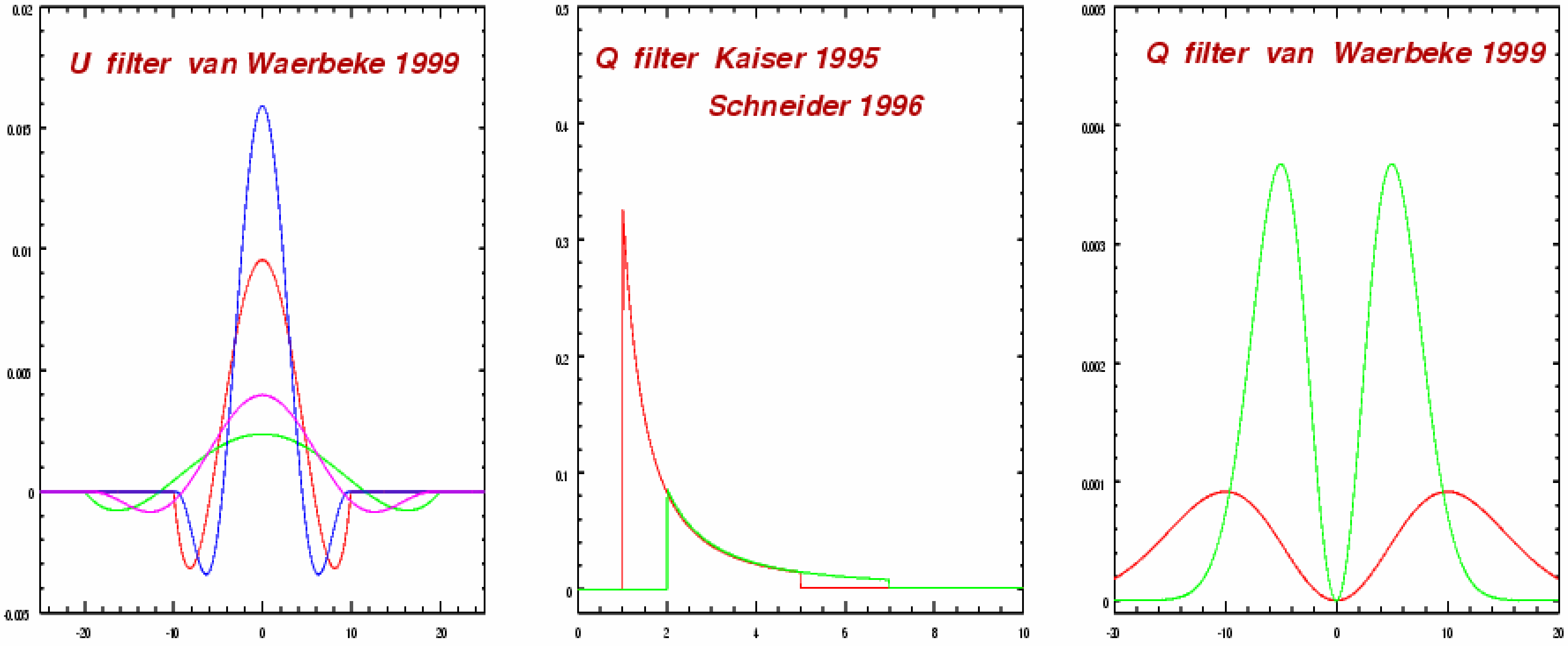
$$M_{\perp}\left(\vec{\theta}_0\right) = \int Q\left(\left|\vec{\theta}\right|\right)\ \gamma_x\left(\vec{\theta};\vec{\theta}_0\right) \mathrm{d}^2\vec{\theta} \tag{148}$$

  Note that $M_{\perp}$ does not represent a physical mass density. In fact it represent a curl component and should be zero in abscence of noise.

Practical usage of M_{ap}: using a discrete sum of N galaxy ellipticities located at position $\vec{\theta}$, inside oa circle of radius R from $\vec{\theta}_0$:

$$M_{ap}\left(\vec{\theta}_0\right) = \frac{\pi R^2}{N} \sum_i \varepsilon_{ti}(\theta)\; Q\left(\left|\vec{\theta} - \vec{\theta}_0\right|\right) \tag{149}$$

where ε_t is the tangential component of the ellipticity with respect tp position $\vec{\theta}_0$.


U filter van Waerbeke 1999
Q filter Kaiser 1995
Schneider 1996
Q filter van Waerbeke 1999

Mass reconstruction

Application to clusters of galaxies

From cluster galaxy observation to cluster mass map

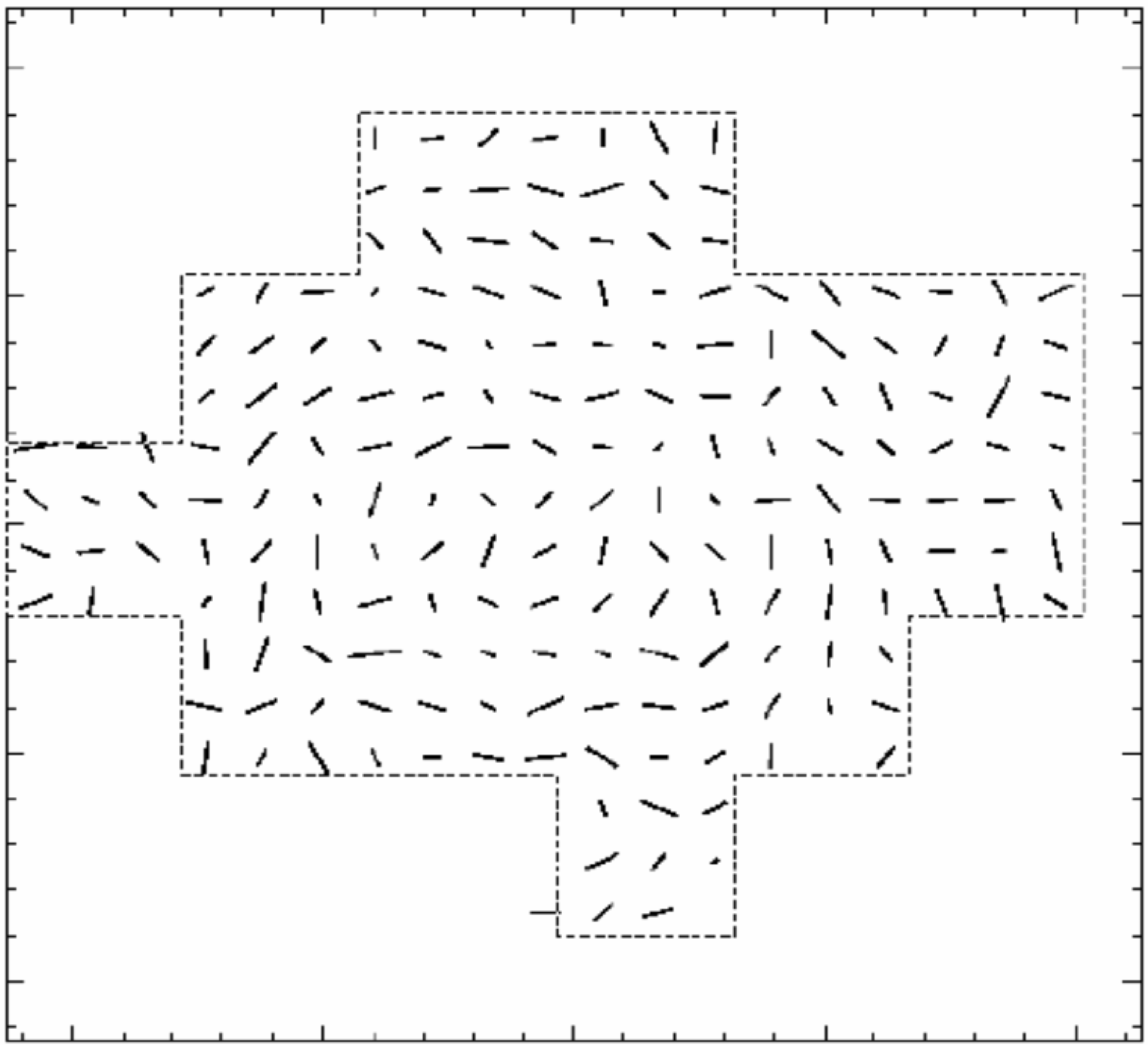
MS1054
HST

Hoekstra
et al 1998

From cluster galaxy observation to cluster mass map

MS1054
HST

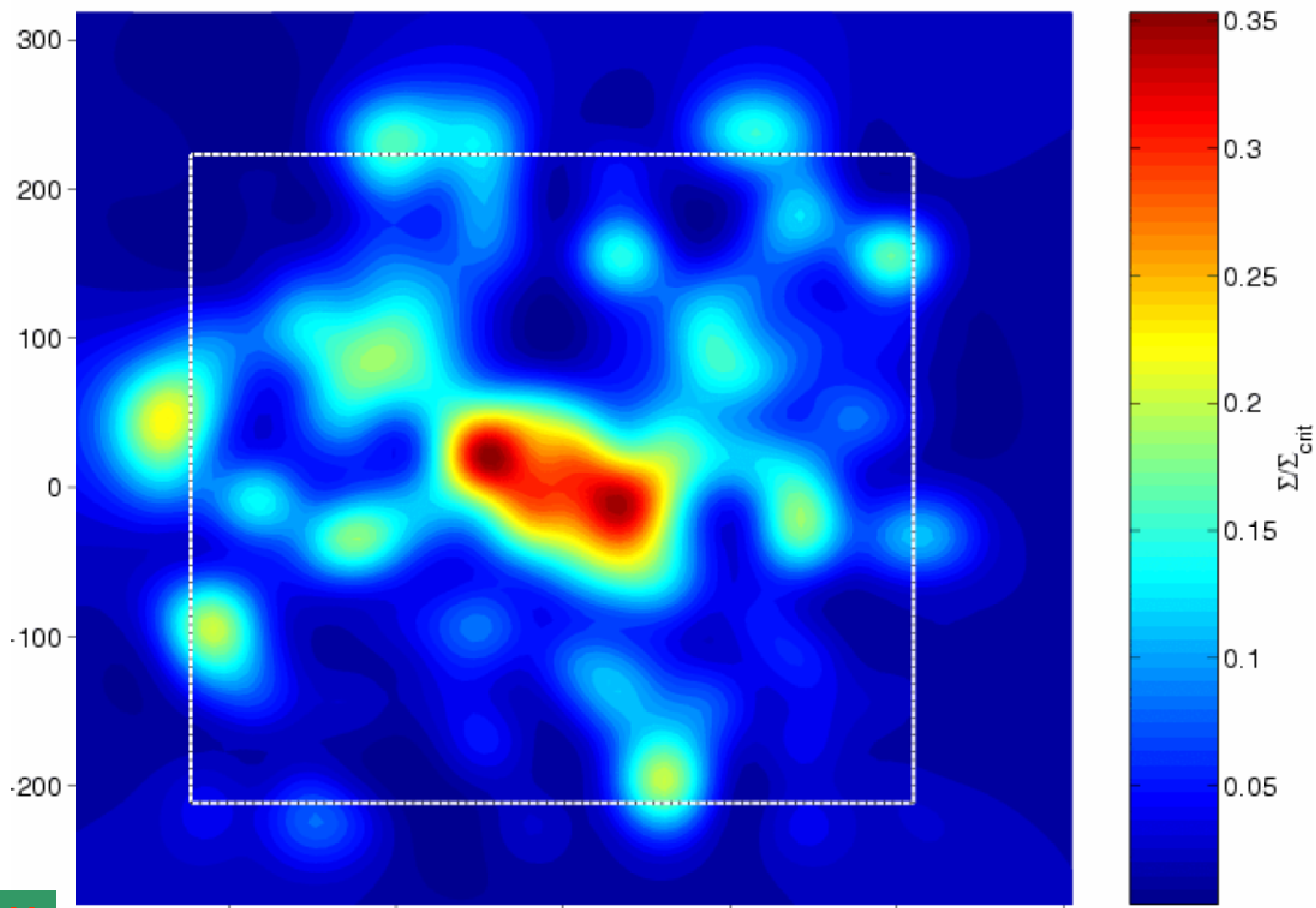
Hoekstra
et al 1998

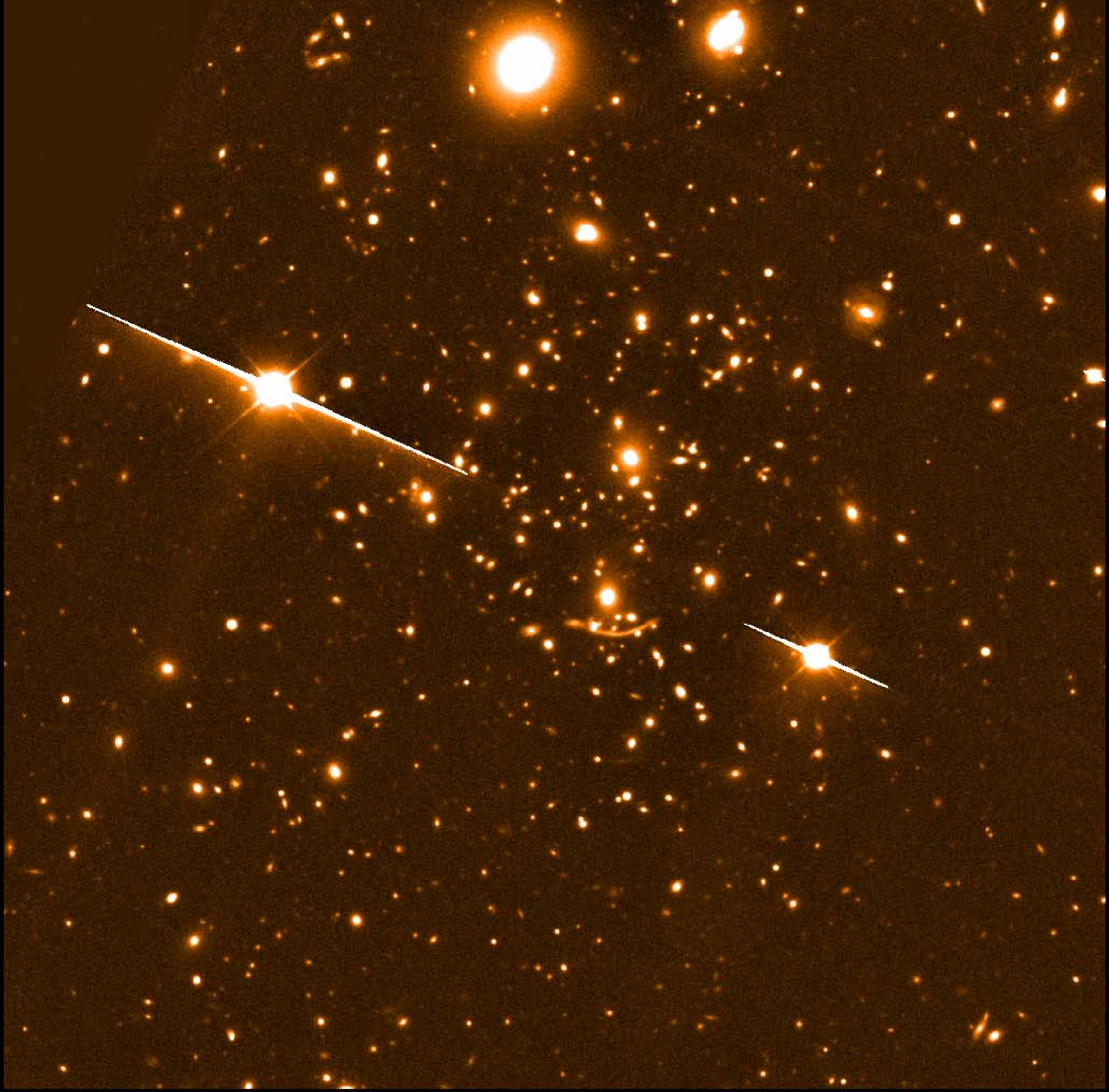


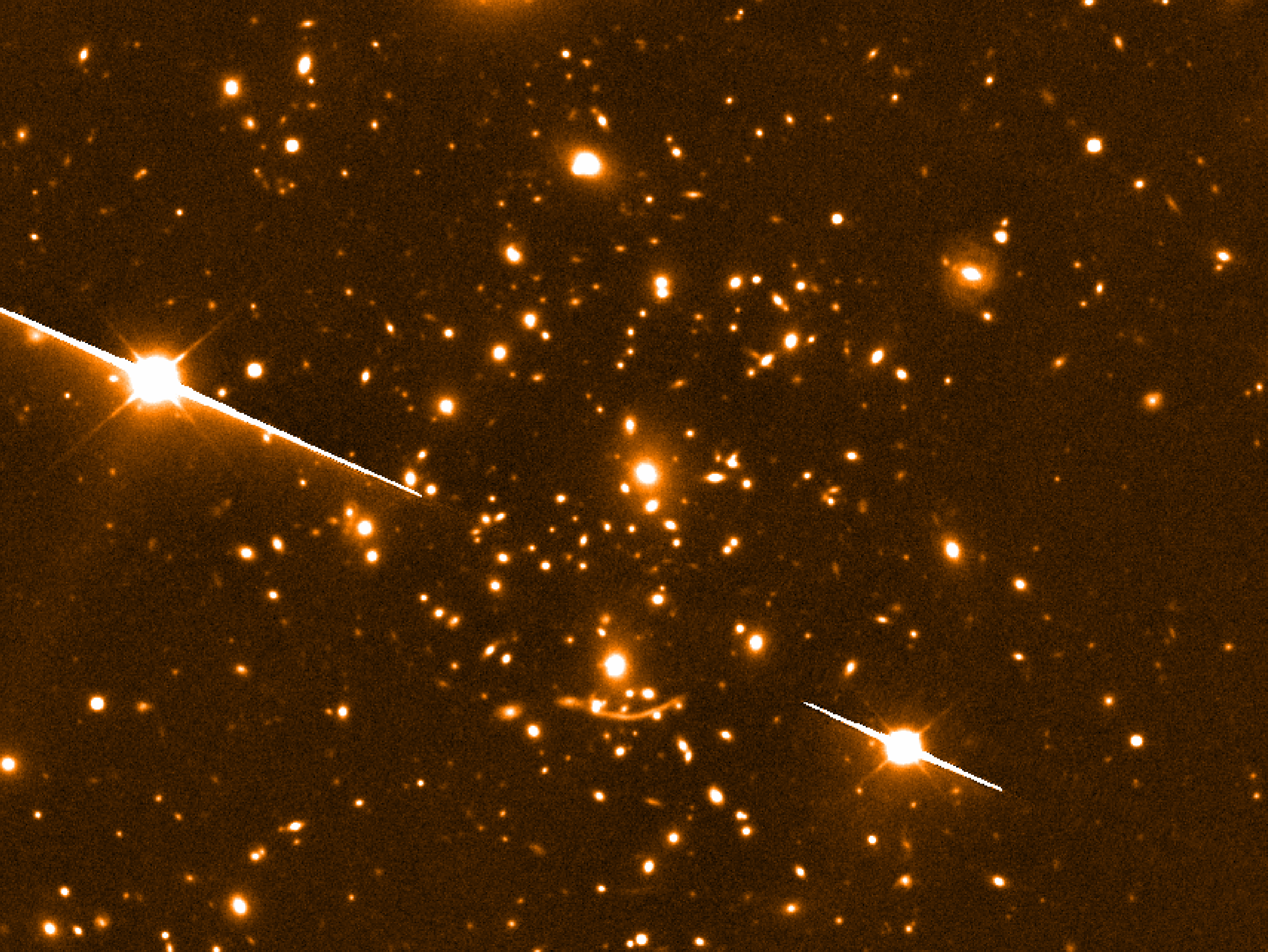
From cluster galaxy observation to cluster mass map

MS1054
HST

Hoekstra
et al 1998

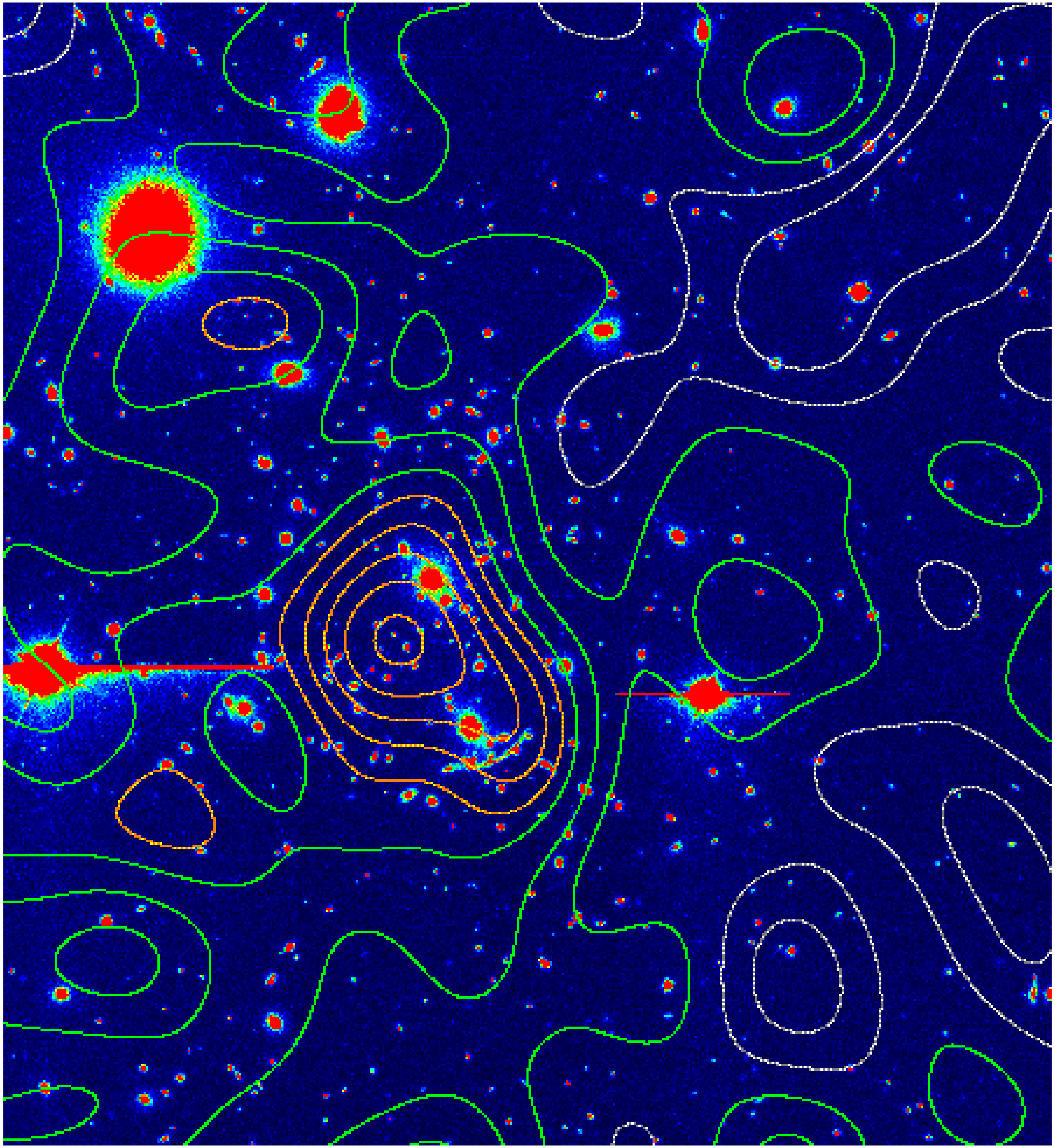






First giant arc discovered in Abell 370

$Z_{cluster} = 0.374$, $Z_{arc} = 0.720$



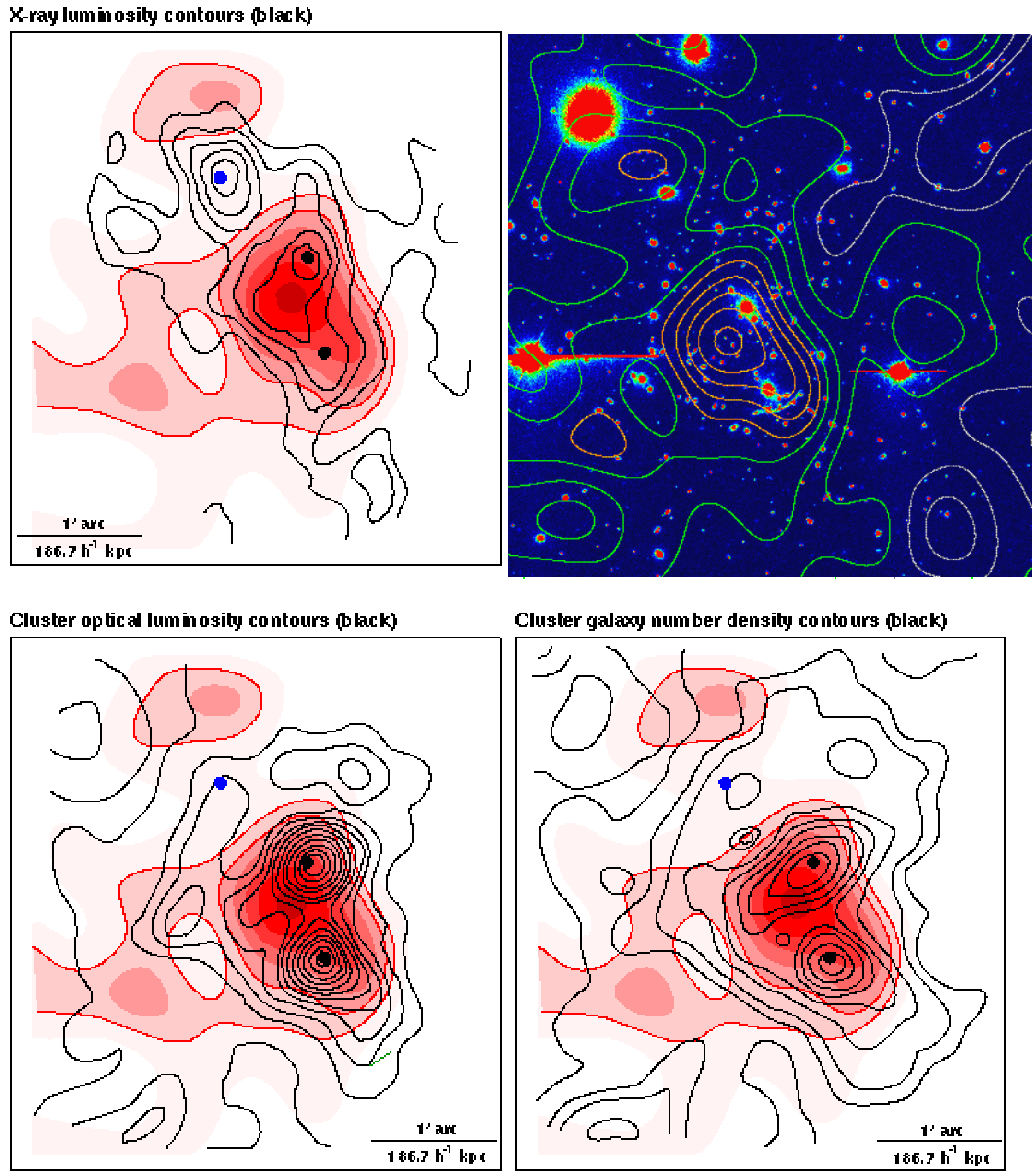
Abell 370

$Z_{cluster}$ =0.374 , Z_{arc}=0.720

Does light traces masses?

X-ray vs. Mass : yes

Red galaxies vs. Mass: yes

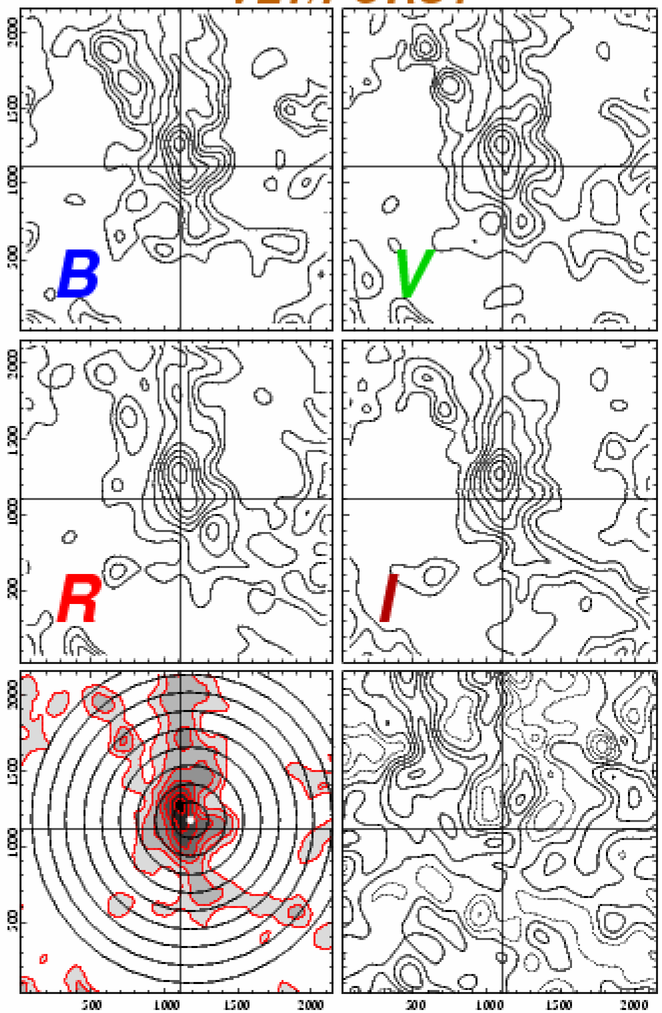


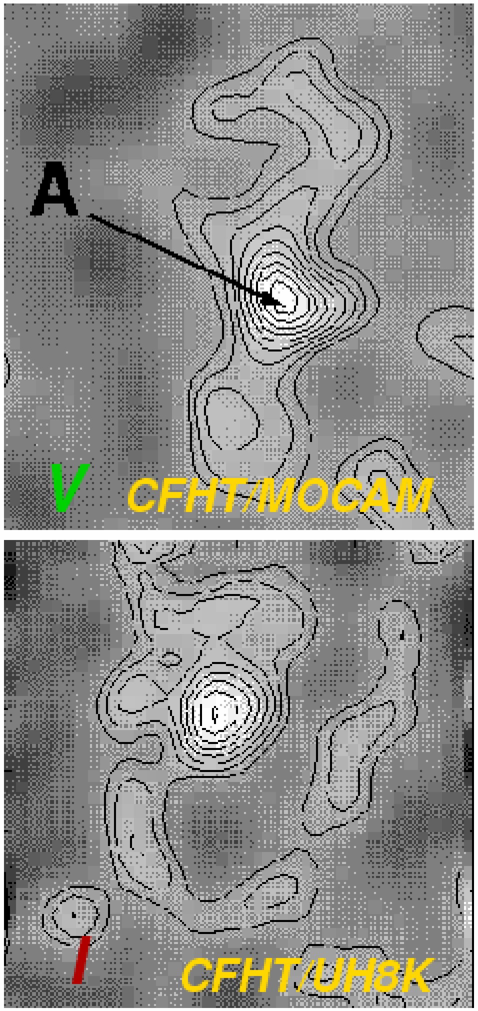
MS1008-1224
(z=0.3)
VLT/FORS
UBRI

Reconstruction in 4 filters | With 2 different instruments

MS1008-1224 ***Abell 1942***

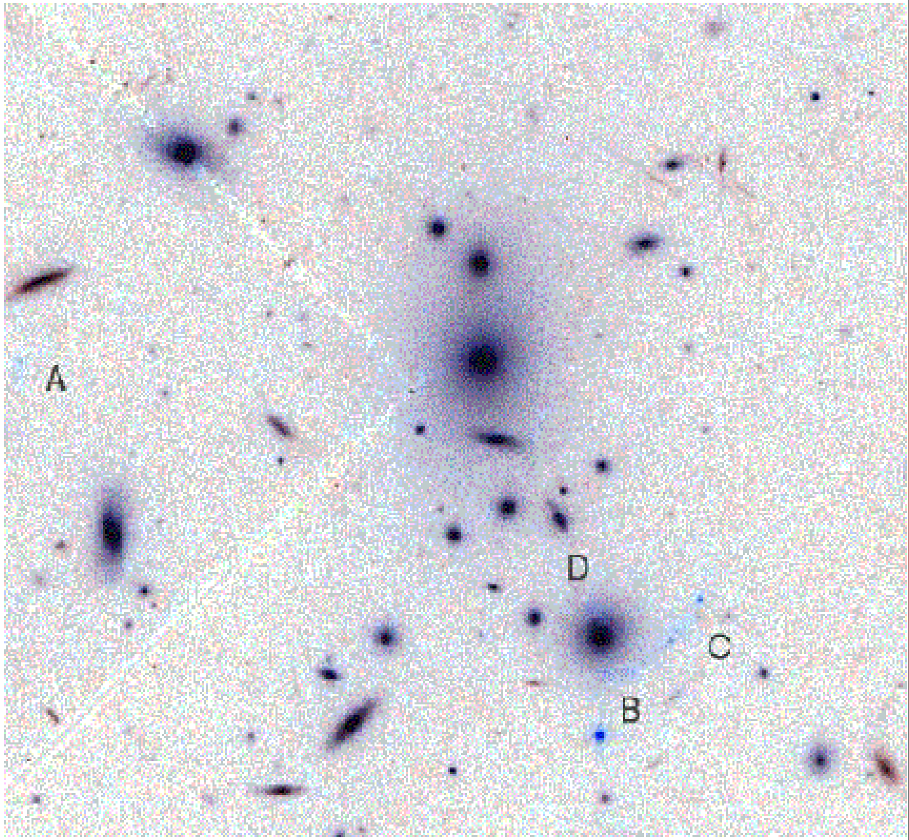
VLT/FORS1



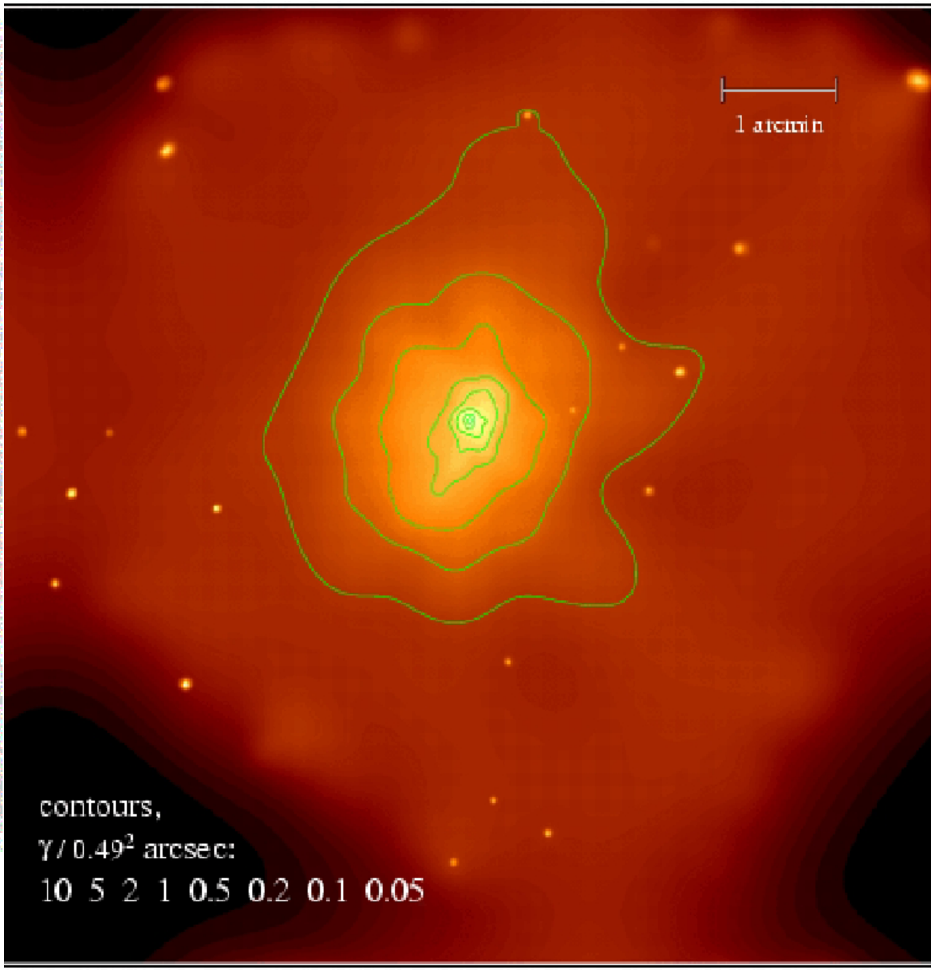


Athreya et al 1999 ***Erben et al 2000***

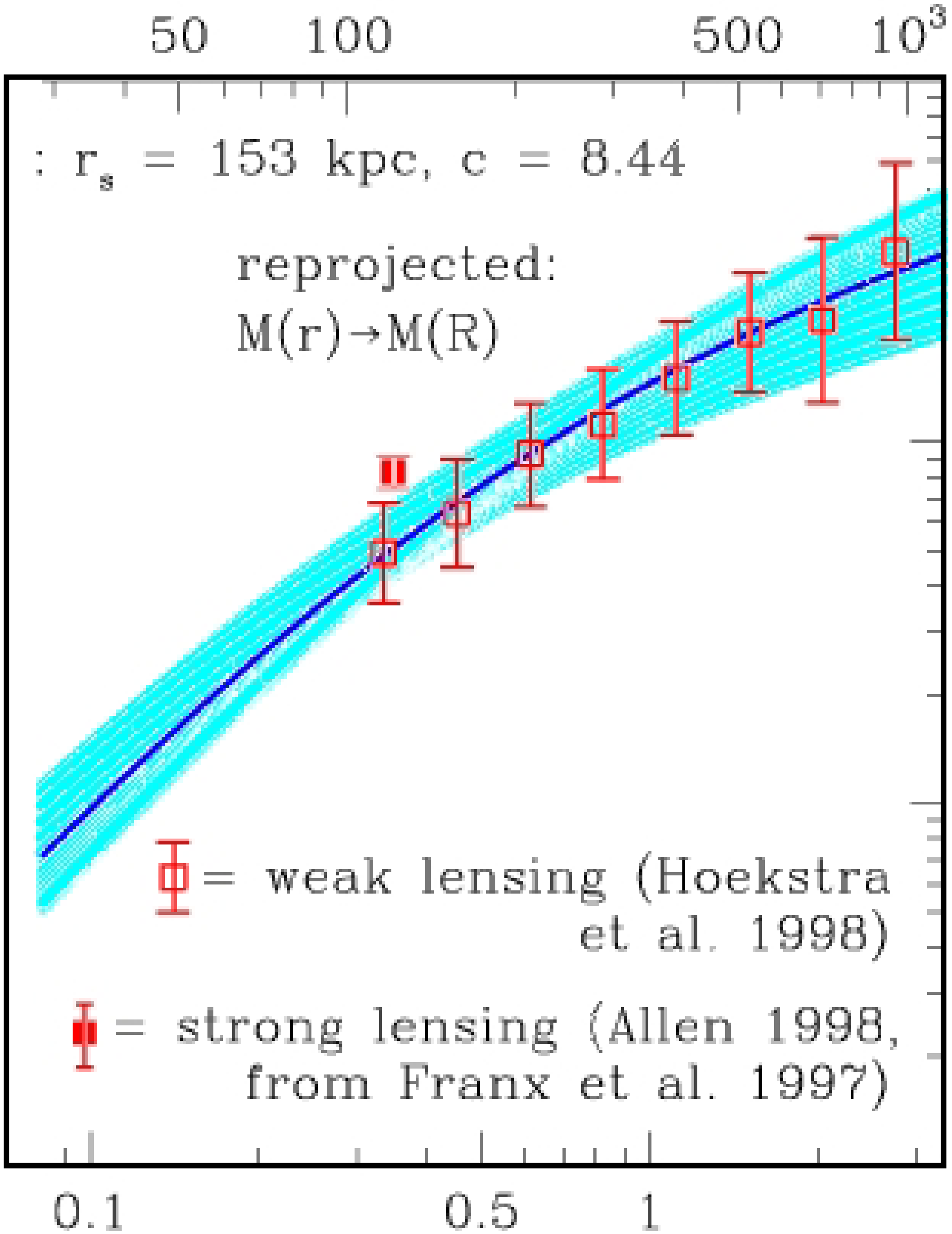
MS1358+6245



HST



Chandra

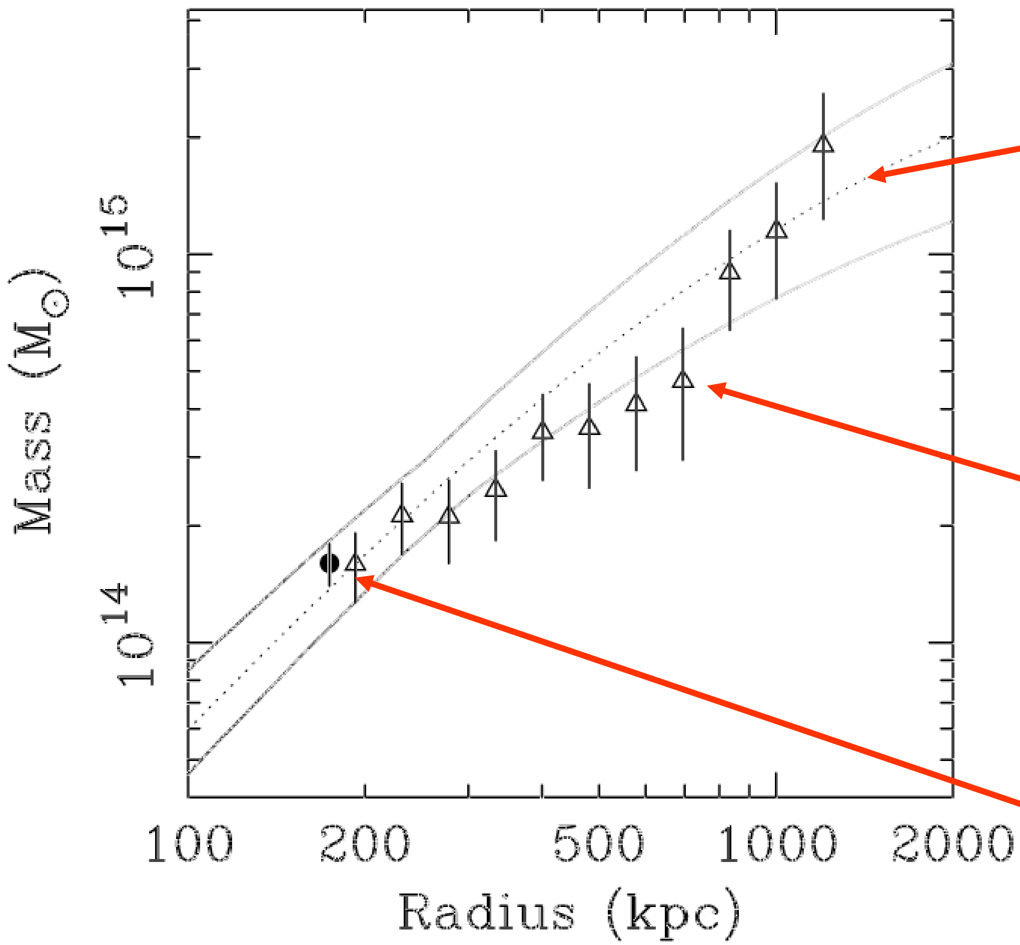


Chandra
+Strong / Weak
lensing

Cluster of galaxies
MS1358+6254

Comparison lensing / X

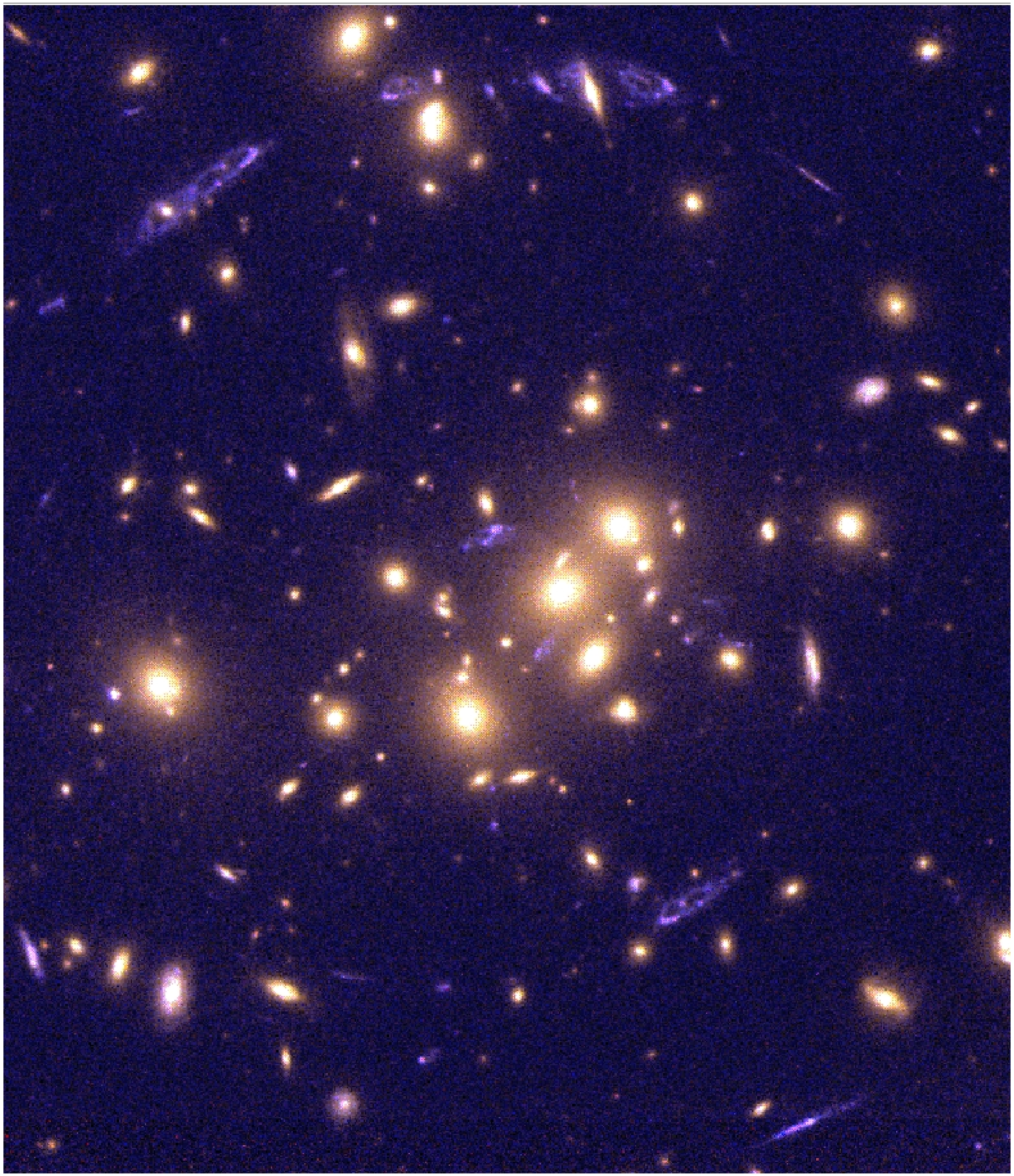
Abell 2390



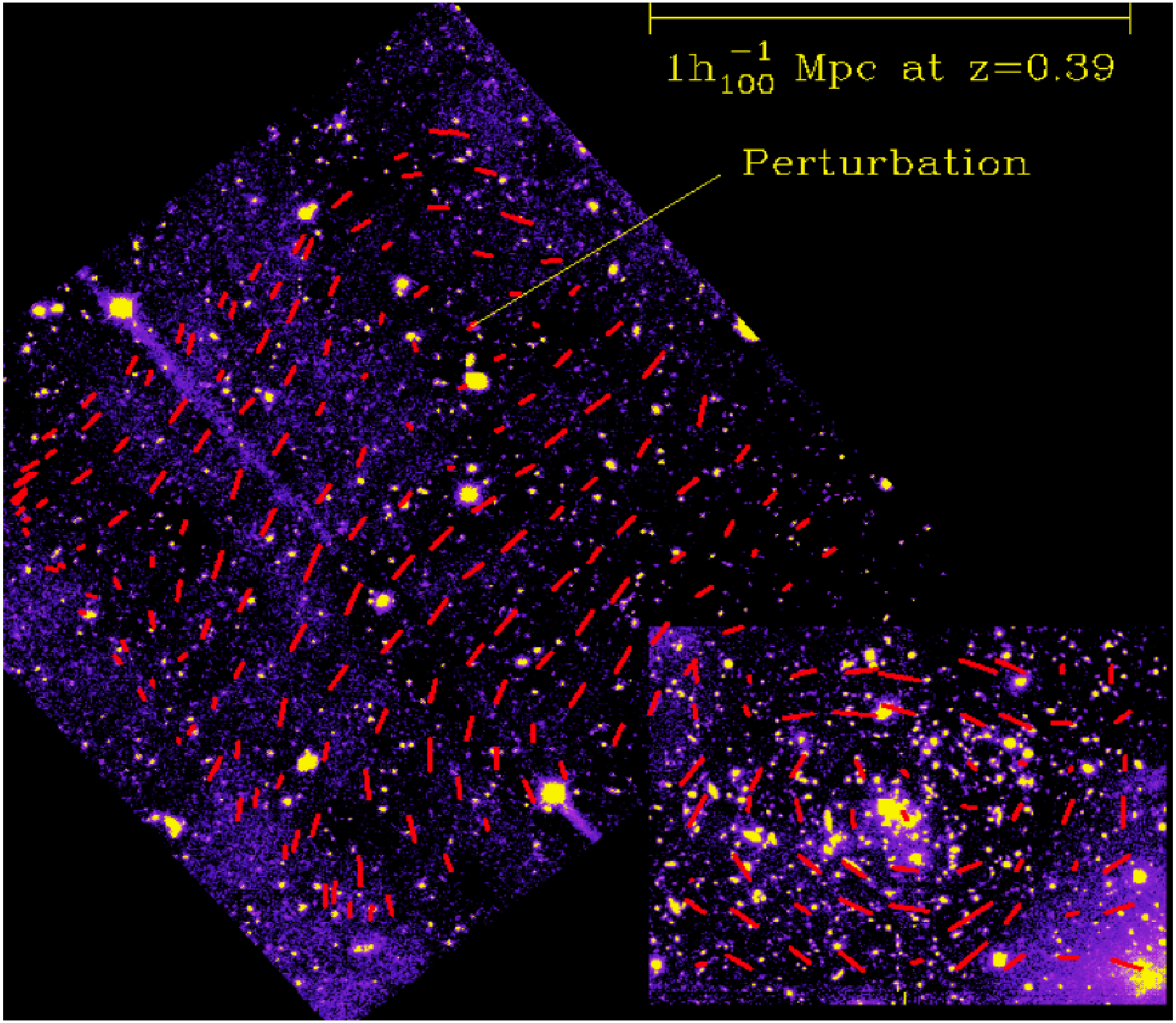
Chandra:
Allen et al 2001

Weak Lensing:
Squires et al 1996

Strong Lensing:
Pierre et al 1999



Cl0024+1654
z=0.55

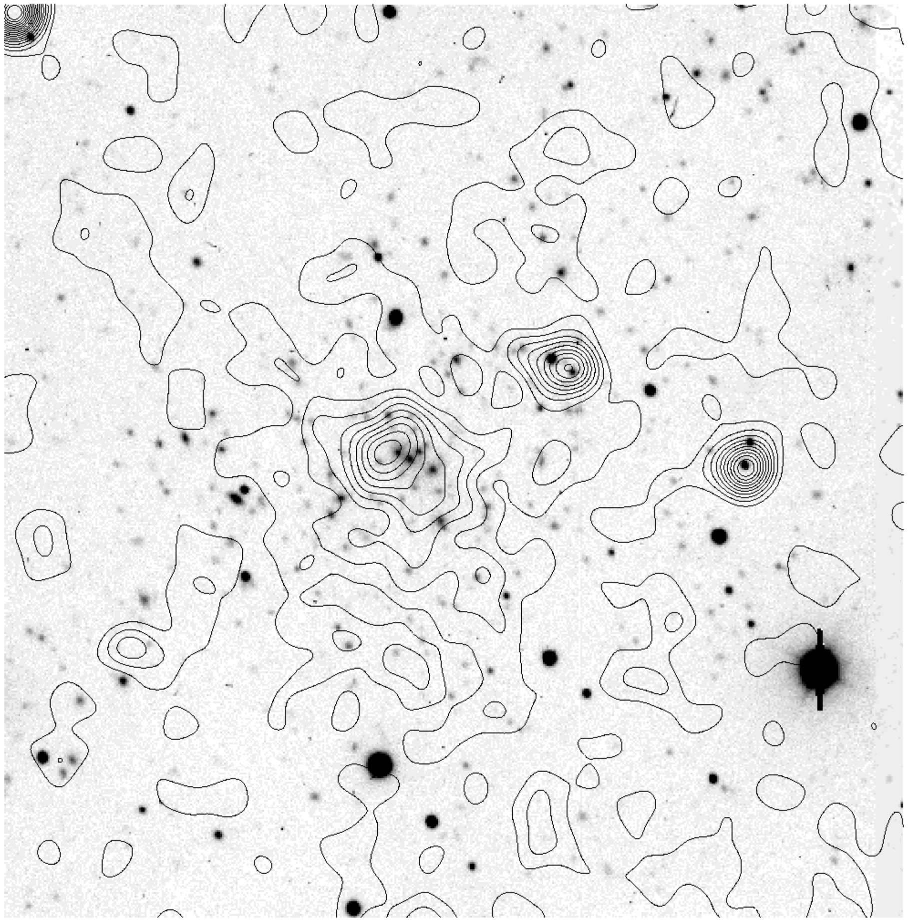


Cl0024+1654
z=0.55

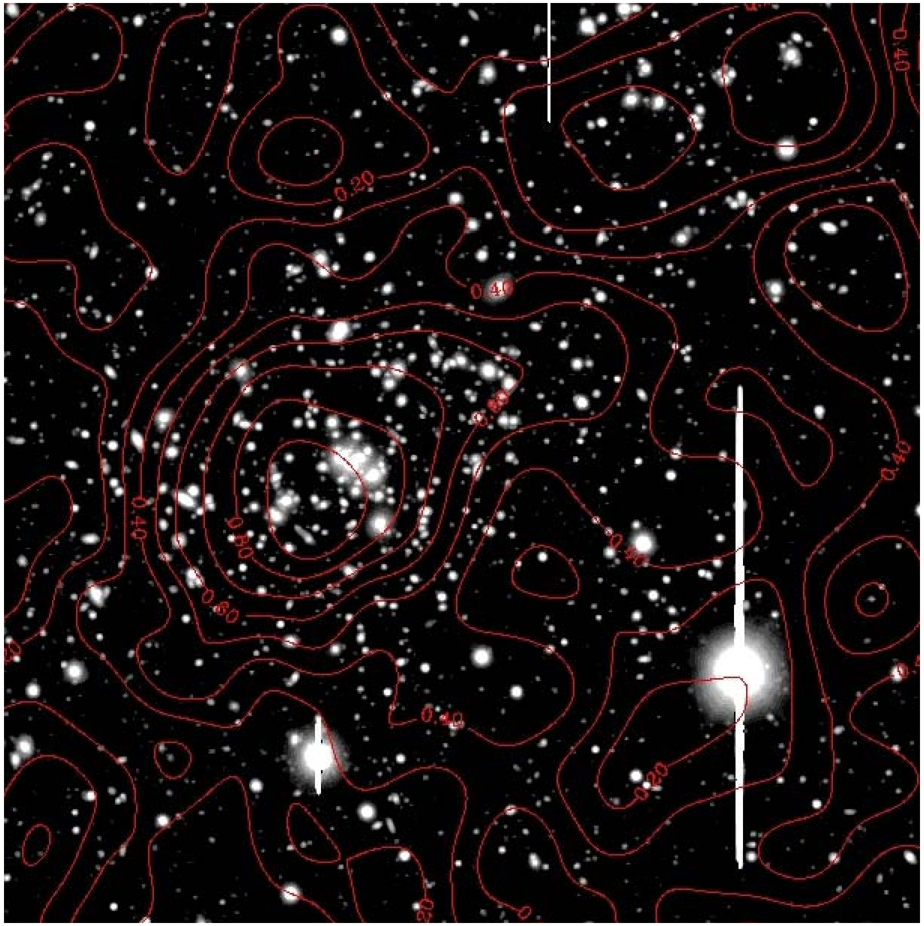
Bonnet, Mellier, Fort 1994

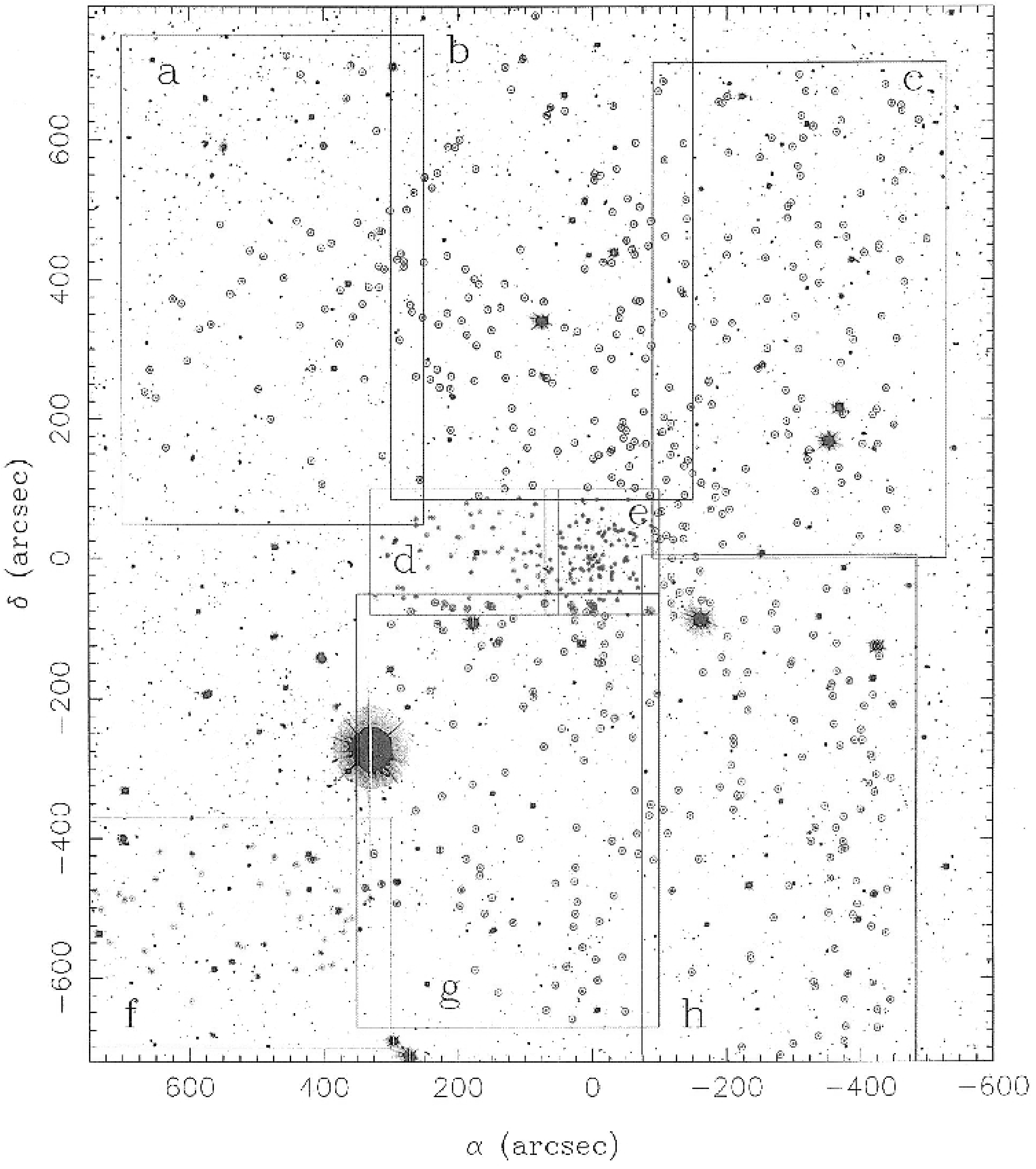
Example: Cl0024+1654 *z=**0.55***

X-gas

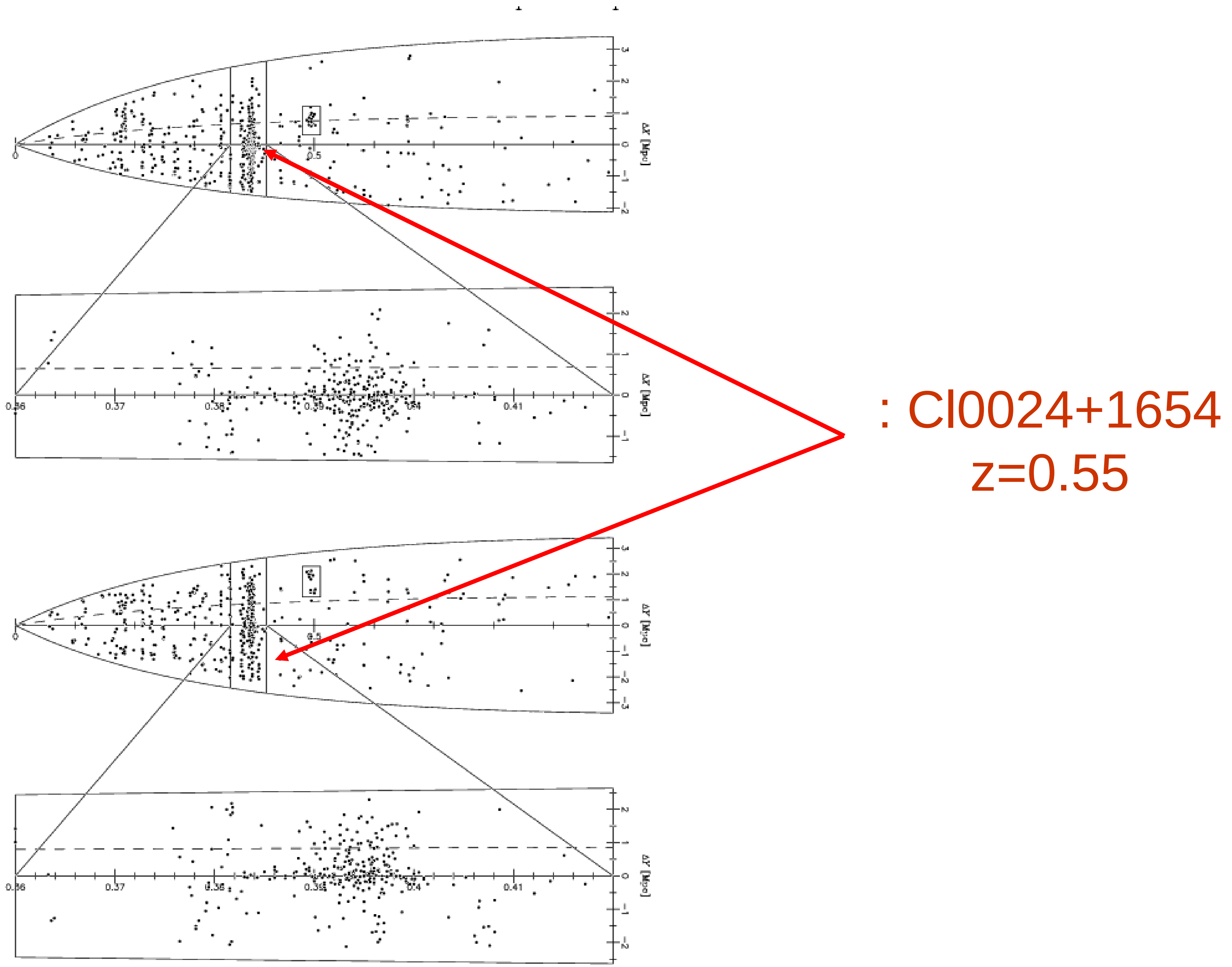


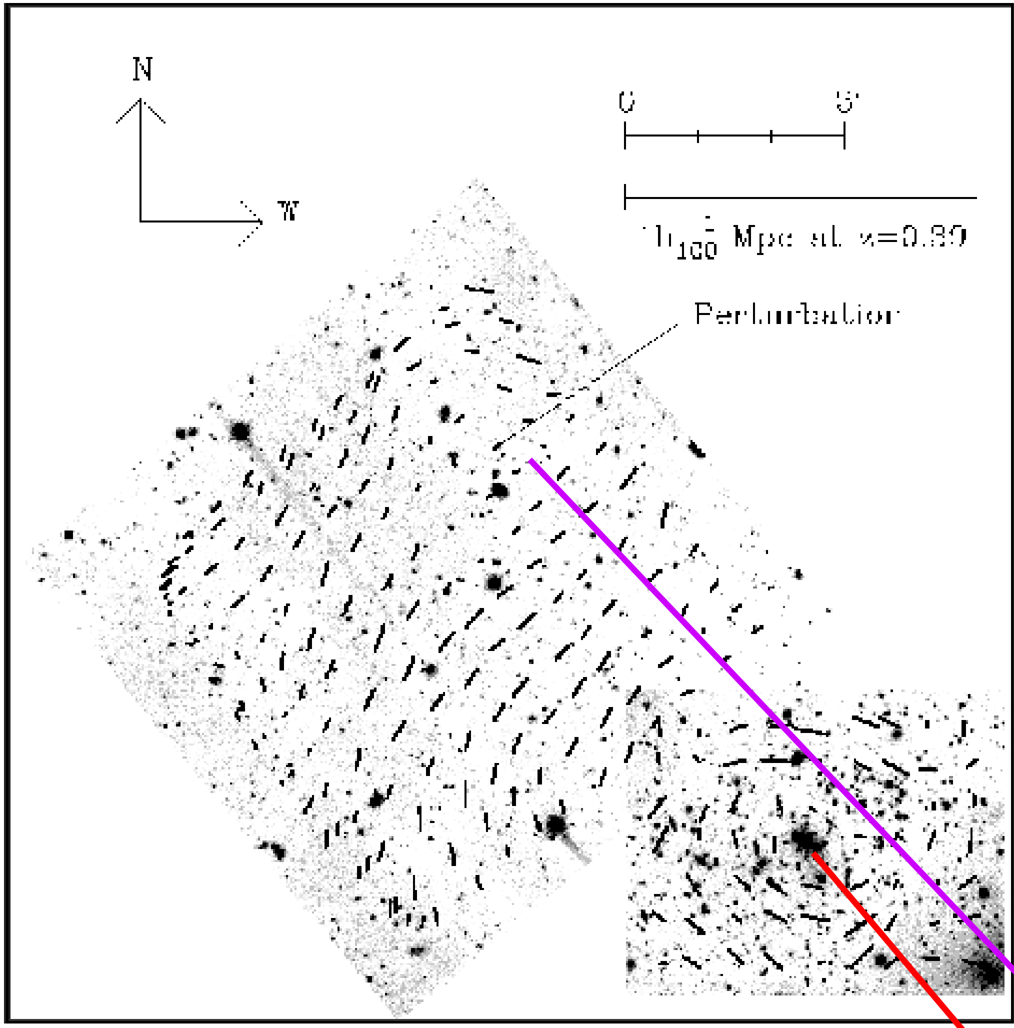
Dark Matter



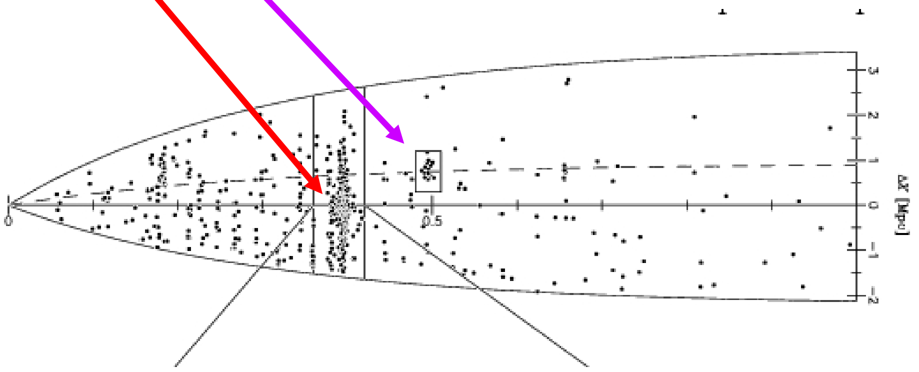


Cl0024+1654
z=0.55


: Cl0024+1654
z=0.55



Cl0024+1654
z=0.55



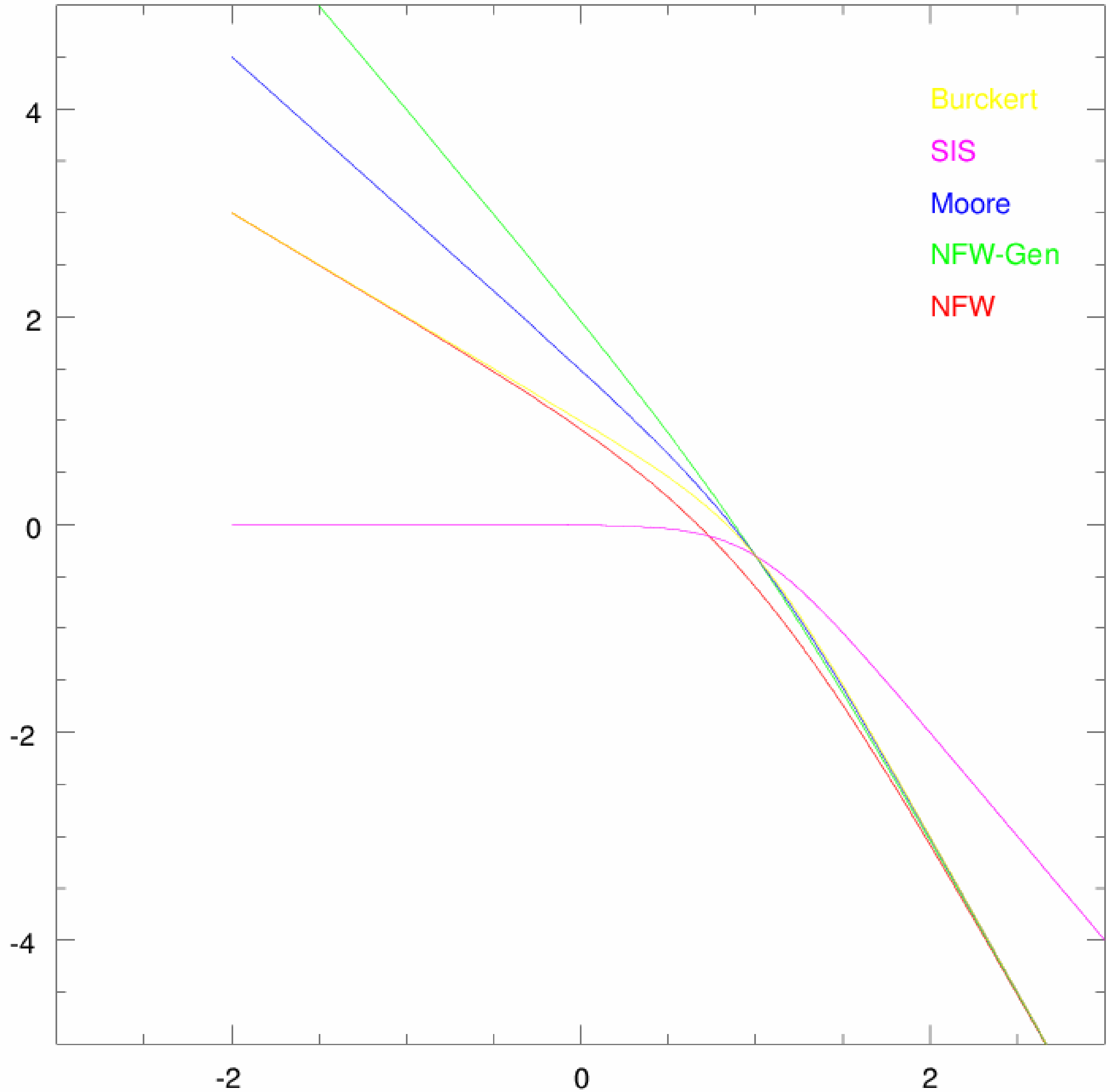
Summary mass from cluster WL reconstruction

- Median σ_v: ~ 1000 km/sec
- Median *M/L* : ~ 350 *h*

Cluster	z	σ_{obs} (kms^{-1})	σ_{wl} (kms^{-1})	M/L (h_{100})	Scale (h_{100}^{-1} Mpc)	Tel.	Ref.
A2218	0.17	1370	1000-1400	≈ 300	0.5	CFHT	Squires et al (19)
			-	310	0.1	HST	Smail et al (20)
A1689	0.18	2400	1200-1500	-	0.5	CTIO	Tyson et al (21)
			-	400	1.0	CTIO	Tyson & Fischer (22)
			1030	-	1.0	ESO/2.2	Clowe & Schneider (23)
A2163	0.20	1680	740-1000	300	0.5	CFHT	Squires et al (25)
A2390	0.23	1090	≈1000	320	0.5	CFHT	Squires et al (24)
< 8 *clusters* >	< 0.2 >	-		< 295 >	1.0	CTIO	Wittman et al (26)
MS1455+22	0.26	1133	-	1080	0.4	WHT	Smail et al (27)
AC118	0.31	1950	-	370	0.15	HST	Smail et al (20)
MS1008-1224	0.31	1054	940	340	0.5	VLT	Athreya et al (28)
			850	≈ 320	0.5	VLT	Lombardi et al (29)
MS2137-23	0.31	-	950	300	0.5	VLT	Gavazzi et al (in prep.)
MS1358+62	0.33	940	780	180	0.75	HST	Hoekstra et al (30)
MS1224+20	0.33	802	-	≈ 800	1.0	CFHT	Fahlman et al (31)
			1300	890	1.0	MDM2.2	Fischer (32)
Q0957+56	0.36	715	-	-	0.5	CFHT	Fischer et al (33)
Cl0024+17	0.39	1250	-	150	0.15	HST	Smail et al (20)
			1300	≈900	1.5	CFHT	Bonnet et al (10)
Cl0939+47	0.41	1080	-	120	0.2	HST	Smail et al (20)
			-	≈250	0.2	HST	Seitz et al (34)
Cl0302+17	0.42	1080		80	0.2	HST	Smail et al (20)
RXJ1347-11	0.45	-	1500	400	1.0	CTIO	Fischer & Tyson (35)
3C295	0.46	1670	1100-1500	-	0.5	CFHT	Tyson et al (21)
			-	330	0.2	HST	Smail et al (20)
Cl0412-65	0.51	-	-	70	0.2	HST	Smail et al (20)
Cl1601+43	0.54	1170	-	190	0.2	HST	Smail et al (20)
MS0016+16	0.55	1230	-	180	0.2	HST	Smail et al (20)
			740	740	0.6	WHT	Smail (36)
			800	-	0.6	Keck	Clowe et al (6)
MS0451	0.55	1371	980	-	0.6	Keck	Clowe et al (6)
Cl0054-27	0.56	-	-	400	0.2	HST	Smail et al (20)
MS2053	0.59	820	730	-	0.5	Keck	Clowe et al (6)
			886	360	0.5	HST	Hoekstra et al (40)
MS1137+60	0.78	884	1190	270	0.5	Keck	Clowe et al (37,6)
RXJ1716+67	0.81	1522	-	190	0.5	Keck	Clowe et al (37)
			1030	-	0.5	Keck	Clowe et al (6)
MS1054-03	0.83	1360	1100-2200	350-1600	0.5	UH2.2	Luppino & Kaiser (38)
			1310	250-500	0.5	HST	Hoekstra et al (39)
			1080	-	0.5	Keck	Clowe et al (6)
< 10 *clusters* >	< 0.5 > –	-			1.0	VLT	White et al (2002)
EDICS	– < 0.8 >	-					and Clowe et al (2002)

Mass profile

NFW, SIS, POW, Generalised-NFW, Moore, Buckert, de Vaucouleur??



Burckert
SIS
Moore
NFW-Gen
NFW
4
2
0
-2
-4
-2
0
2

The Navarro-Frenk-White profile

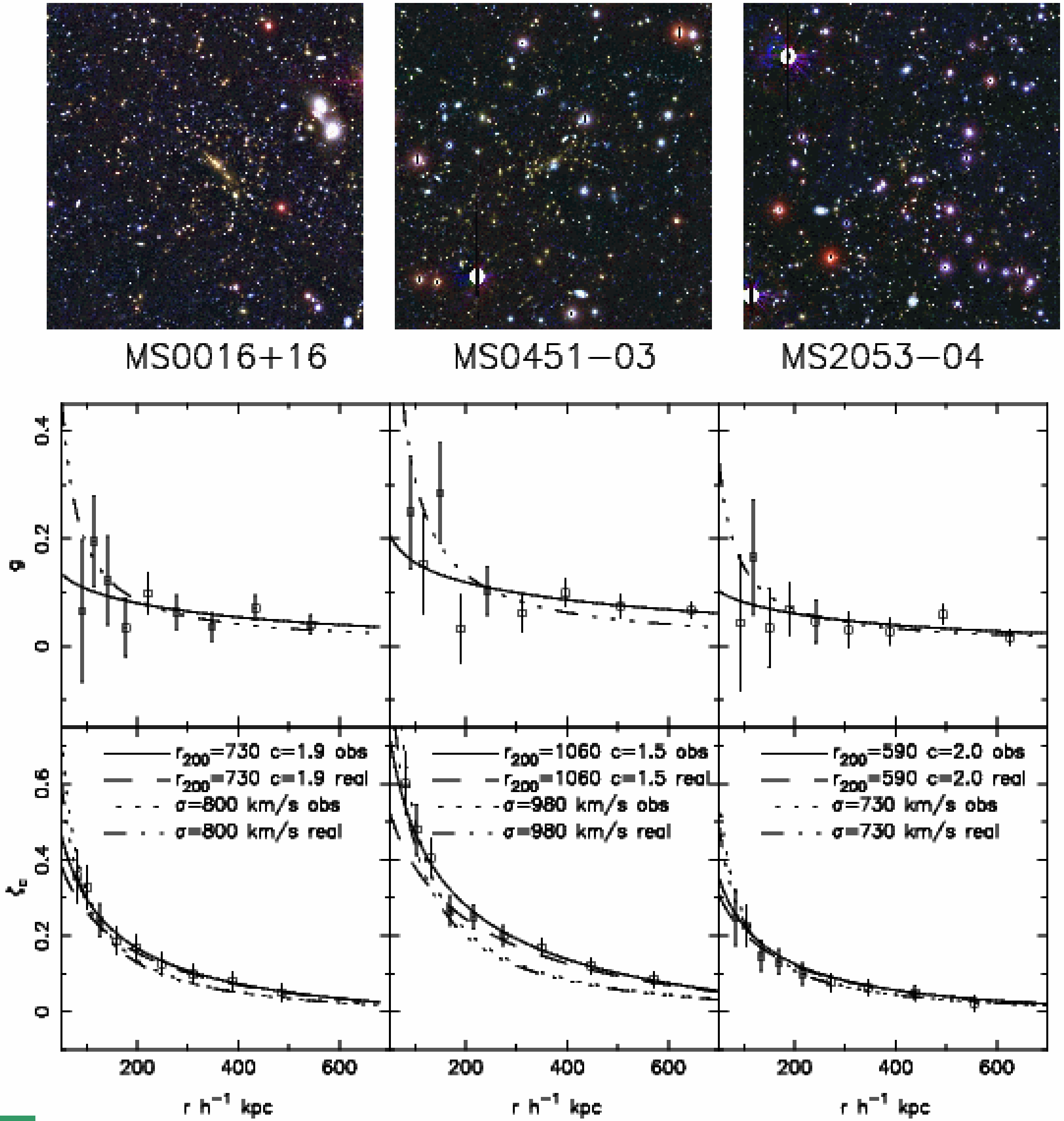
$$\rho(r) = \frac{\delta_c \; \rho_{rc}(z)}{(r/r_s)\; (1 + r/r_s)^2}$$

$$\rho_{rc}(z) = \frac{3H(z)^2}{8\pi G}$$

$$\delta_c = \frac{200}{3} \frac{C^3}{\ln(1+C) - C/(1+C)}$$

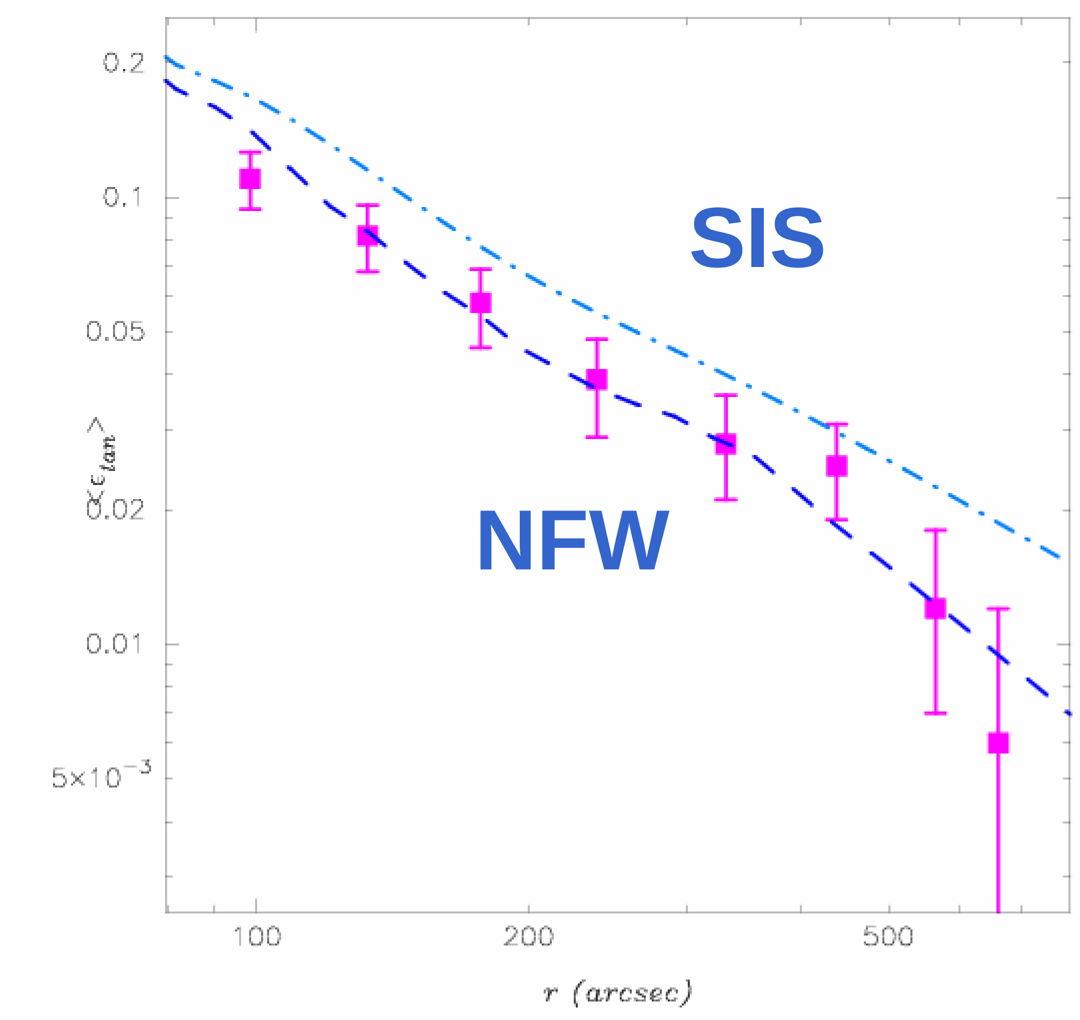
Virial Radius: r_{200}. Radius inside which the mean mass density equals $200\rho_{rc}(z)$.

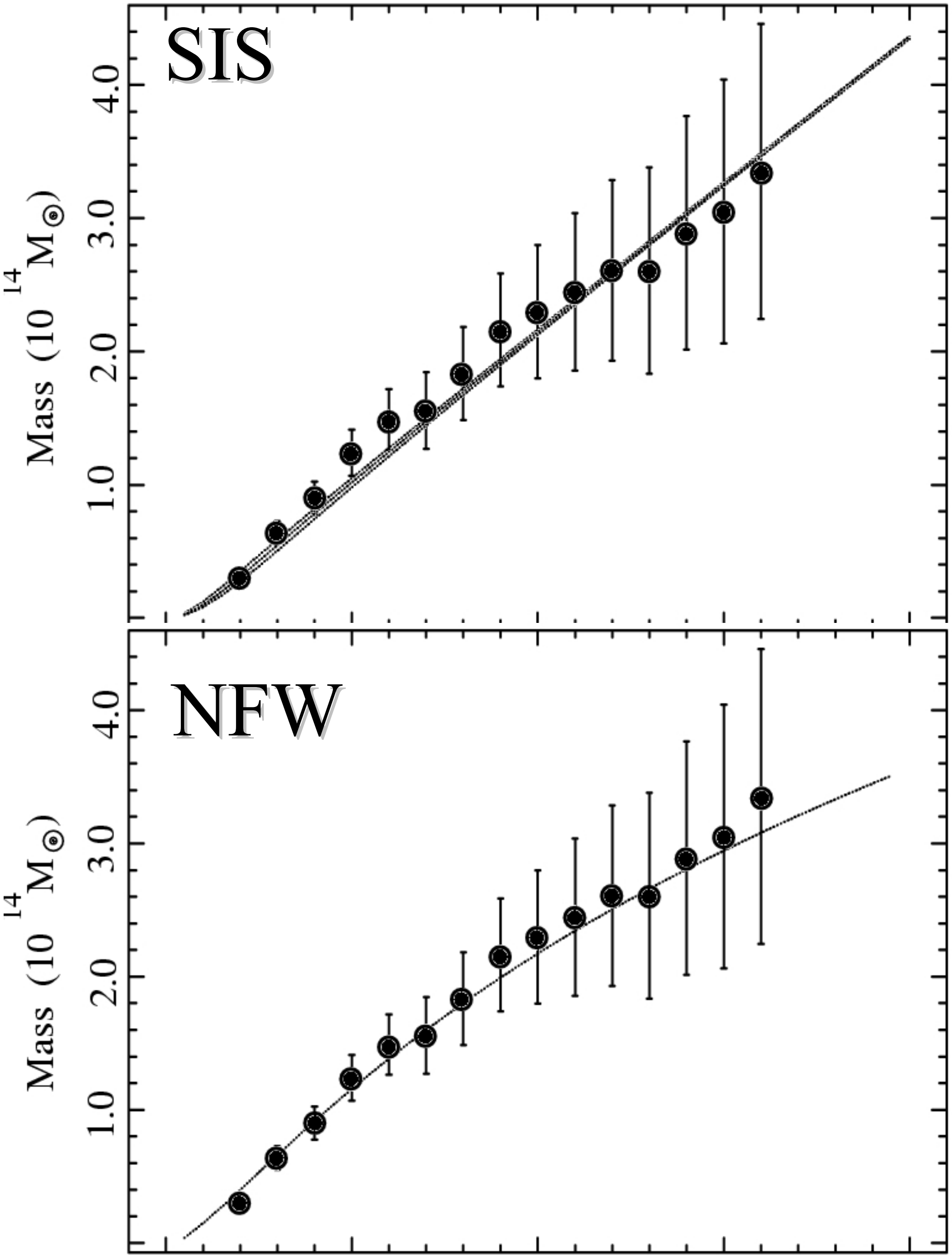
Clowe et al 2000



Cl0024 HST

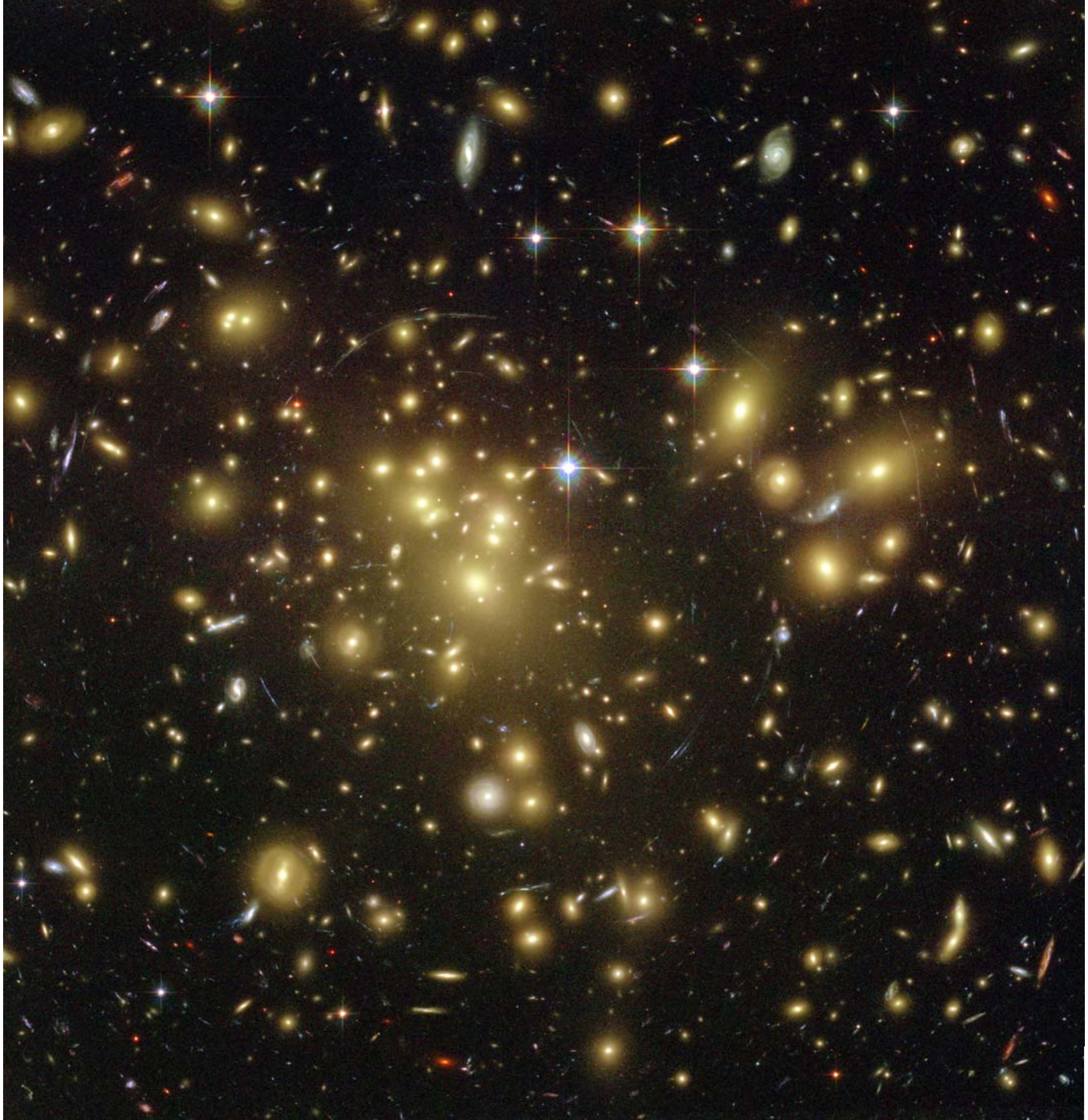
Kneib et al 2003



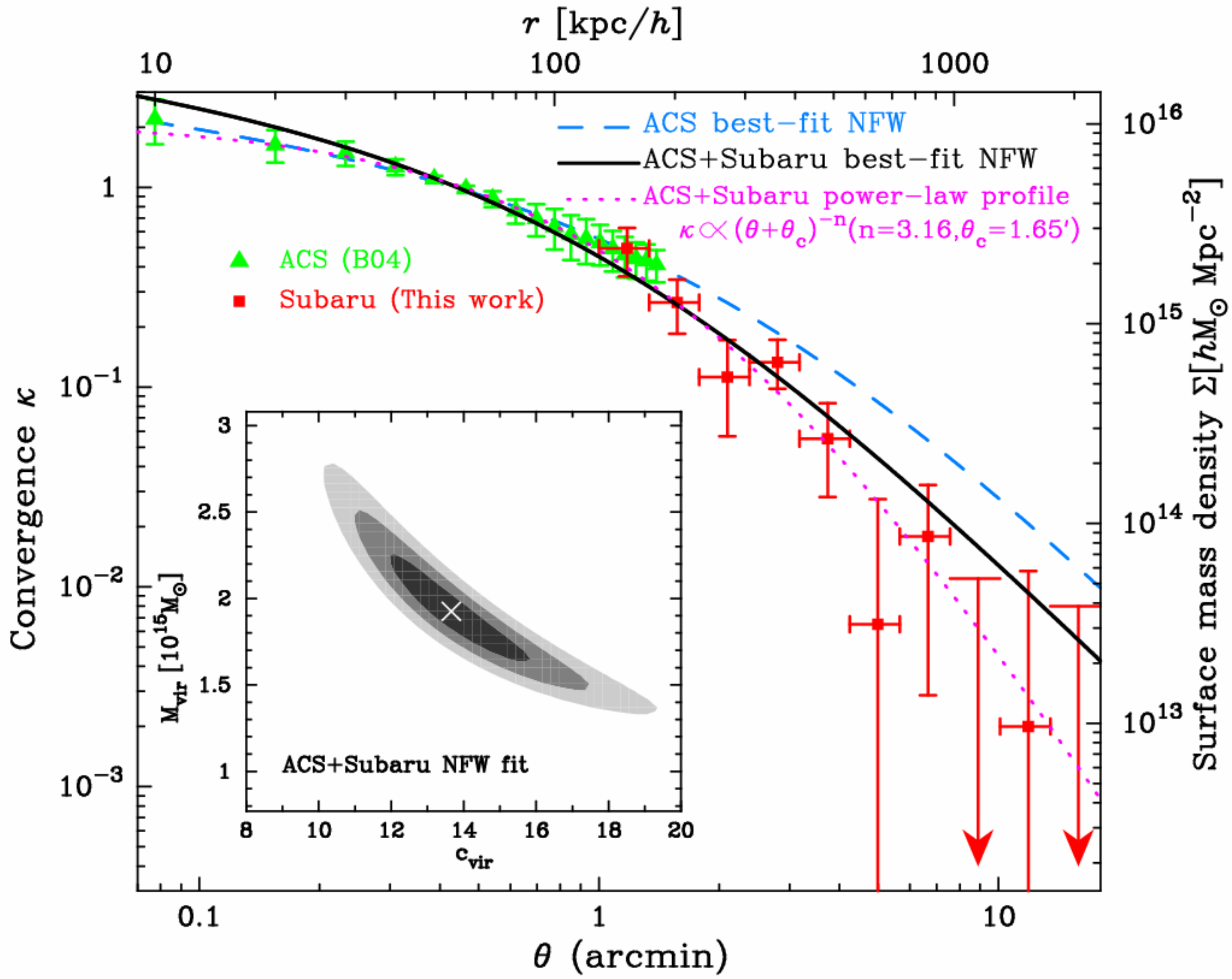


MS1008-1224 VLT :

Athreya et al 2002 , Lombardi et al 2001

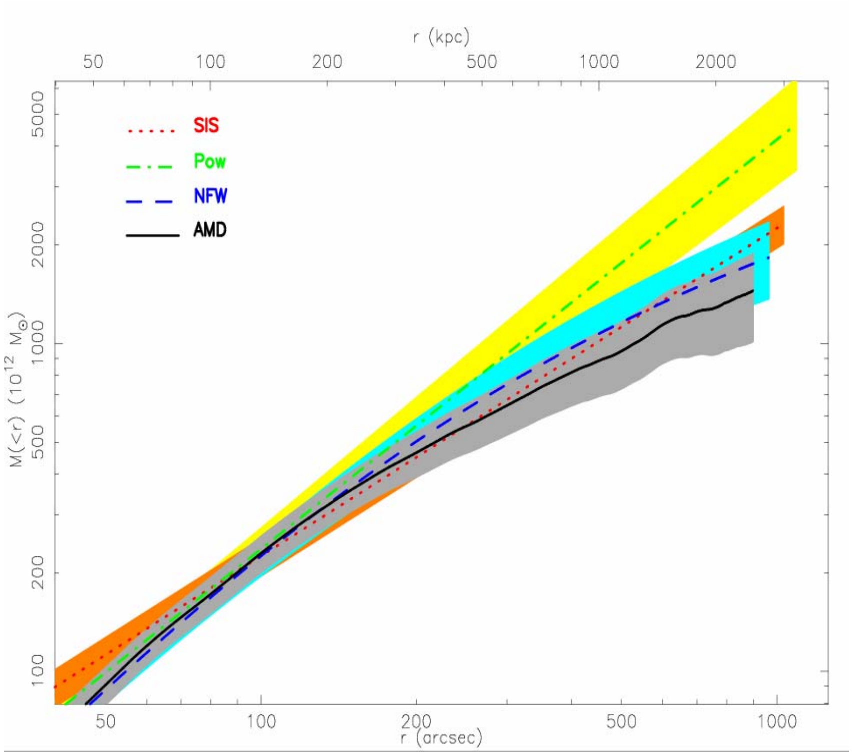


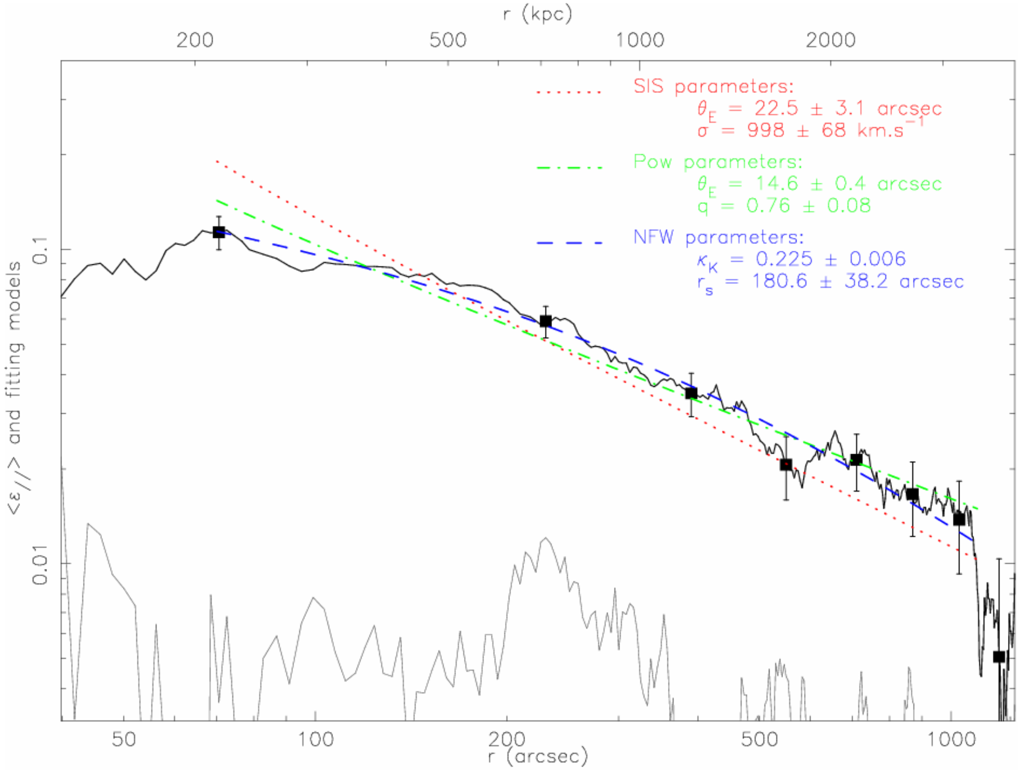
Abell 1689 HST/ACS



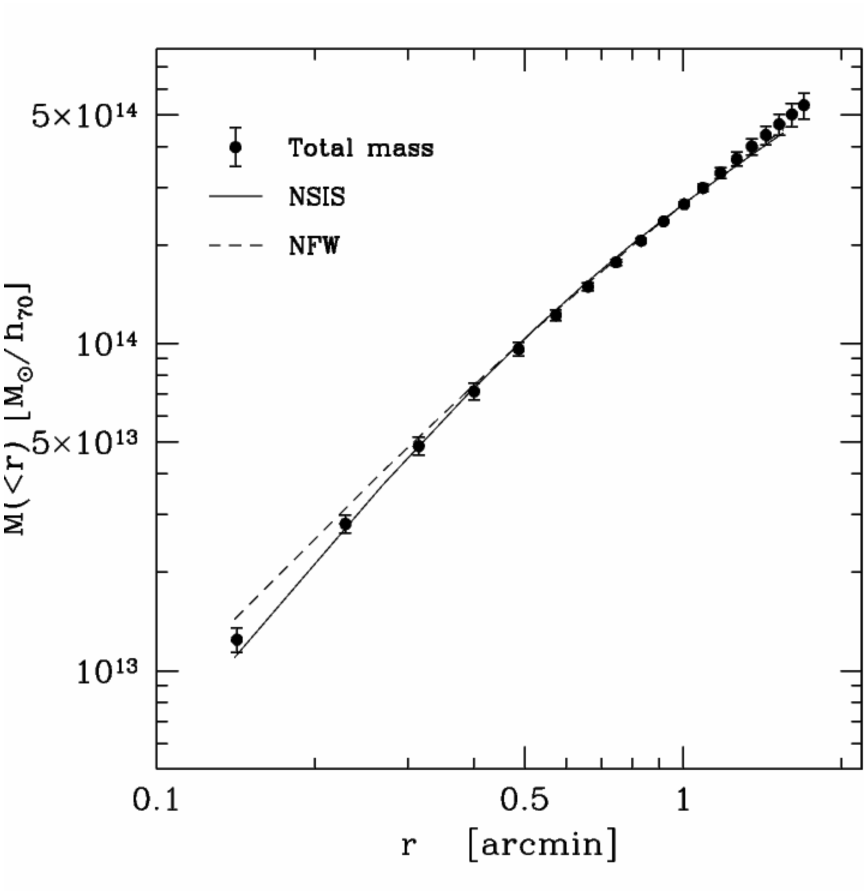
Abell 1689 ; CFH12k

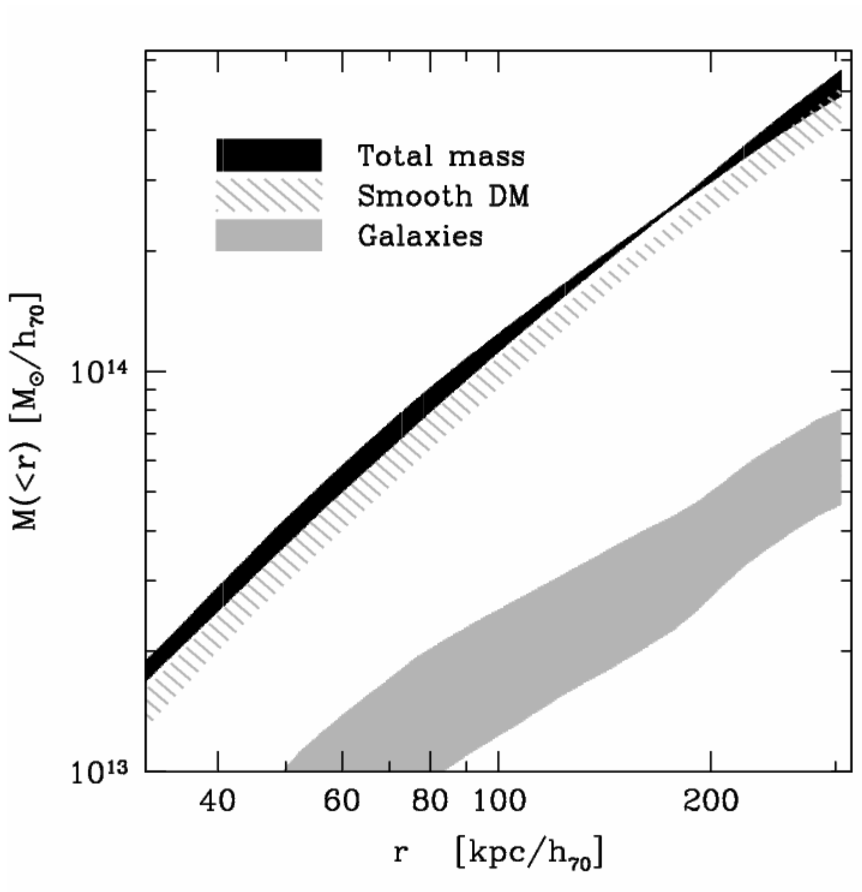
Bardeau et al 2004





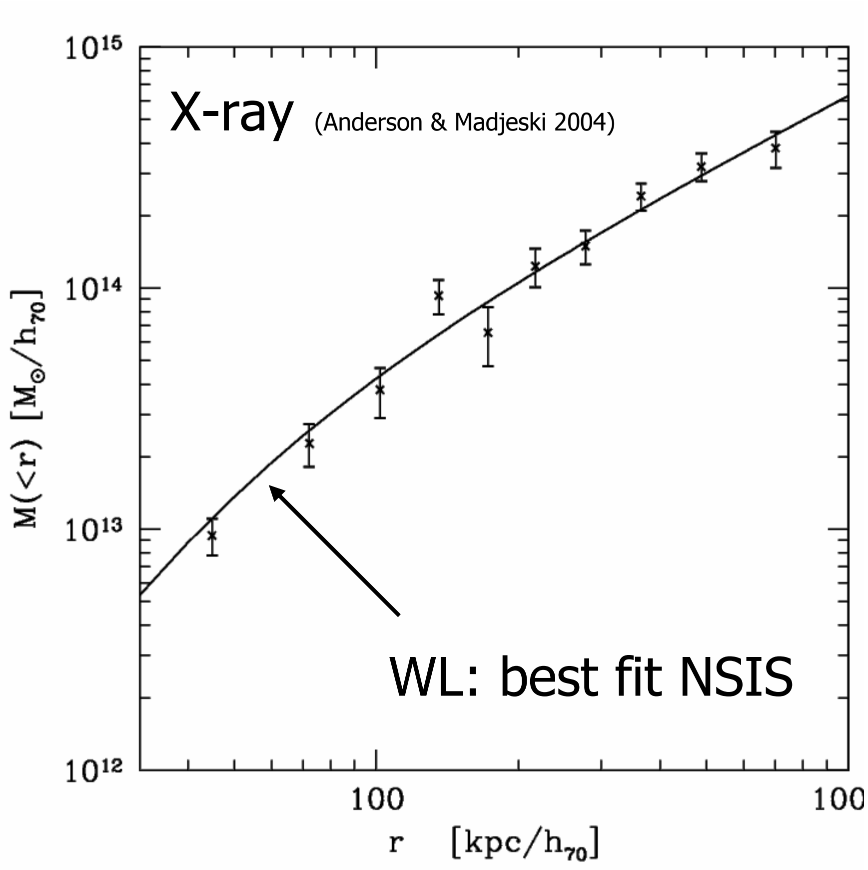
Abell 1689 HST/ACS revisited

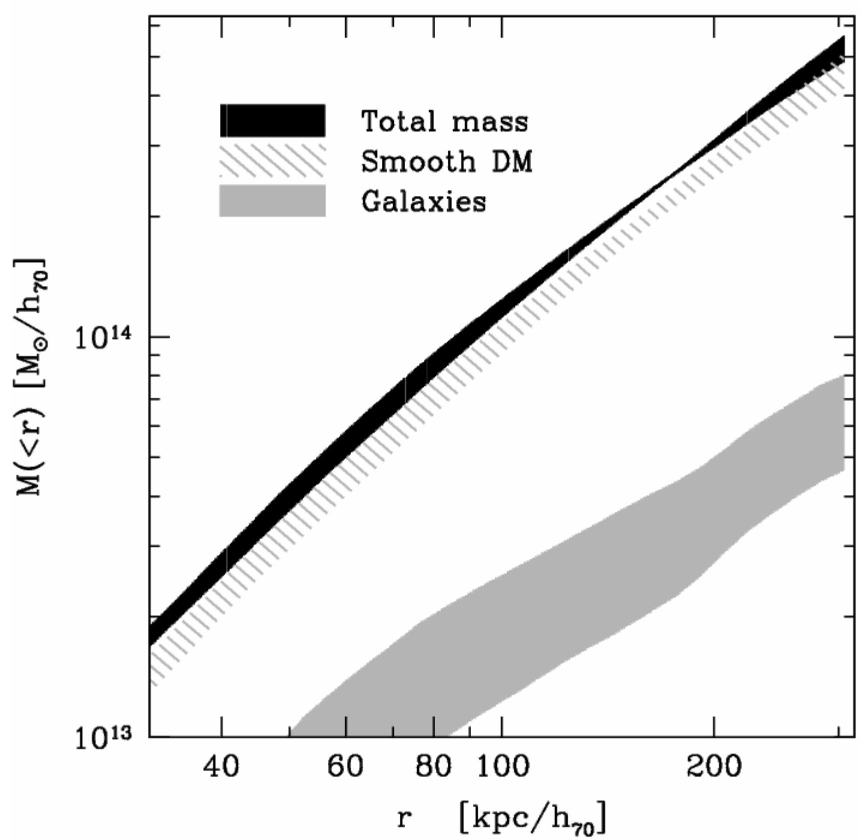




Halkola, Seitz, Panella 2006

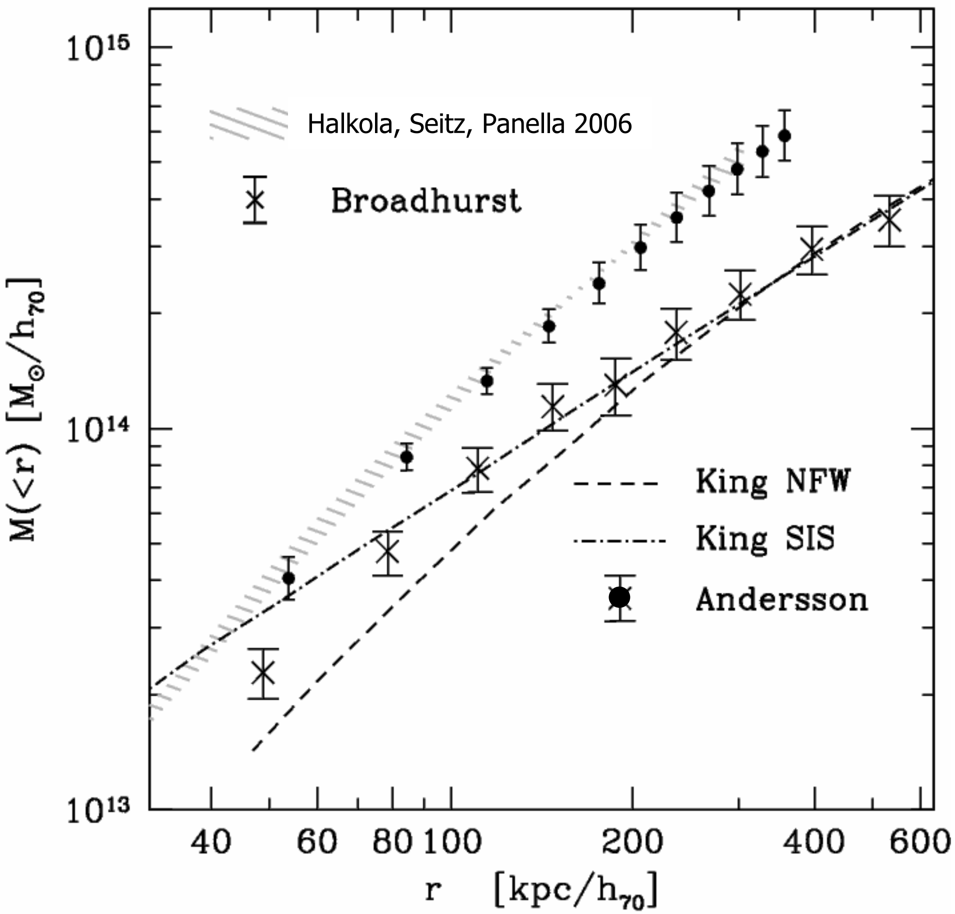
Abell 1689 HST/ACS revisited





Halkola, Seitz, Panella 2006

Abell 1689 HST/ACS revisited



Halkola, Seitz, Panella 2006

NFW Parameters

Method	C	r_{200} (Mpc)	Reference
SL	6.0±0.5	2.82±0.11	this work
SL	$6.5^{+1.9}_{-1.6}$	2.02	Broadhurst et al. 2005a
X-ray	$7.7^{+1.7}_{-2.6}$	1.87±0.36	Andersson & Madejski 2004
WL	4.8	1.84	King et al. 2002

NSIE Parameters

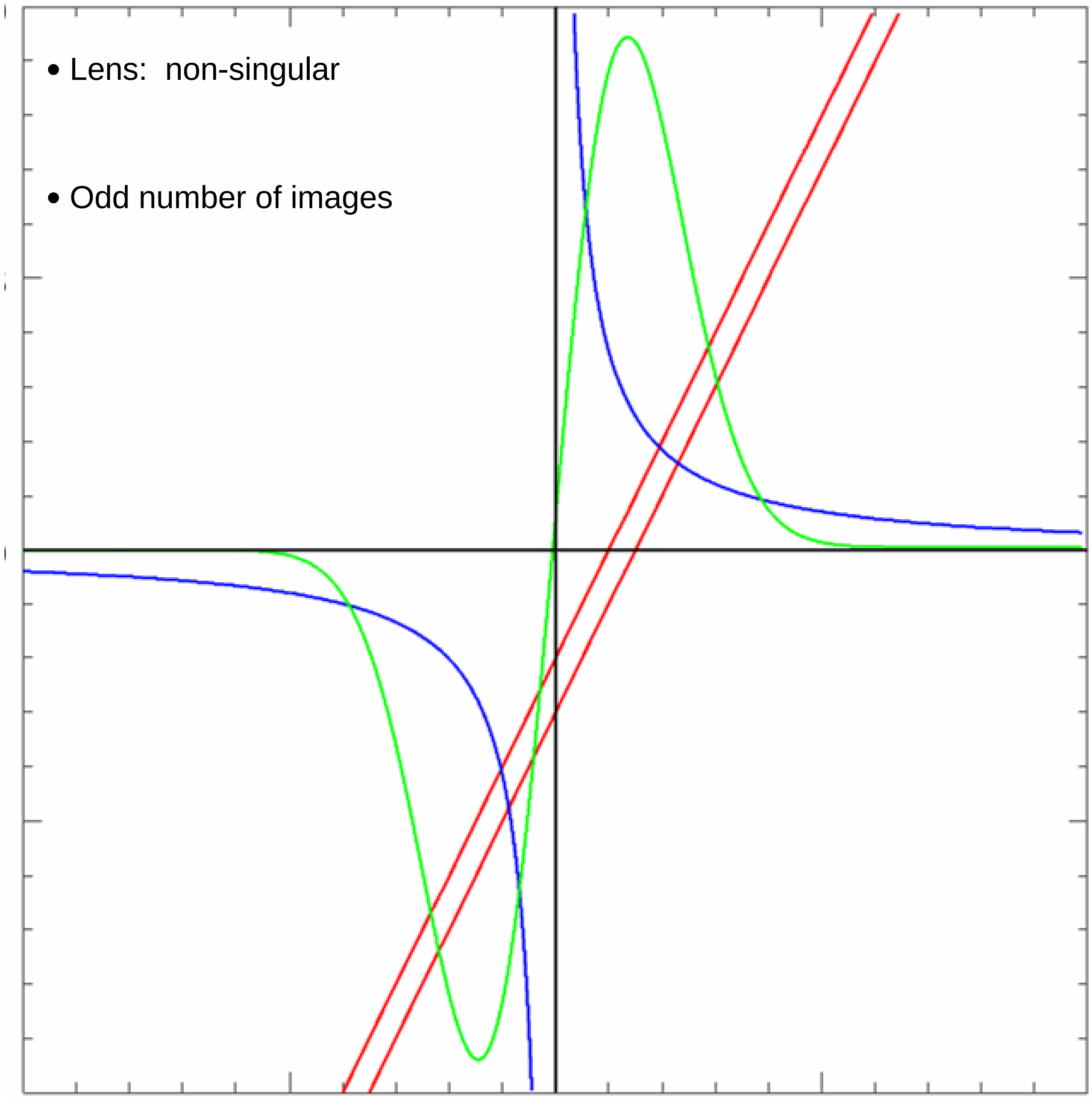
Method	σ (km/s)	r_c (kpc)	Reference
SL	1514±18	71±5	this work
SL	1390	60	Broadhurst et al. 2005a
X-ray	918±27	SIS	Andersson & Madejski 2004
X-ray	1190	27	Andersson & Madejski 2004*
WL	998^{+33}_{-42}	SIS	King et al. 2002
LOSVD	1429^{+145}_{-96}	-	Girardi et al. 1997

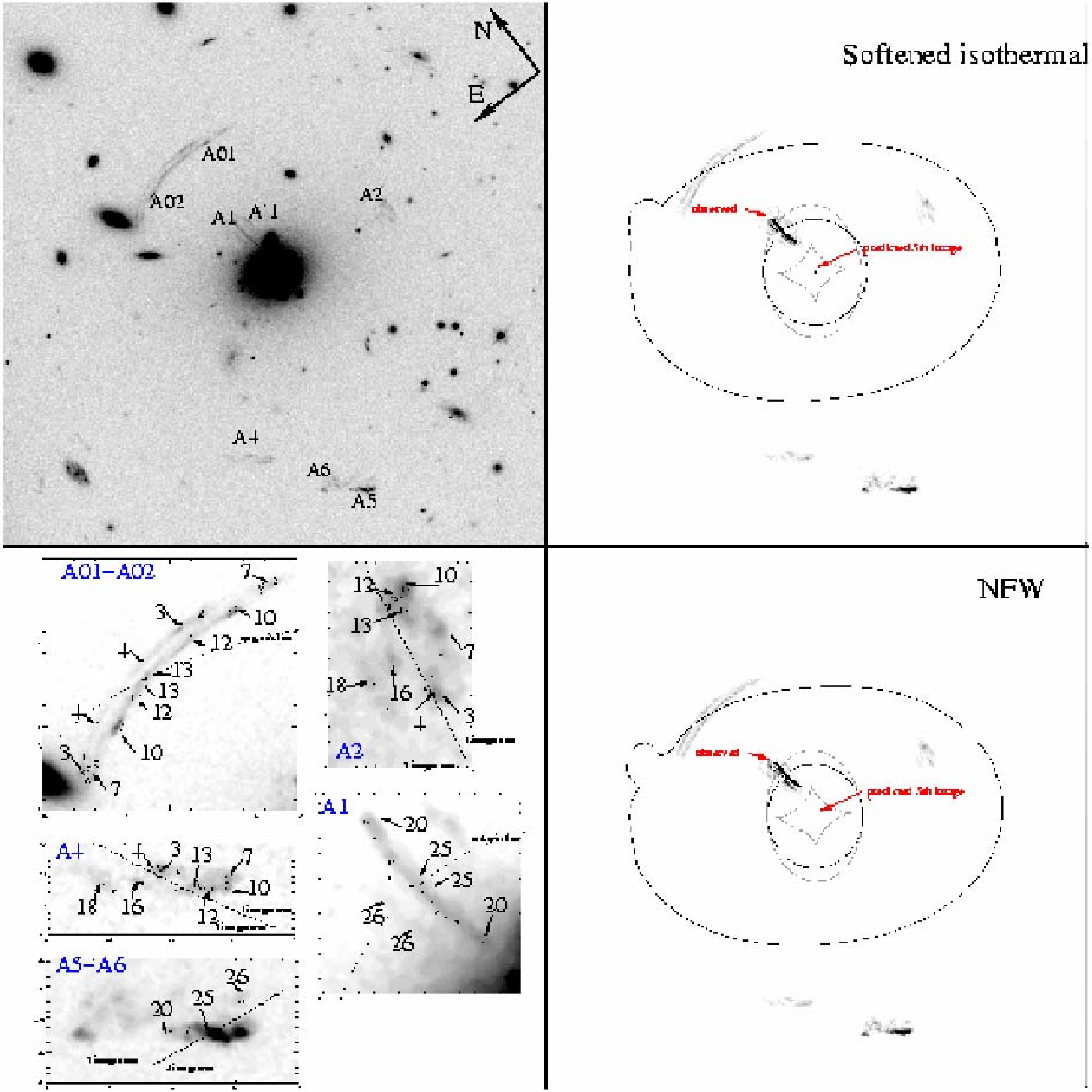
* data from Andersson & Madejski 2004, fitting done in this work.

Table 6. Comparison between mass estimates for Abell 1689 from different methods. The mass measured by Andersson & Madejski (2004) are underestimates of the total mass if the cluster is undergoing a merger. For our work the mass at r=0.25 Mpc/h_{70} is an extrapolation since the multiple images do not extend to such large clustercentric radii.

M(<r) (10^{15} $M_\odot$ h_{100}^{-1})	r (Mpc h_{100}^{-1})	Reference
0.14 ± 0.01	0.10	this work, Model III
0.082 ± 0.013	0.10	Andersson & Madejski (2004)
0.43 ± 0.02	0.24	Tyson & Fischer (1995)
0.20 ± 0.03	0.25	Andersson & Madejski (2004)
0.37 ± 0.06	0.25	this work, Model III
0.48 ± 0.16	0.25	Dye et al. (2001)

- Lens: non-singular
- Odd number of images

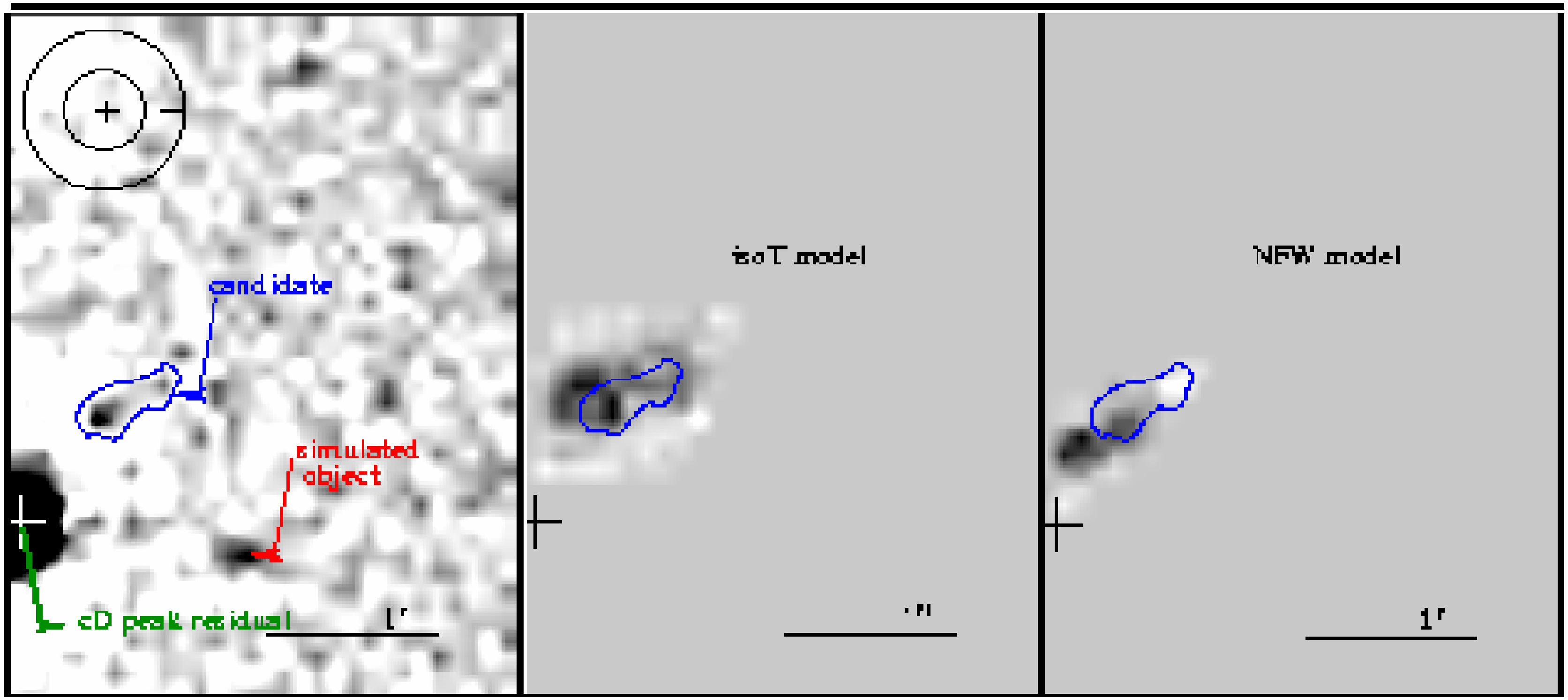




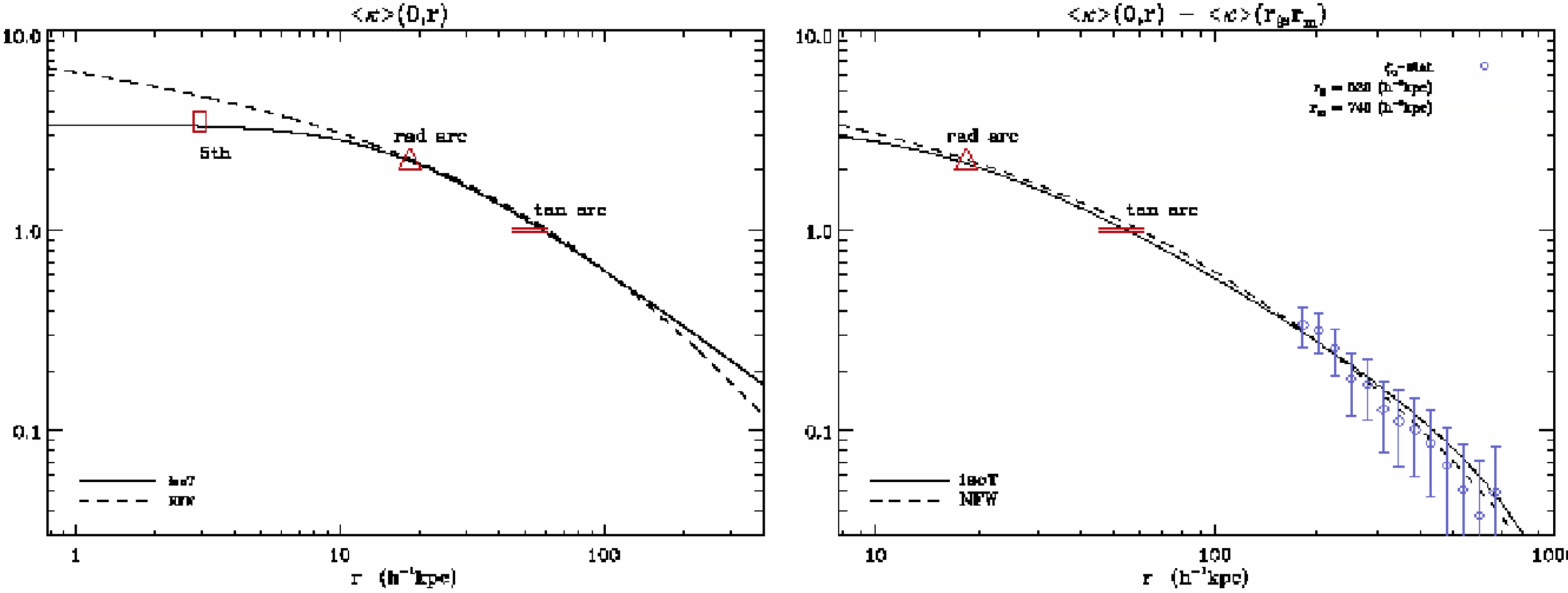
MS2127-23

Gavazzi et al 2004

MS2137-23 Gavazzi et al 2004



MS2137-23: strong + weak with the 5th image



Gavazzi et al 2004

Summary

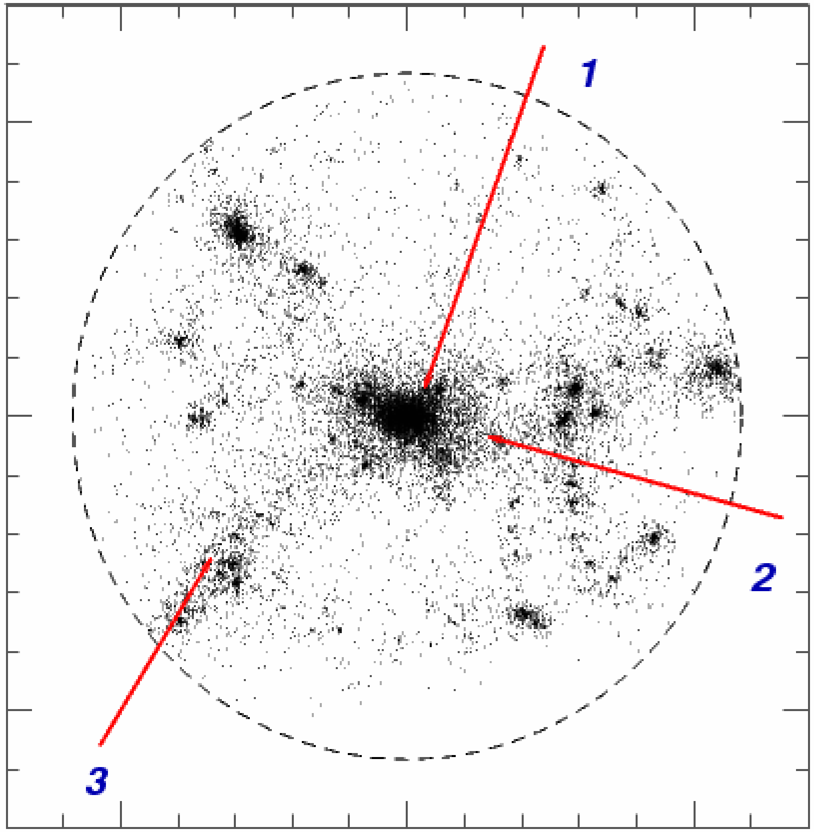
- Most Weak and/or Strong lensing data agree with NFW and SIS

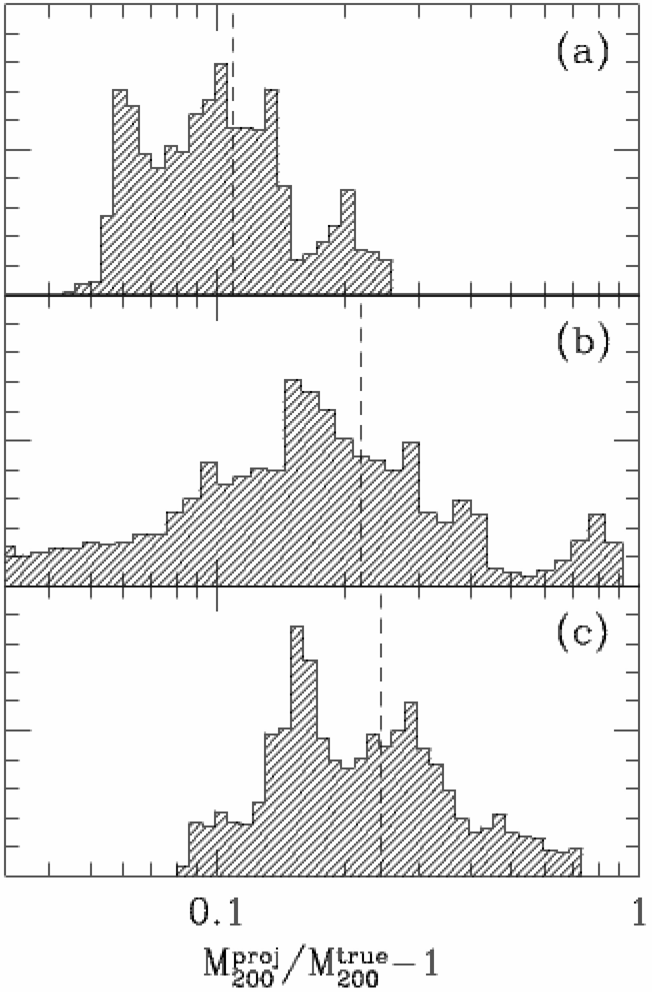
- NFW not rejected (all data compatible with CDM predictions)

Projection effects:

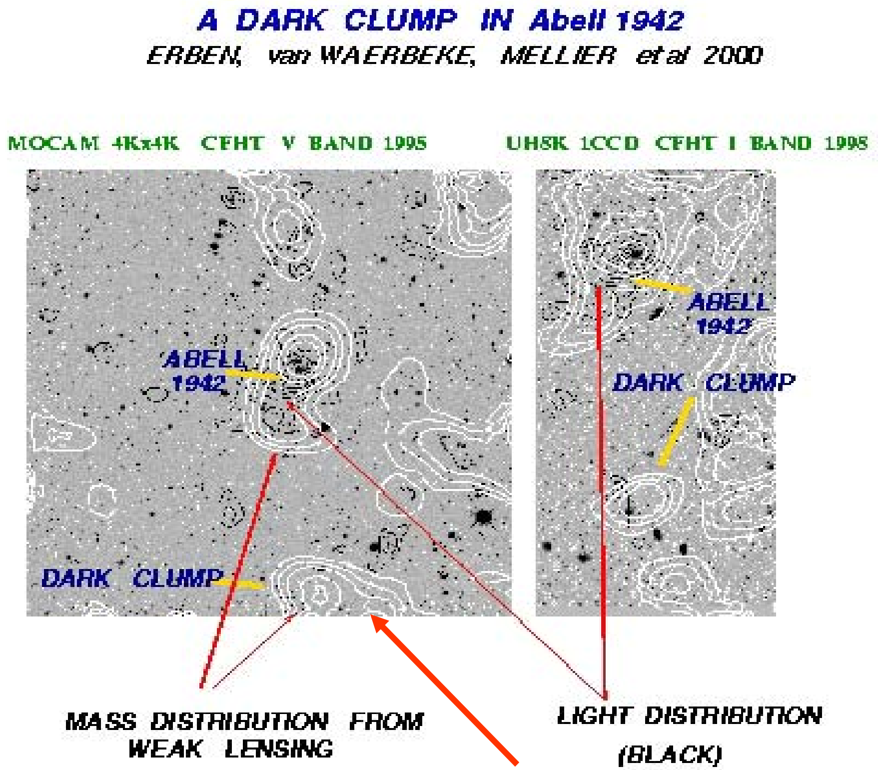
non negligible contamination

Metzler et al 2000

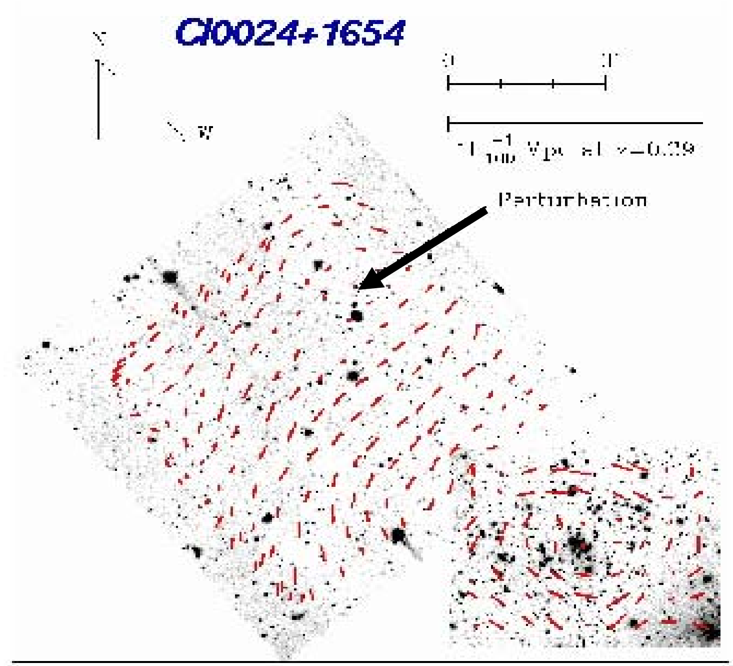




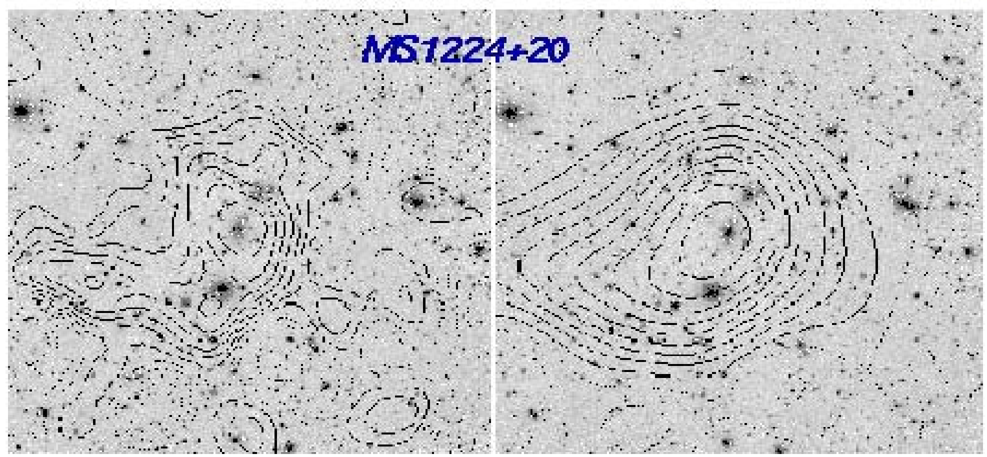
Dark Cluster ?



No high-z cluster detected (Gray et al 2000)



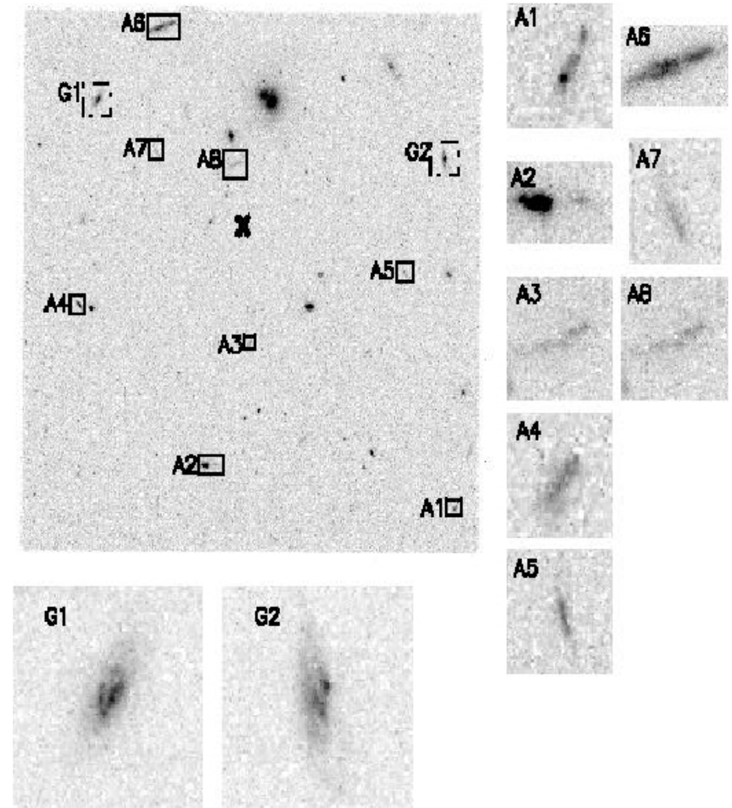
Bonnet, Mellier, Fort 1994



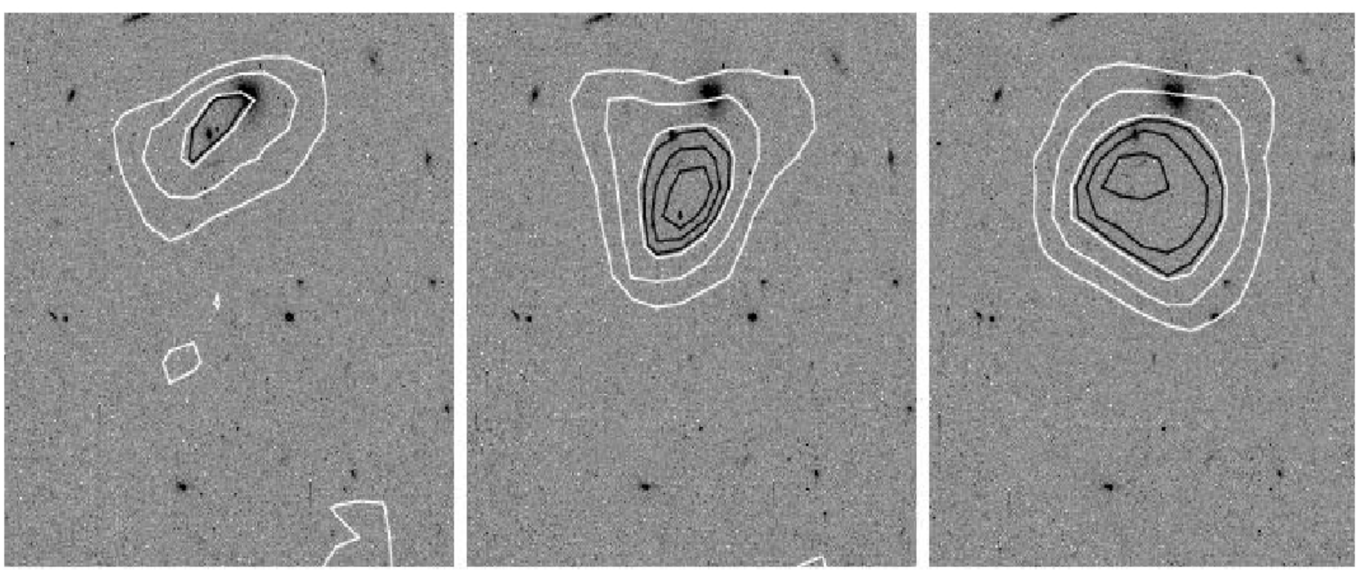
Fischer 2000 (see also Fahlman et al 1994)

Dark Clusters ?

An impressive candidate with a remarkable alignment of galaxies and coherent mass map…..



Miralles, Erben, Haemmerle et al 2002



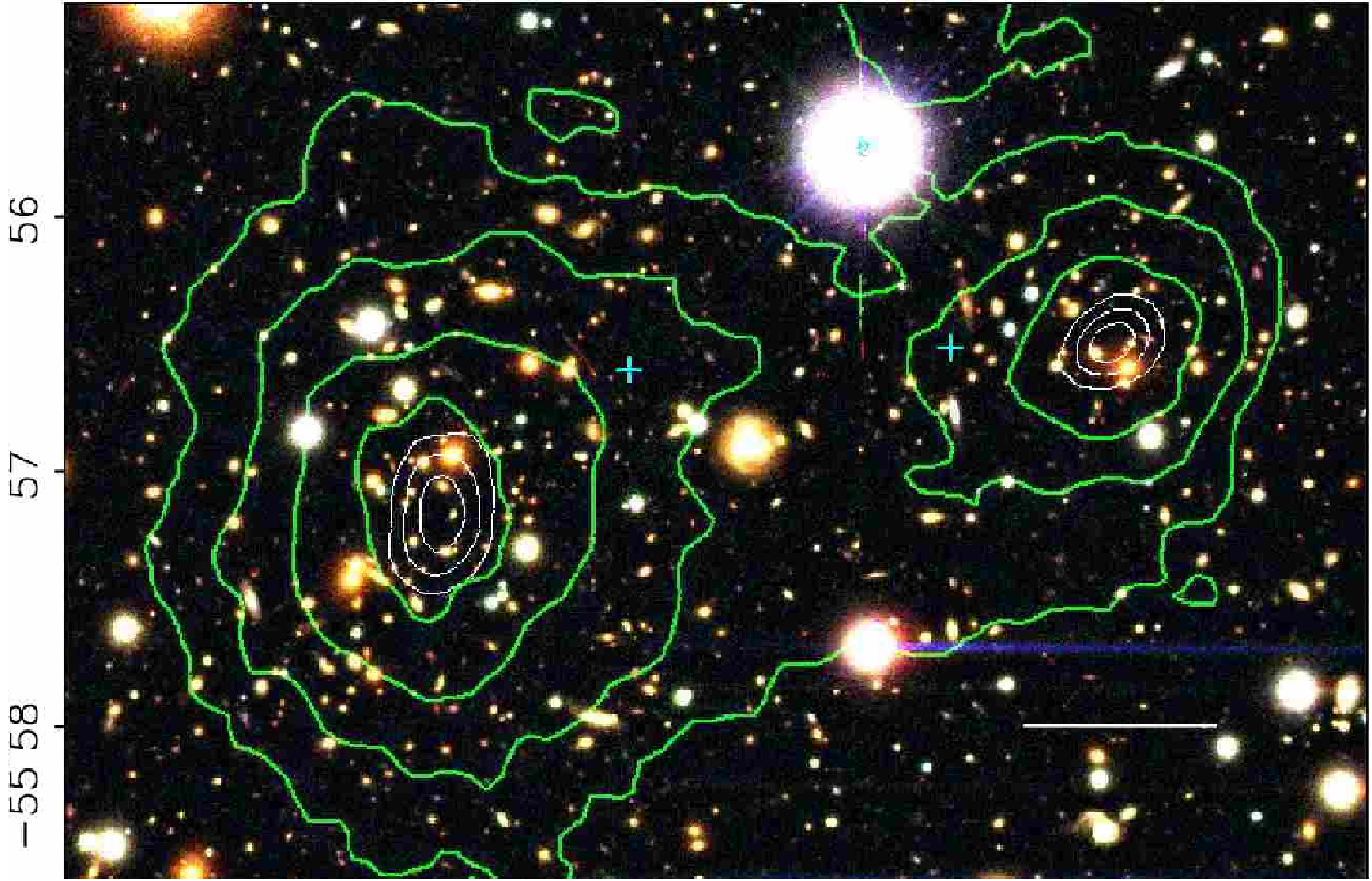
HST/STIS data

Dark cluster…

The « Bullet » cluster

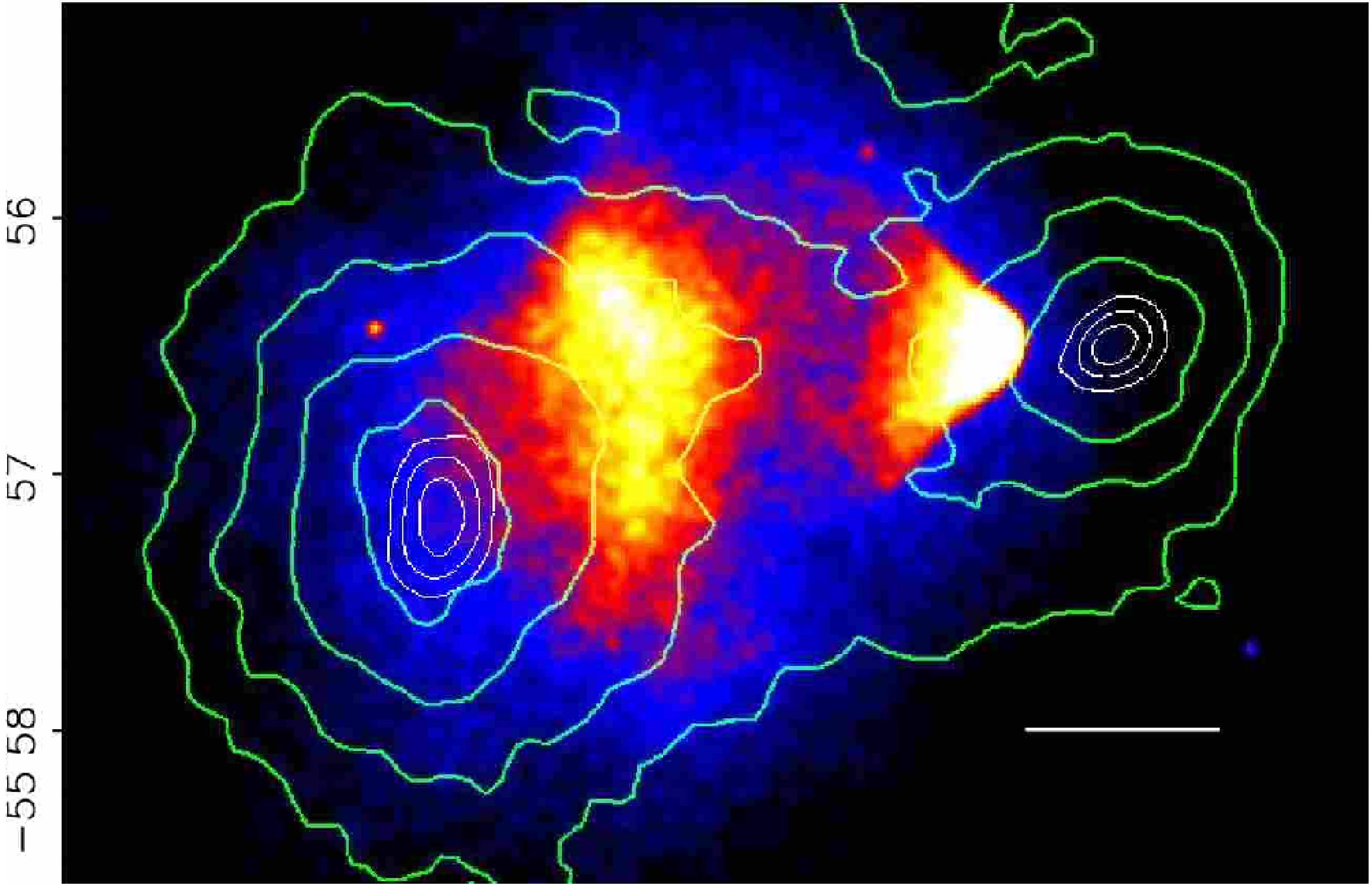
Bullet cluster

Clowe et al 2006



Bullet cluster

Clowe et al 2006



Bullet cluster : example of the interest of lensing

Clowe et al 2006

Mass map from Ellipticity->convergence: total matter

X-ray: baryonic matter

Bullet: baryonic matter decoupled from total mass

-> Clowe et al (2006): Dark matter exists!

Q1: X-ray emissivity is thermal bremsstralhung? … or shock?

Q2: If shock, X-ray from baryons should be visible anyway and center mass positions, but not visible at the kappa center positions

Q3: Is ellipticity->gamma-> kappa calibration correct?

Q4: Is kappa=mass density (MOND: Berkenstein; Angus et al 2006)

Robust statement: the cluster is dominated by collisionless matter, but could be CDM or neutrino with mass~2eV?